Probabilistic Logic Programming

IRSS 2014

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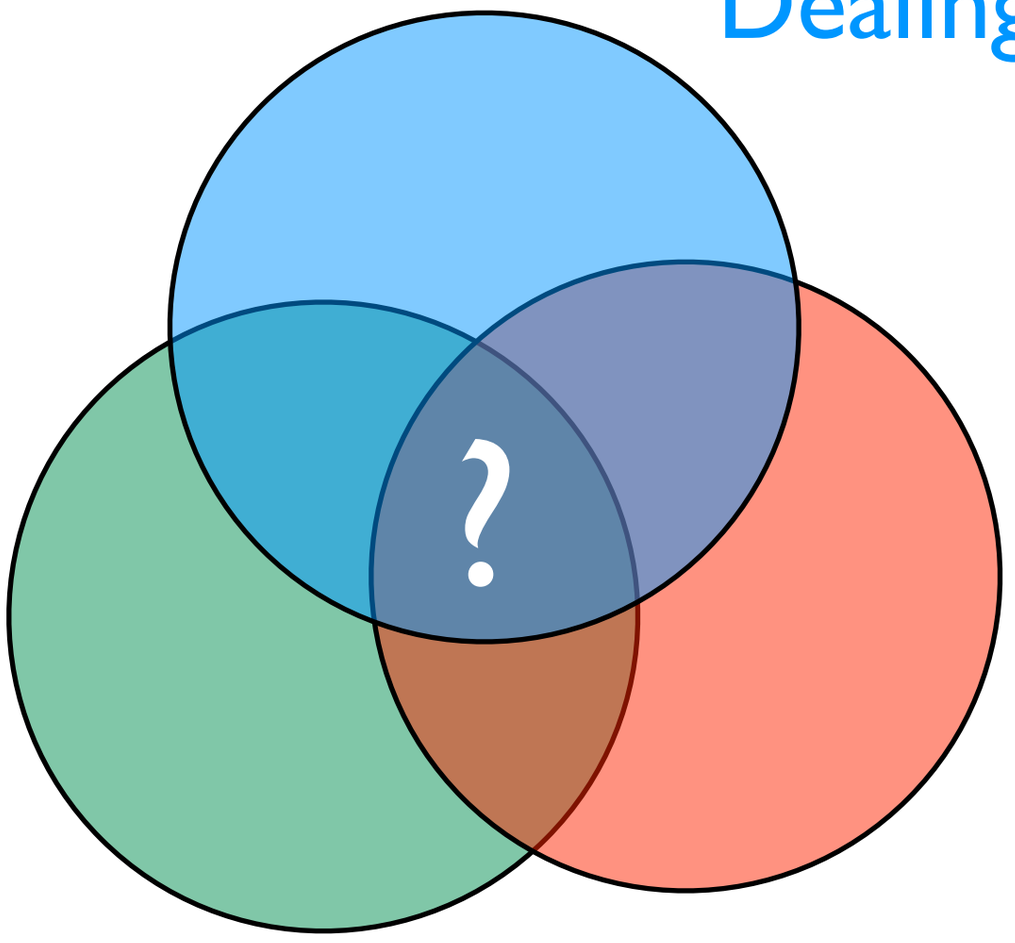
Probabilistic Logic Programming

Thanks to Luc De Raedt & many others working on PLP and especially ProbLog!

A key question in AI:

Dealing with uncertainty

Reasoning with relational data



Learning

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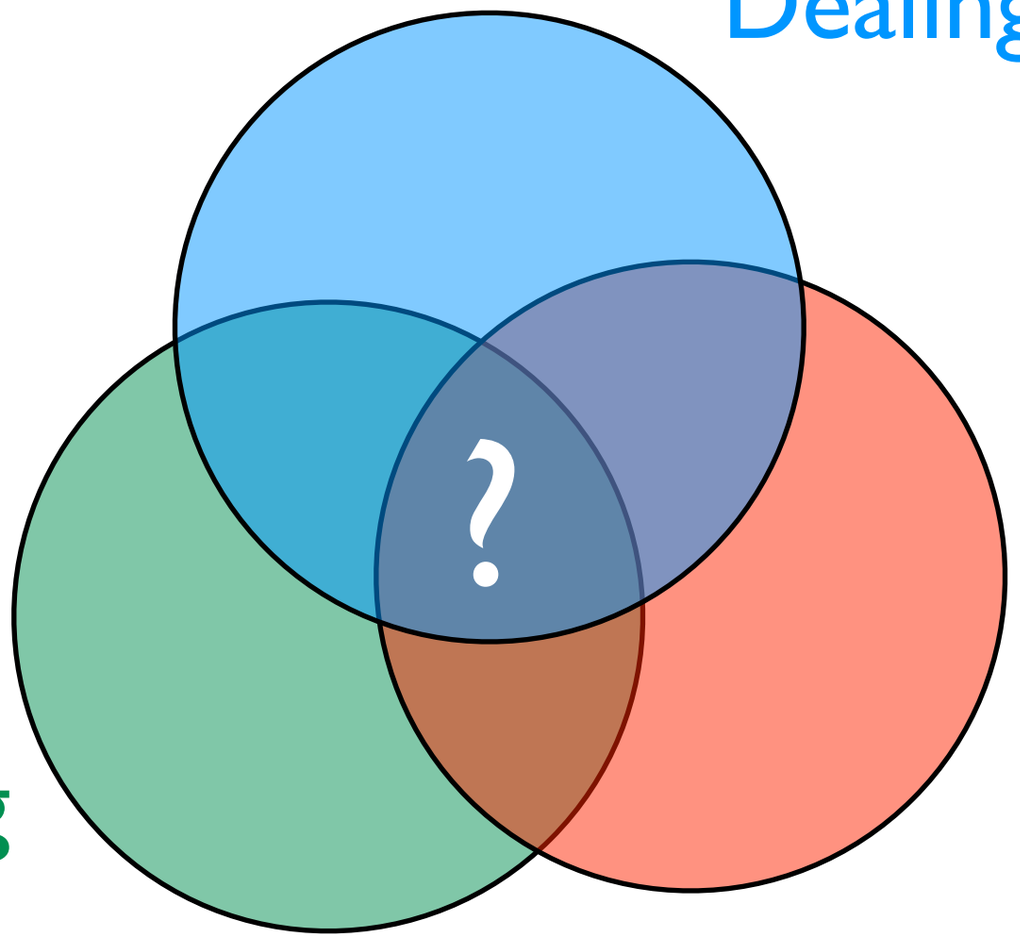
Dealing with uncertainty

Reasoning with relational data

- logic
- databases
- programming
- ...



Learning

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Dealing with uncertainty

- probability theory
- graphical models
- ...

Reasoning with relational data

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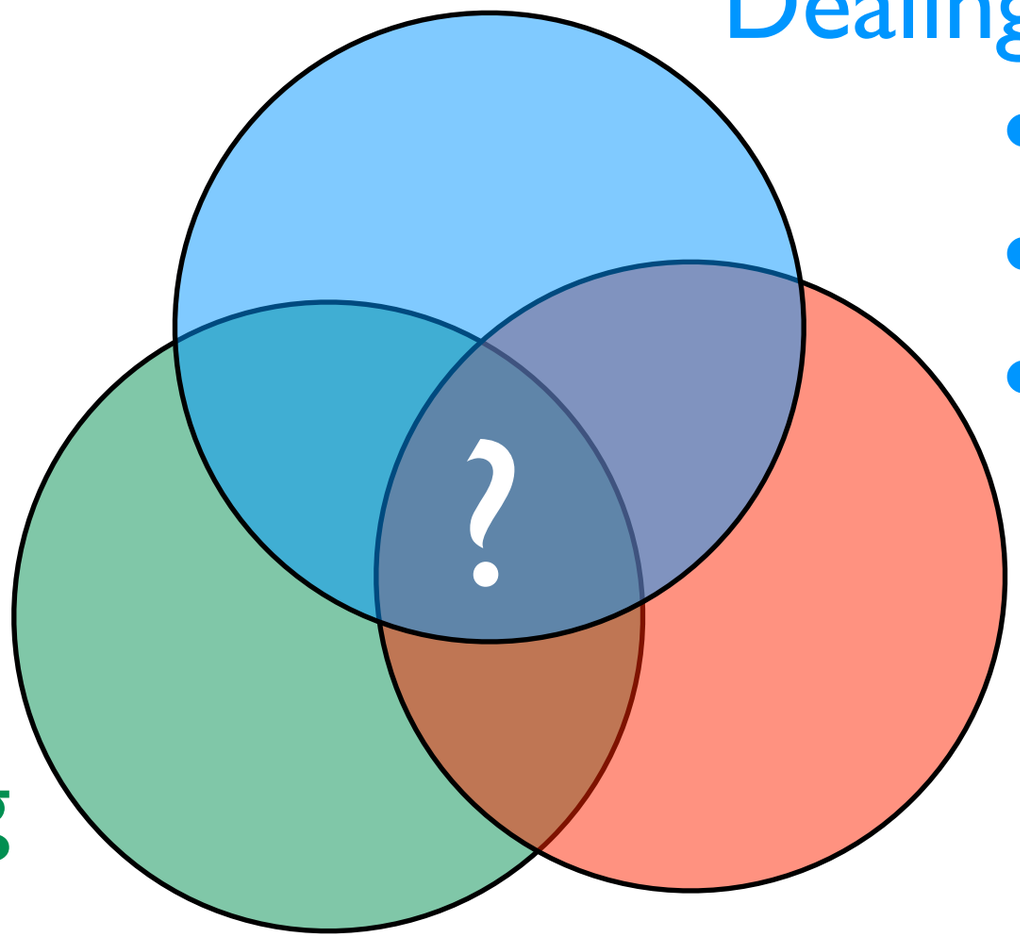

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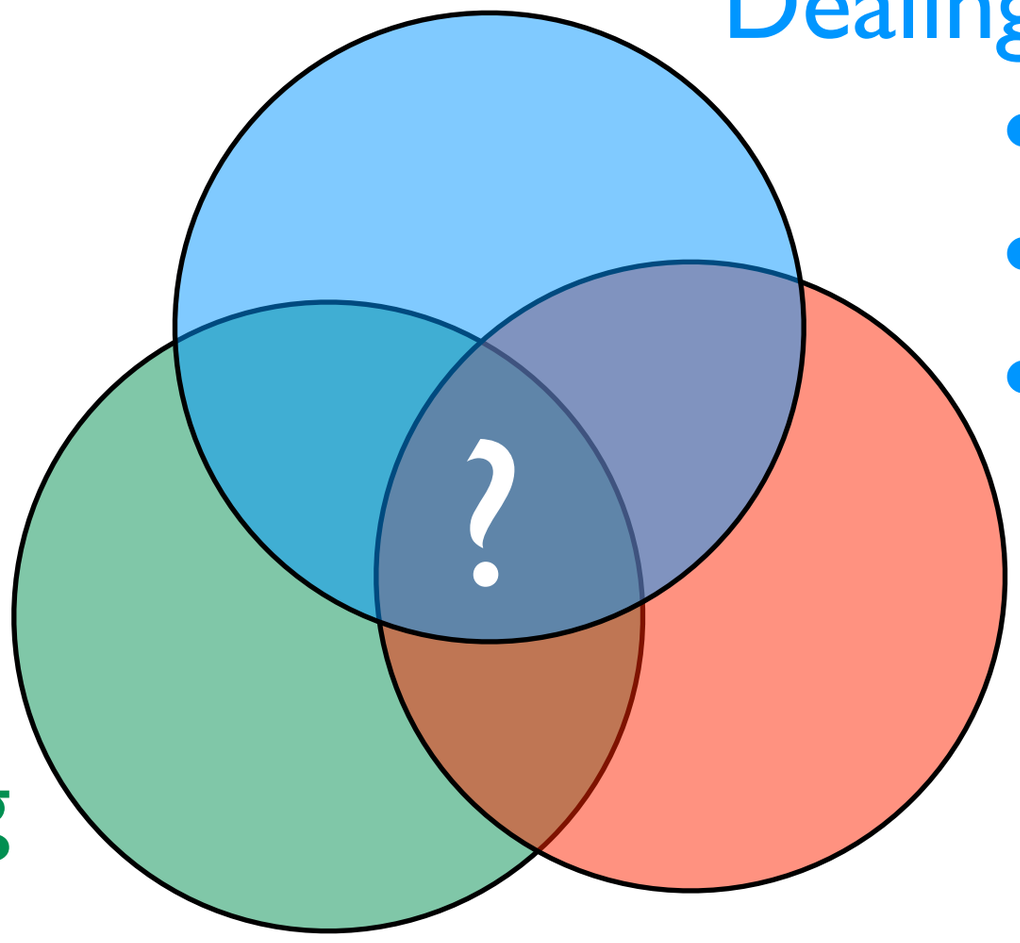

Learning

- parameters
- structure

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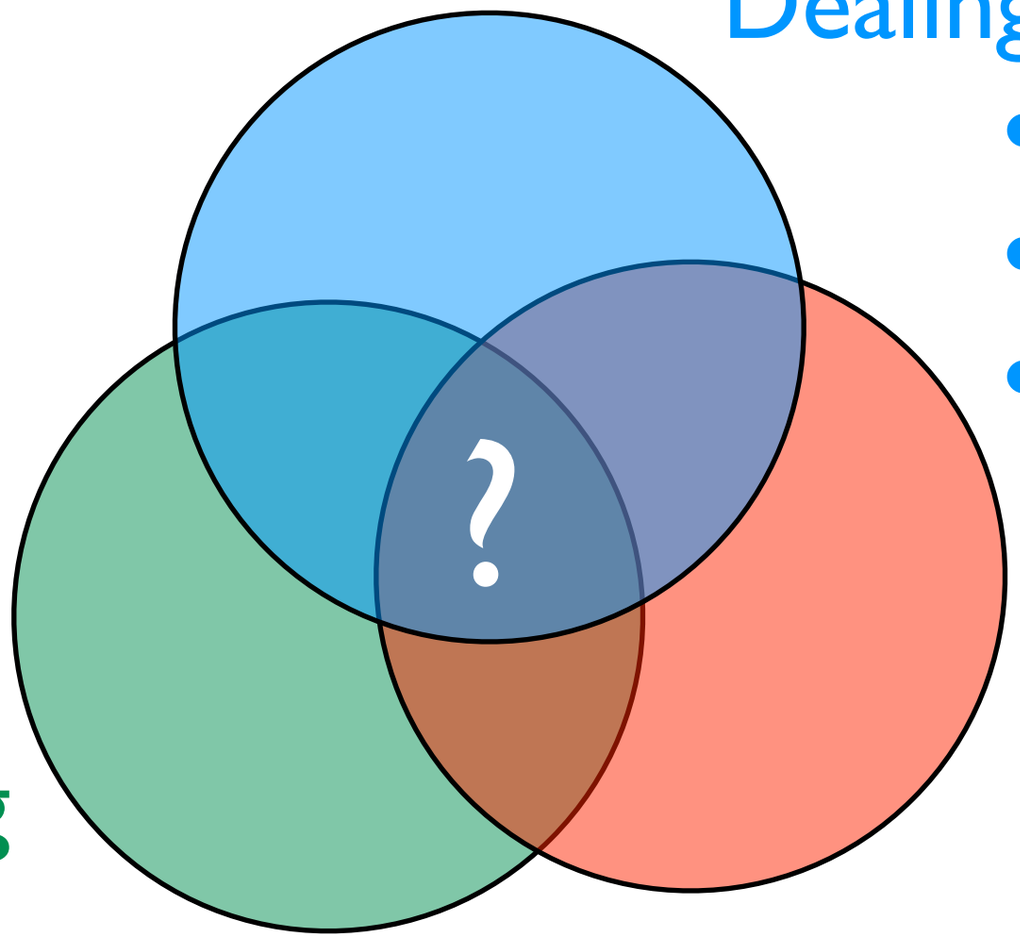

Dealing with uncertainty

- probability theory
- graphical models
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Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

NELL: http://rtw.ml.cmu.edu/rtw/

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instances for many different relations

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instances for many different relations

degree of certainty

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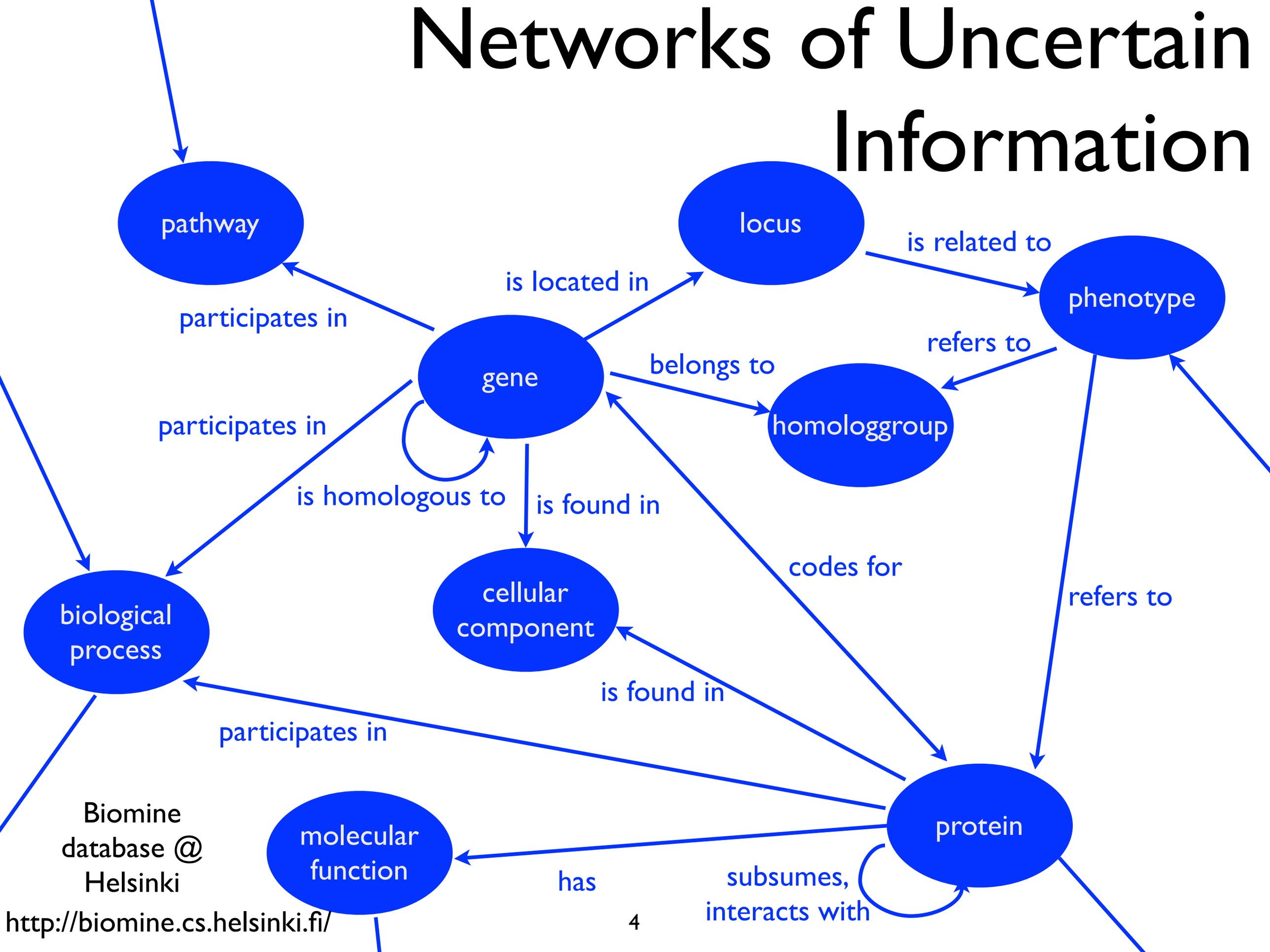
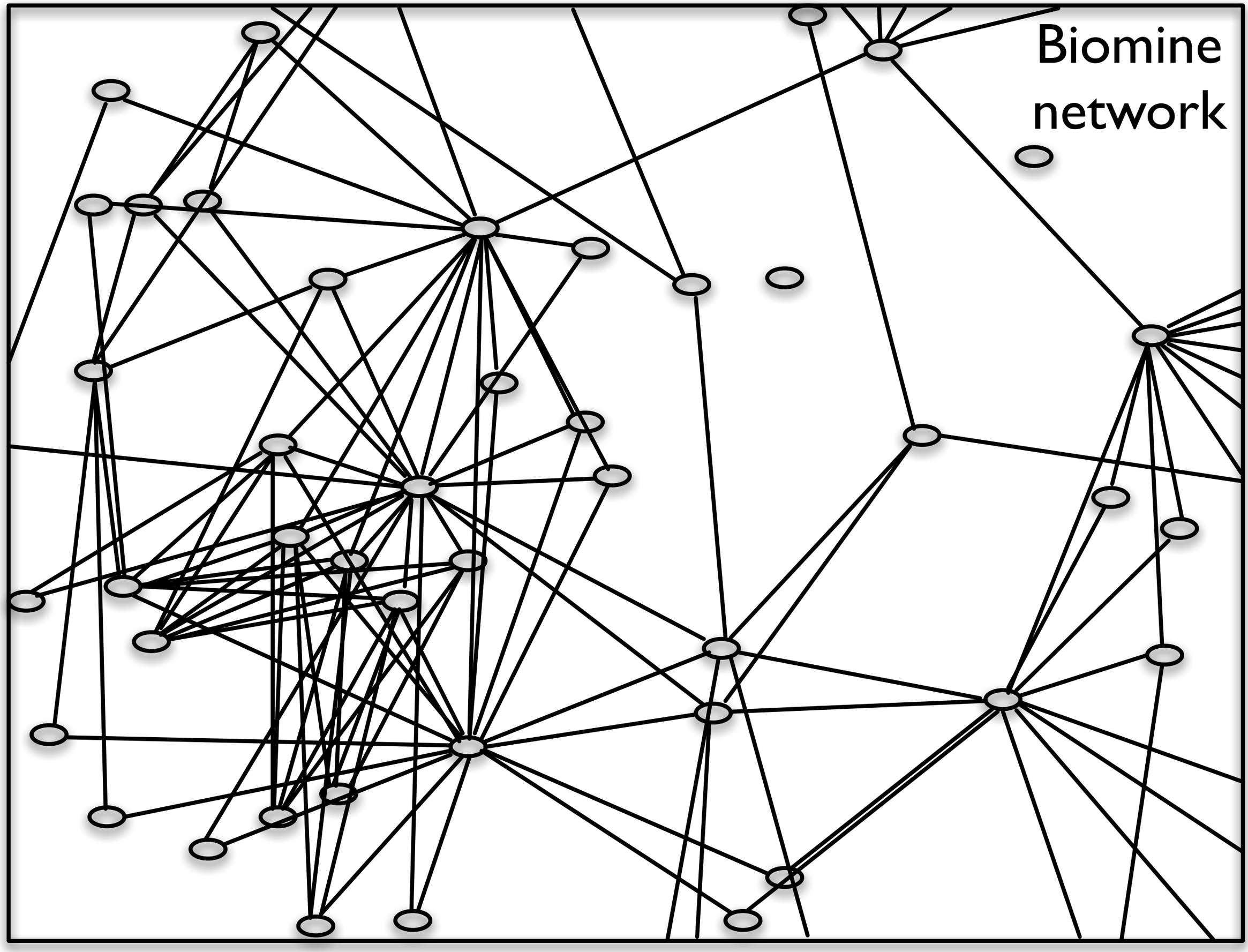
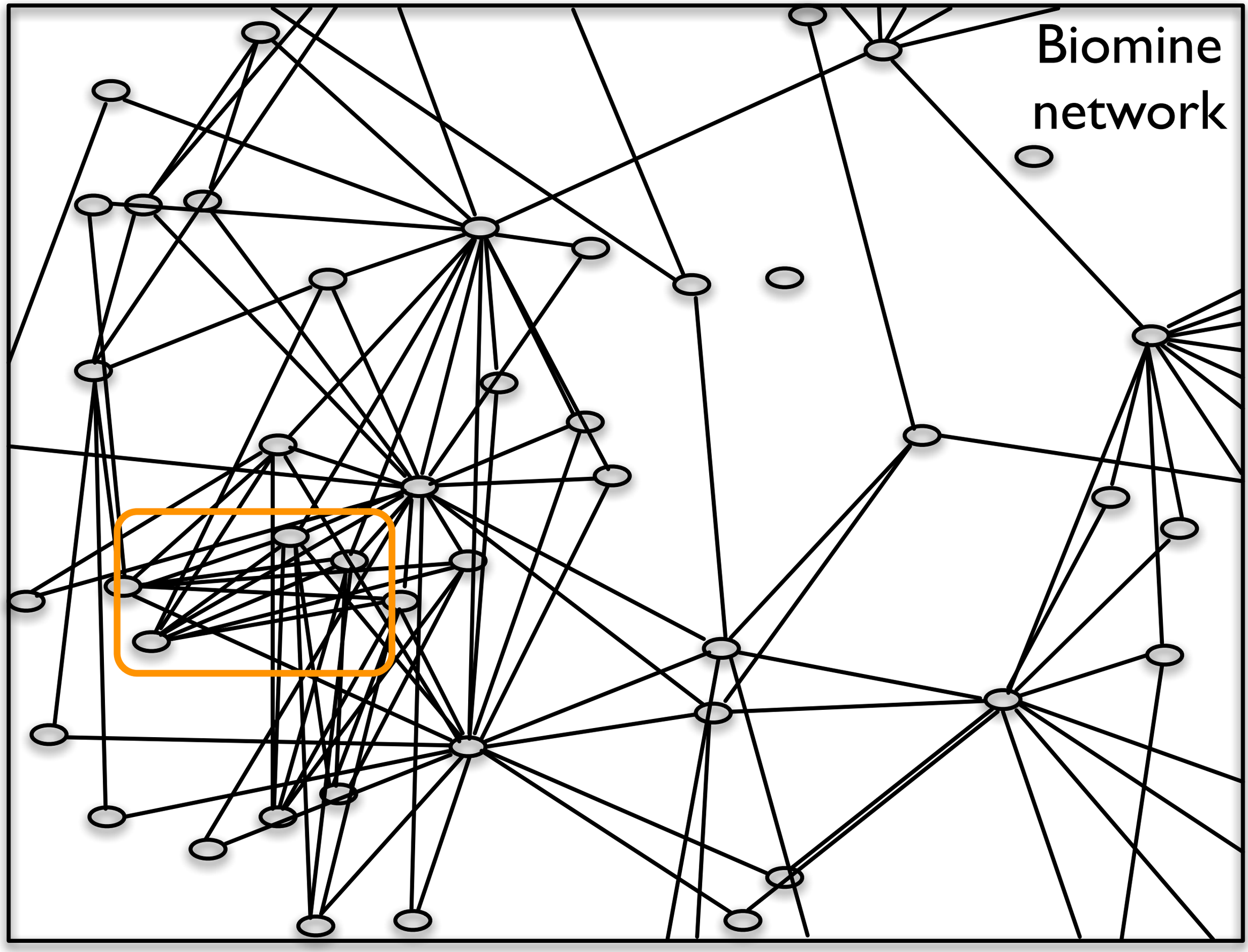
Networks of Uncertain Information



Biomine database @ Helsinki

http://biomine.cs.helsinki.fi/


Biomine
network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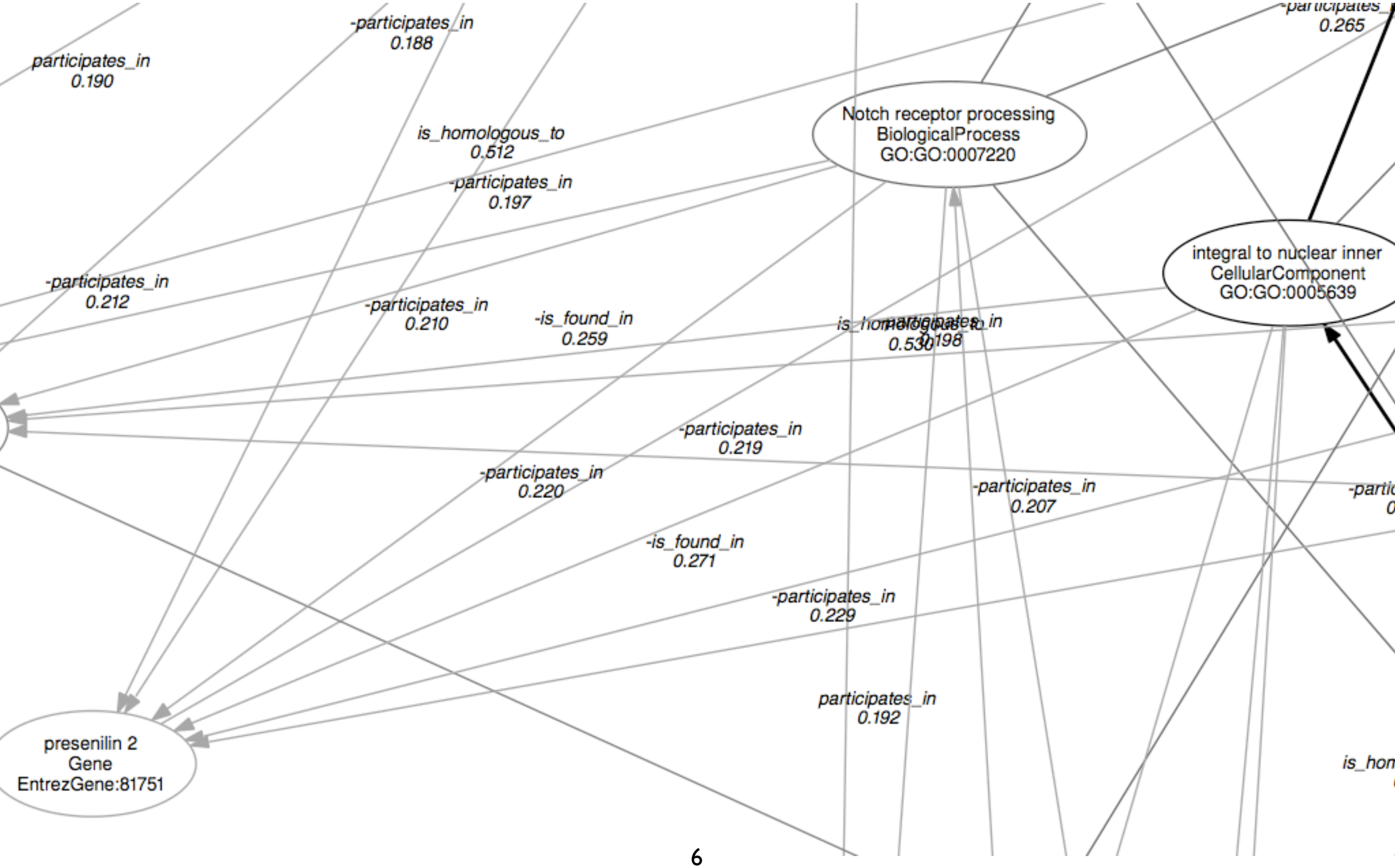
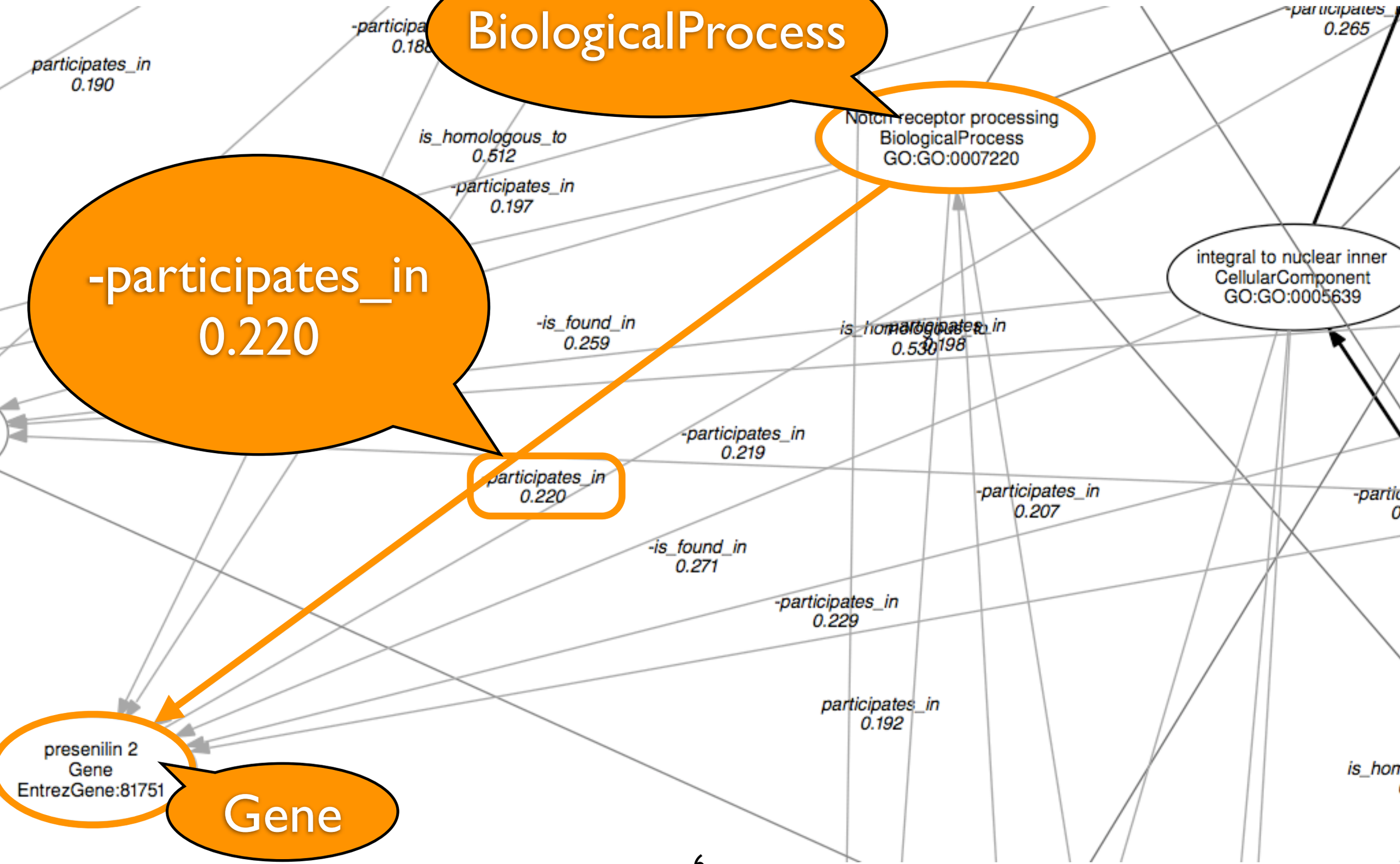


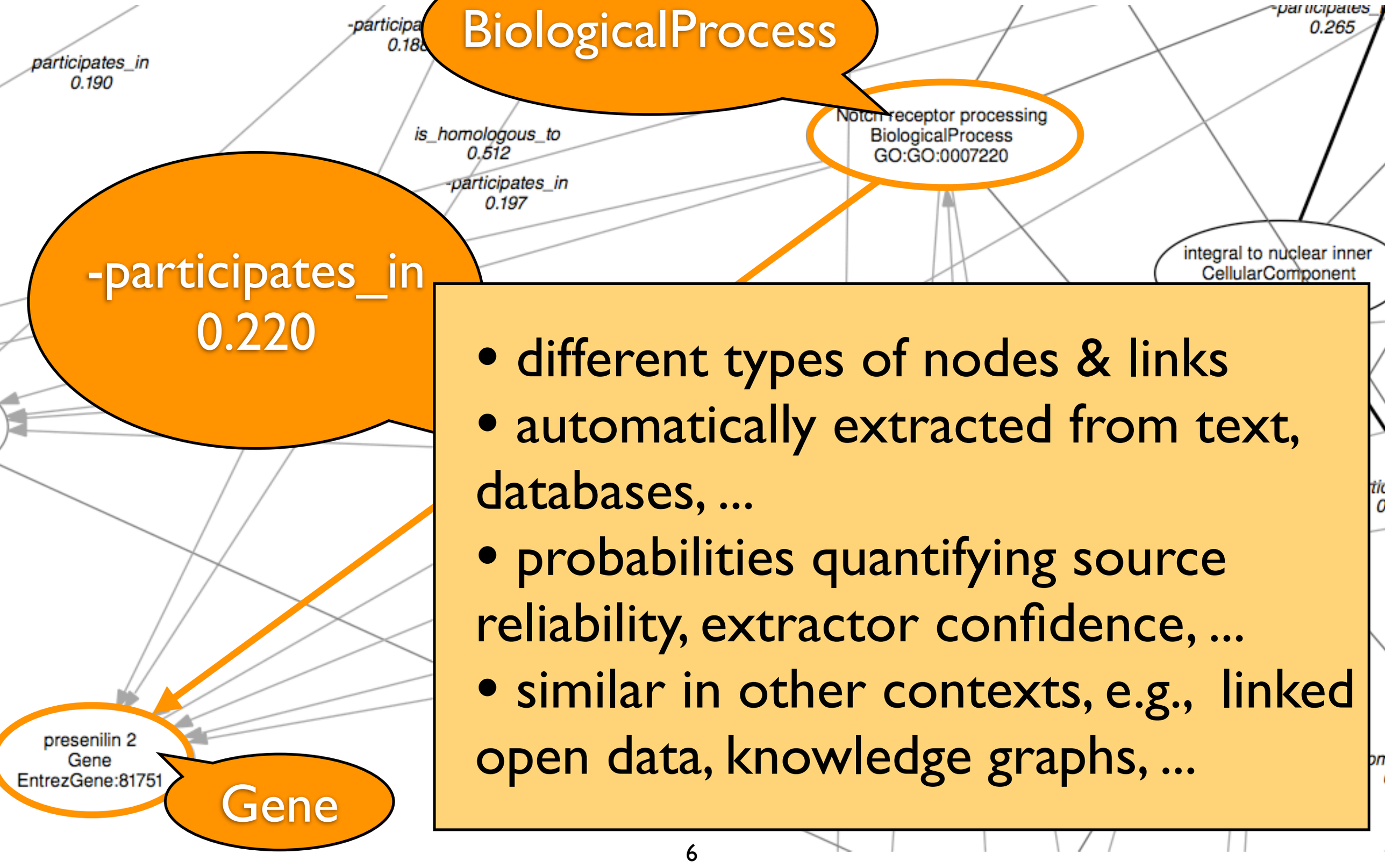
Notch receptor processing
BiologicalProcess
GO:GO:0007220

presenilin 2
Gene
EntrezGene:81751

Biomine Network



Biomine Network

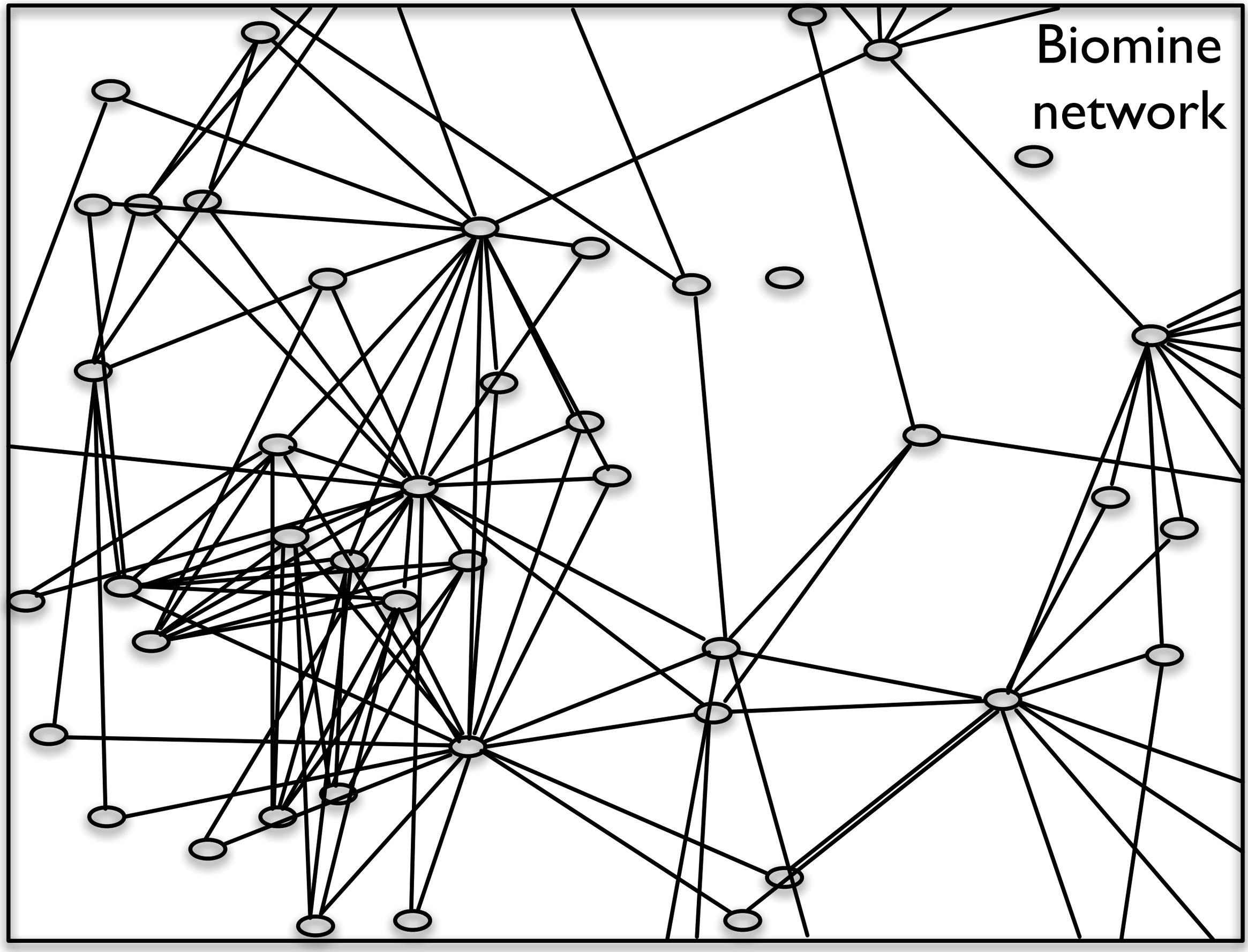


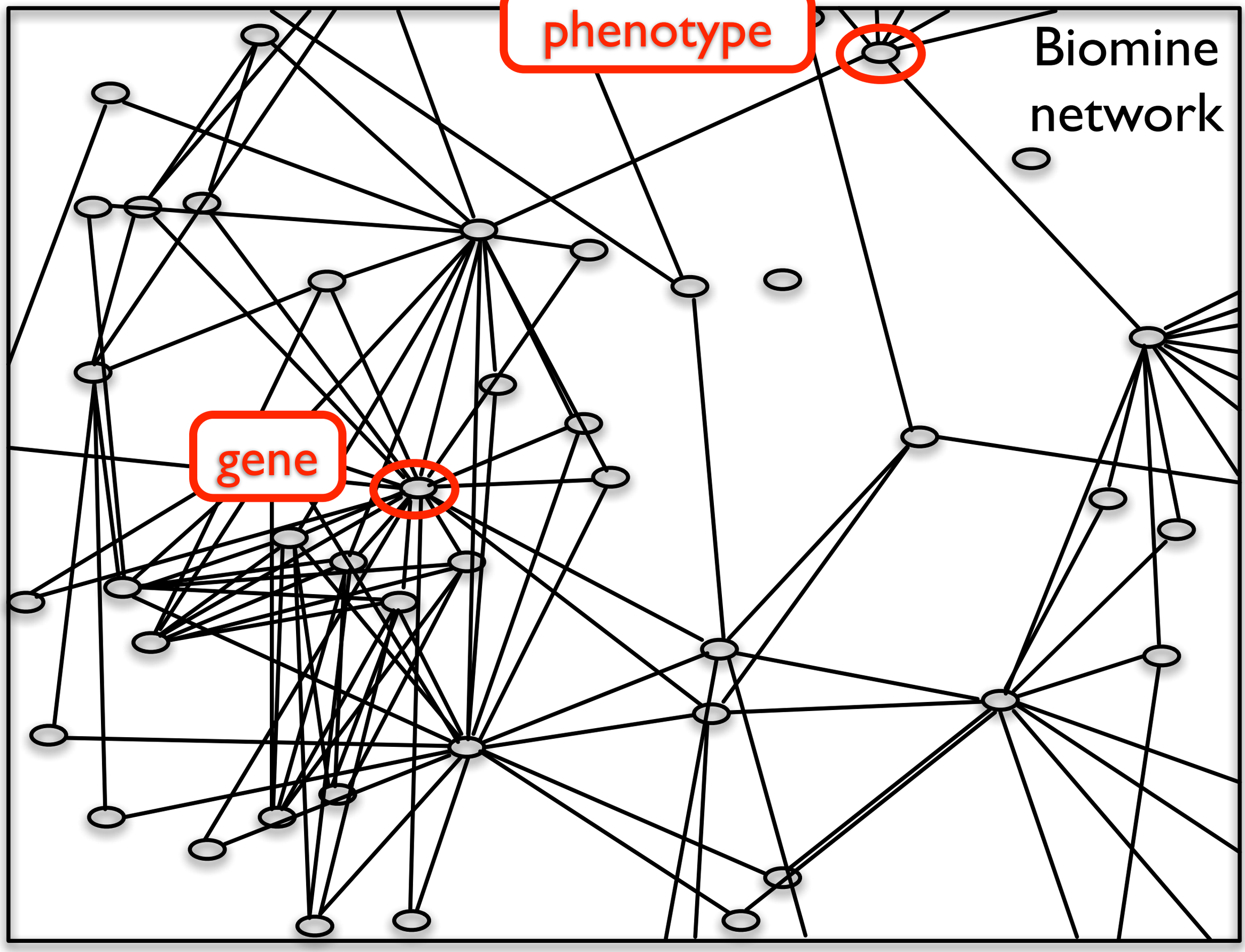
BiologicalProcess

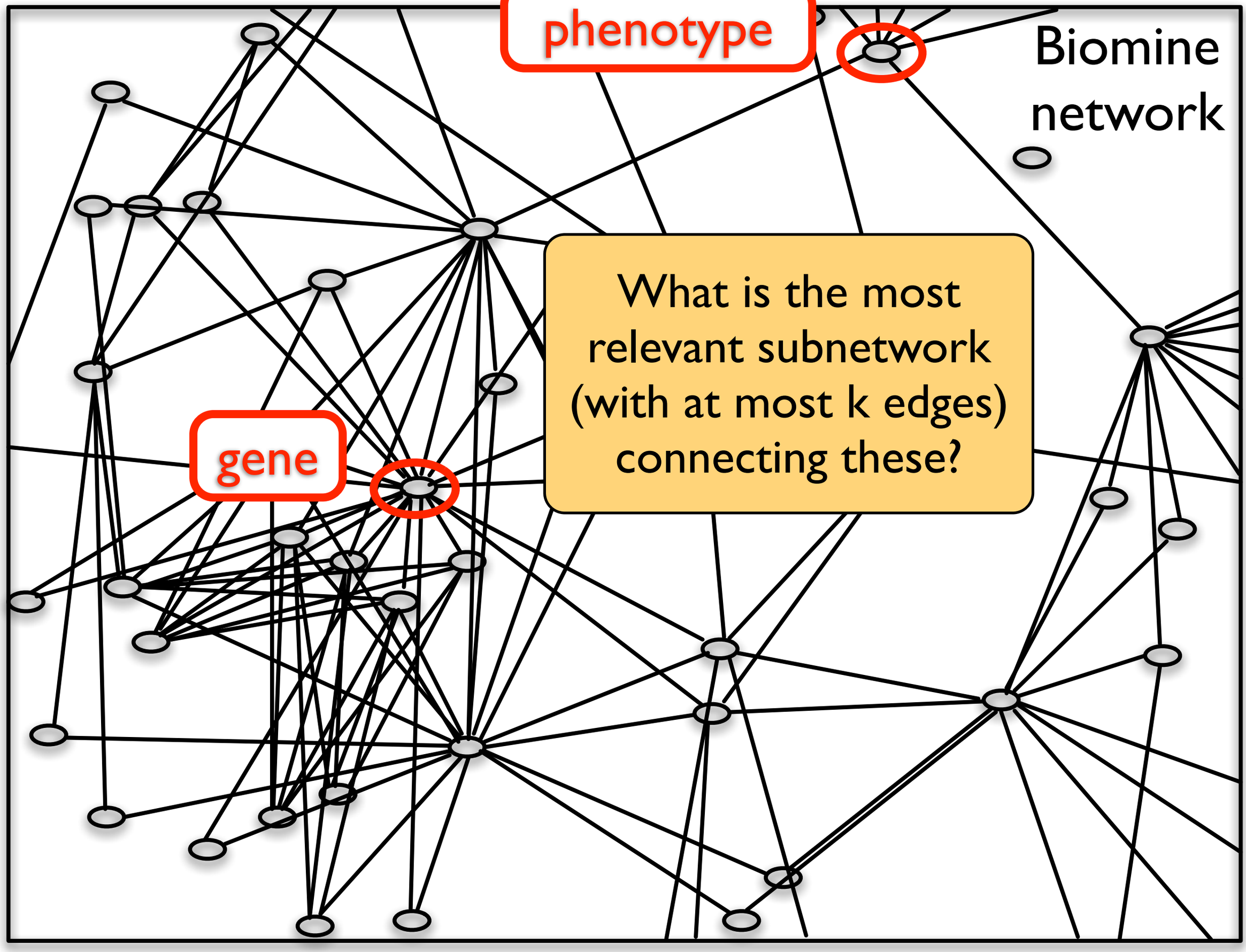
-participates_in
0.220

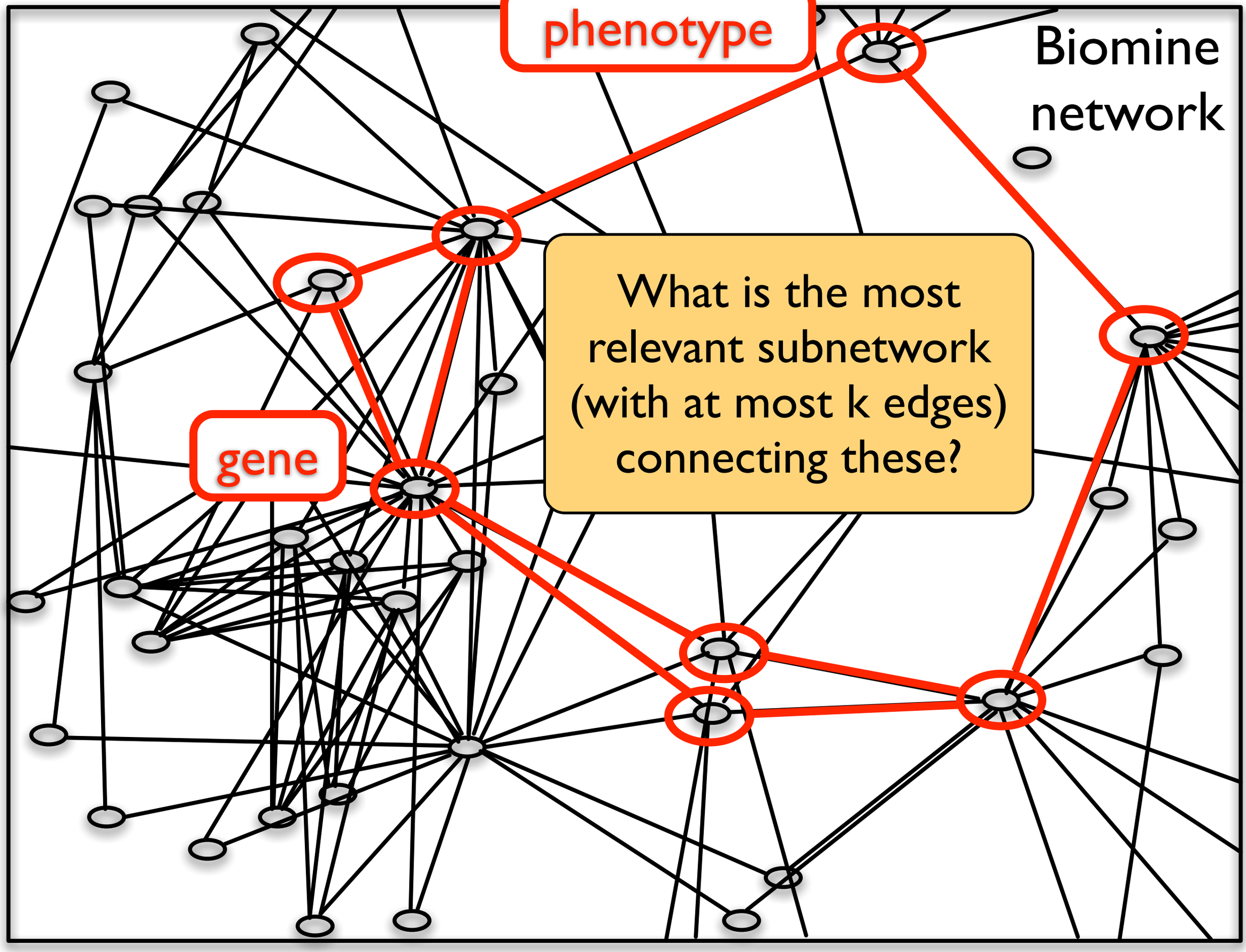
Gene

- different types of nodes & links
- automatically extracted from text, databases, ...
- probabilities quantifying source reliability, extractor confidence, ...
- similar in other contexts, e.g., linked open data, knowledge graphs, ...


Biomine
network


phenotype
Biomine
network
gene


phenotype
Biomine network
gene
What is the most relevant subnetwork (with at most k edges) connecting these?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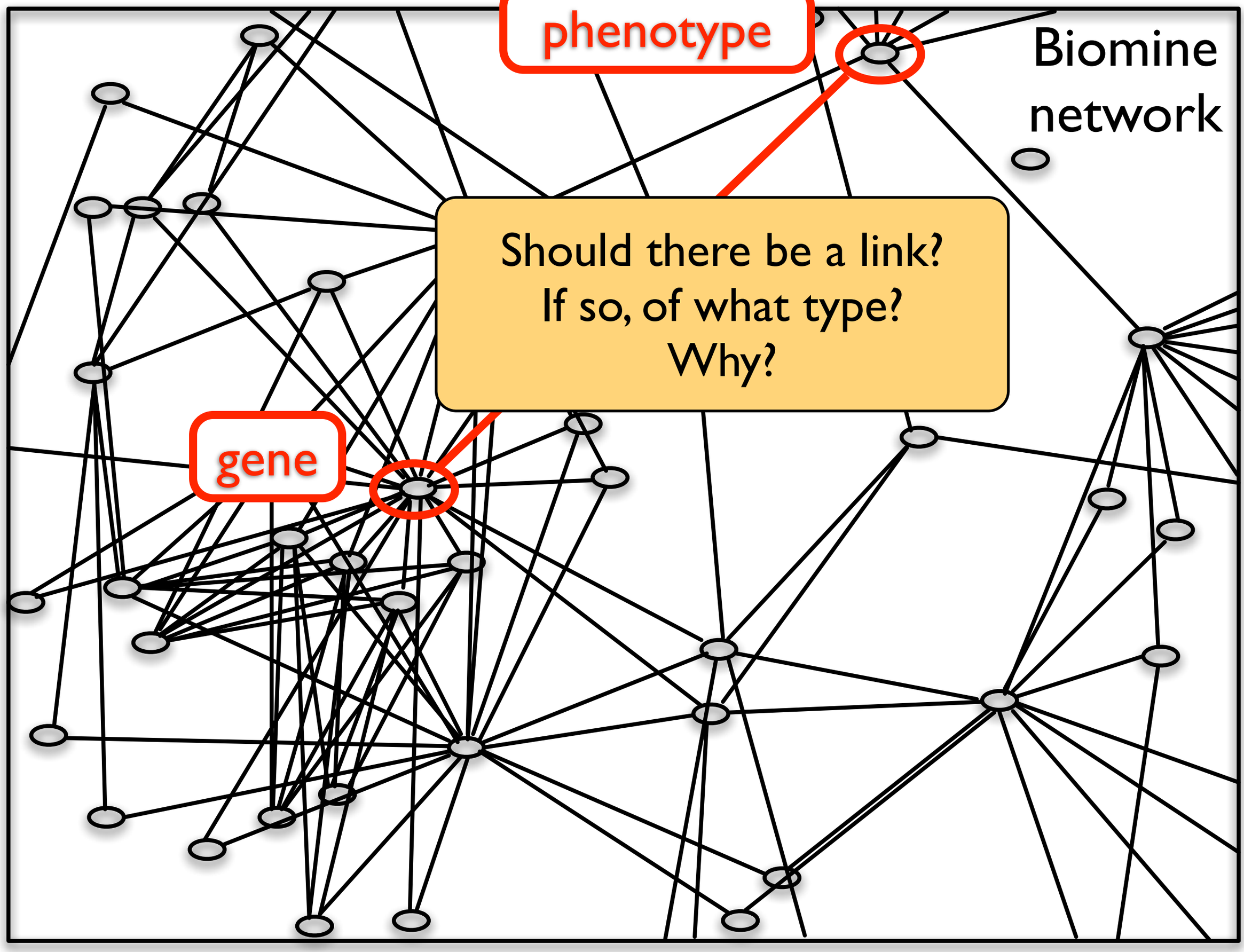
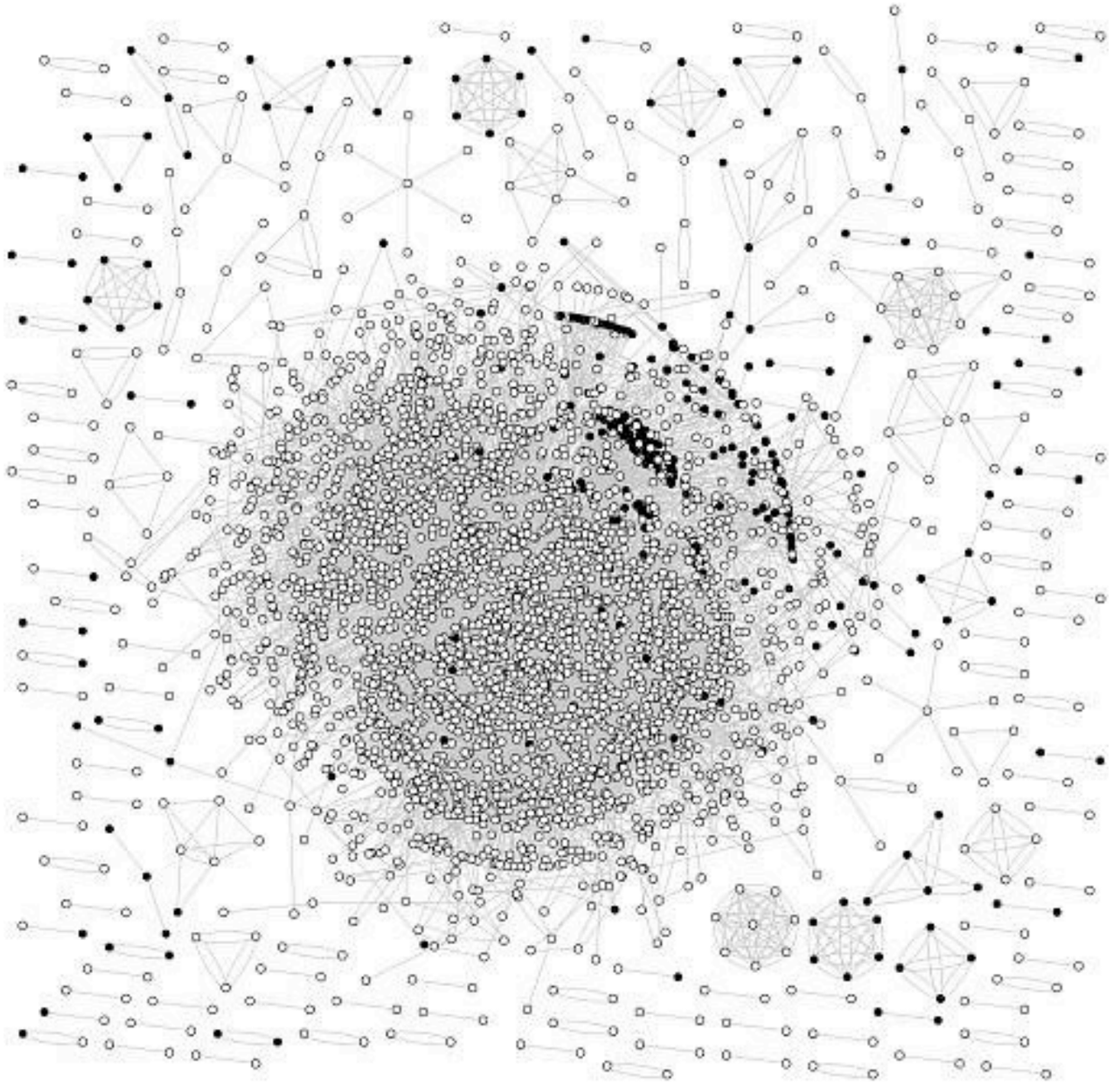

phenotype
Biomine network
Should there be a link?
If so, of what type?
Why?
gene

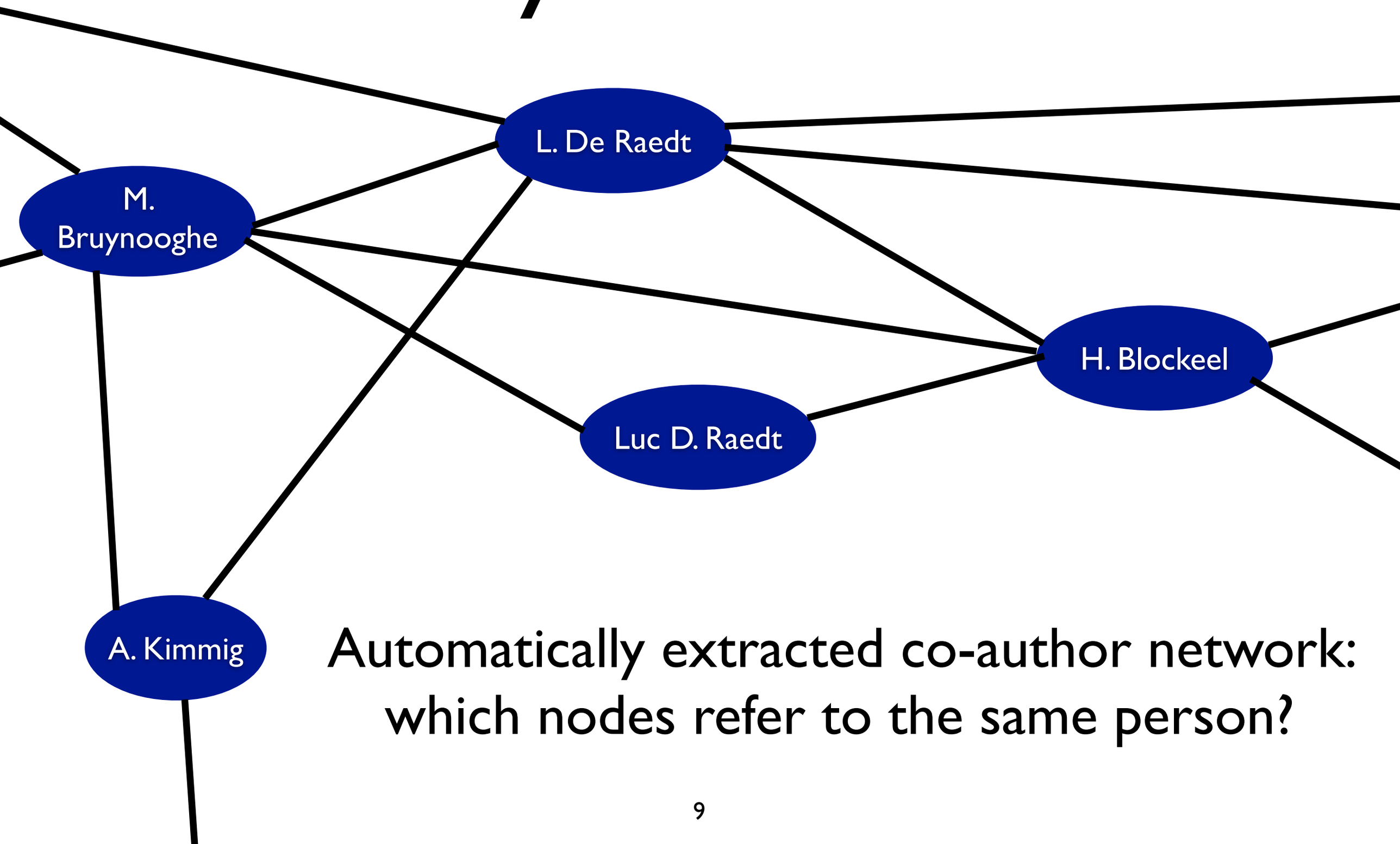
Node Classification



Can we predict the type of a node given information on its neighbors?

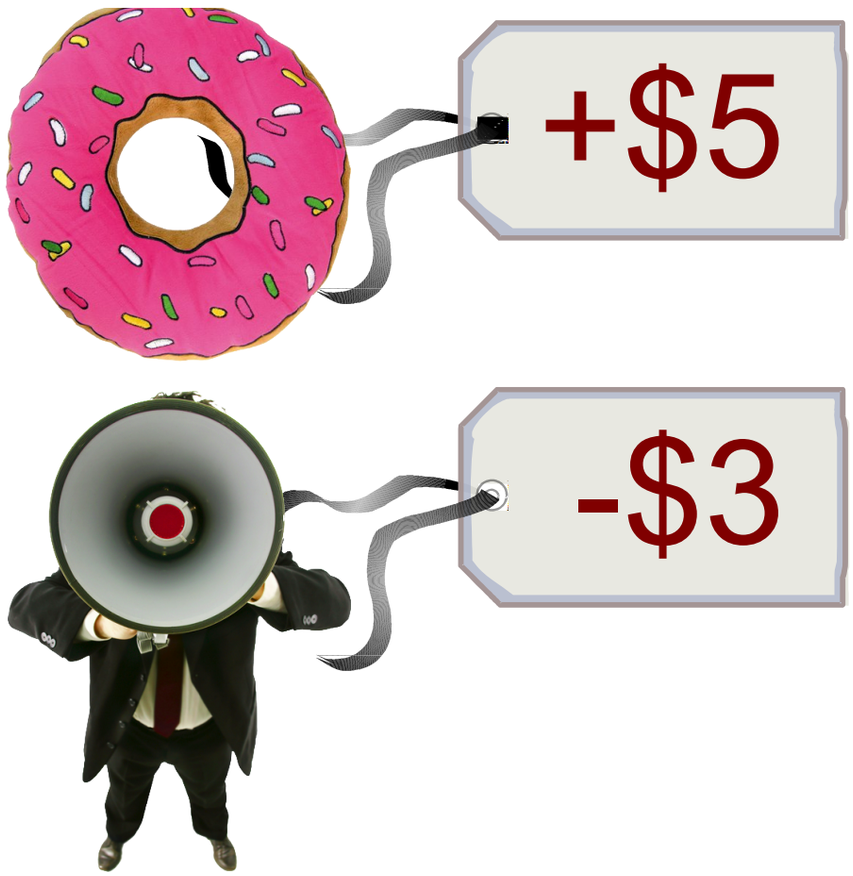
e.g., the type of a webpage given its links and the words on the page?

Entity Resolution

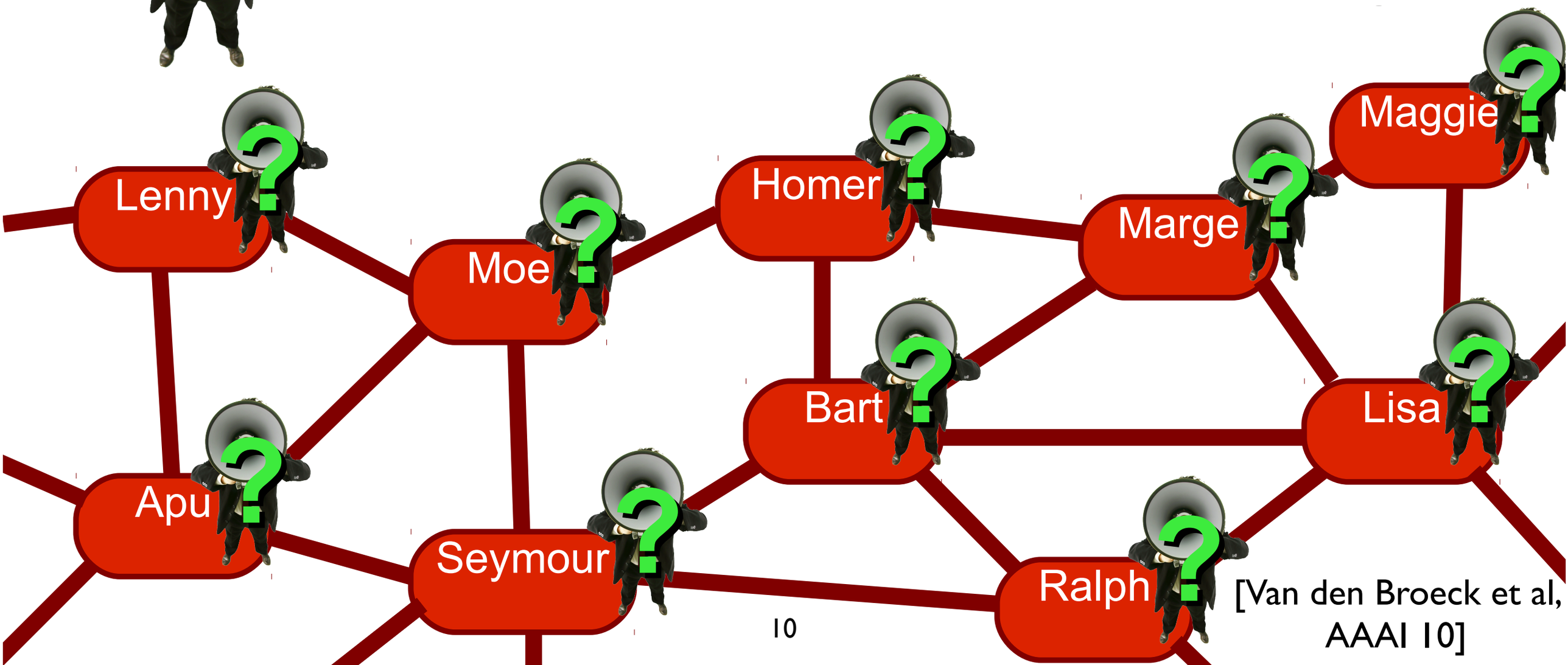


Automatically extracted co-author network: which nodes refer to the same person?

Viral Marketing

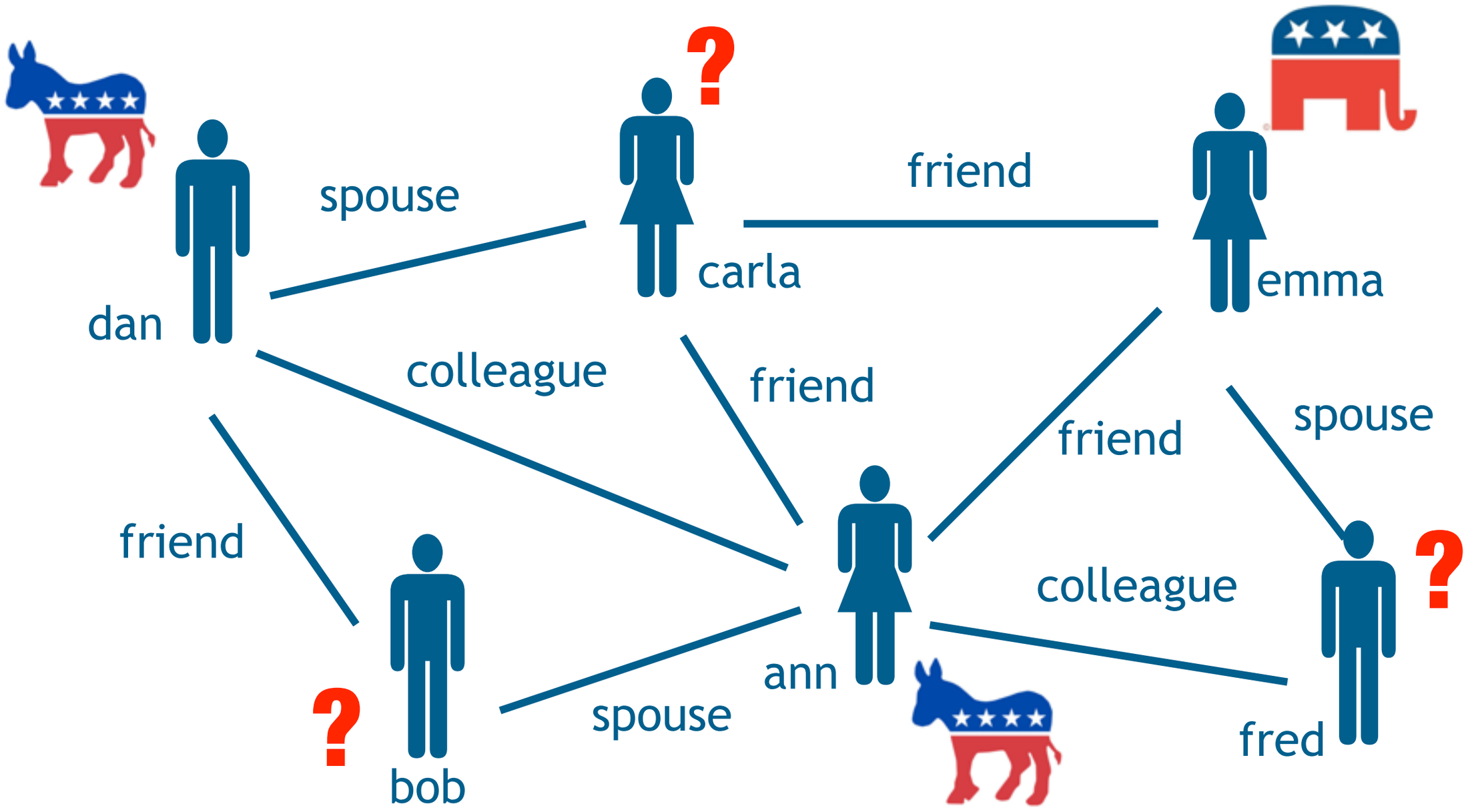


Which advertising strategy maximizes expected profit?



[Van den Broeck et al, AAAI 10]

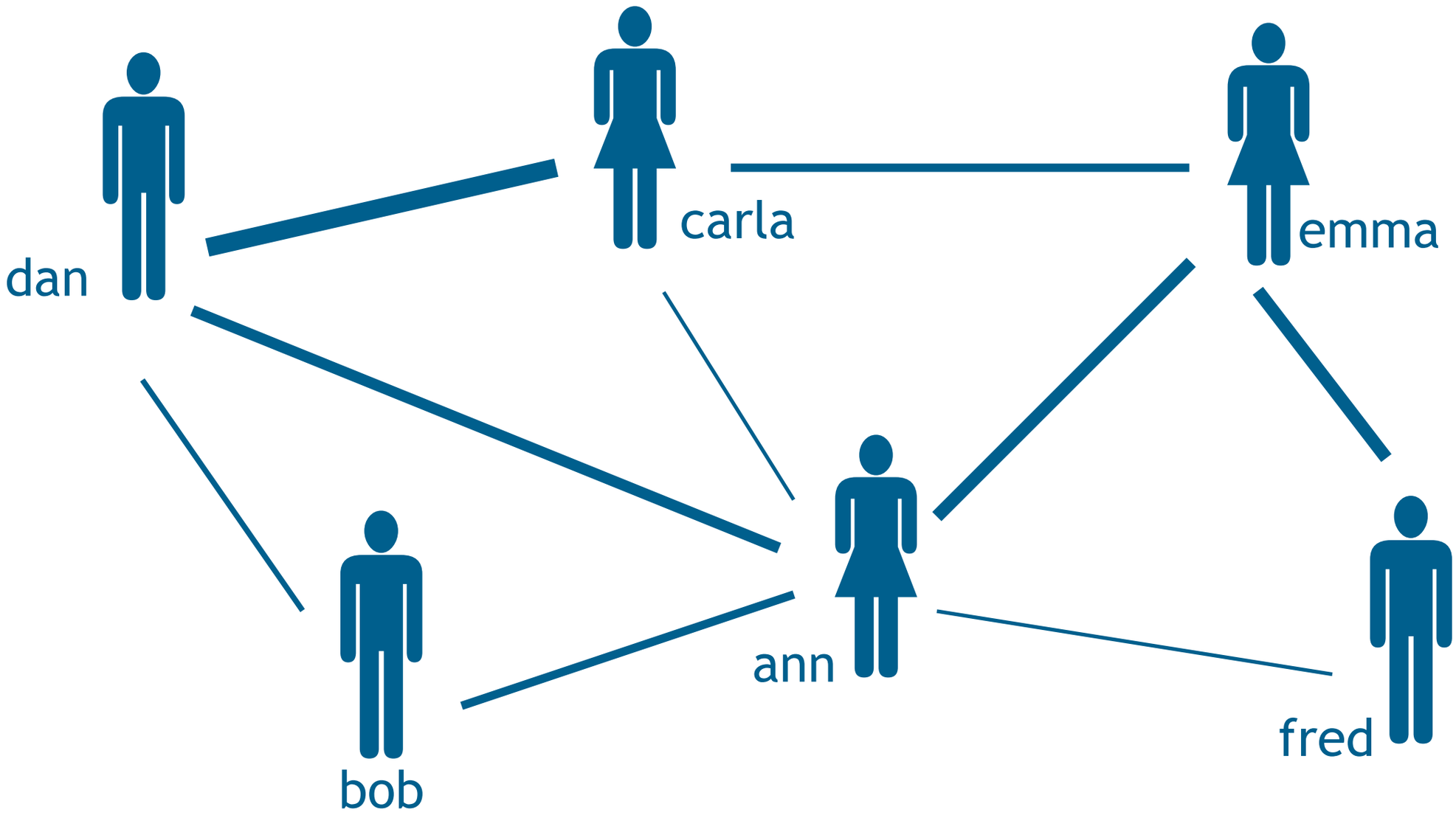
Voter Opinion Modeling



Can we predict preferences?

[Bach et al, NIPS 12]

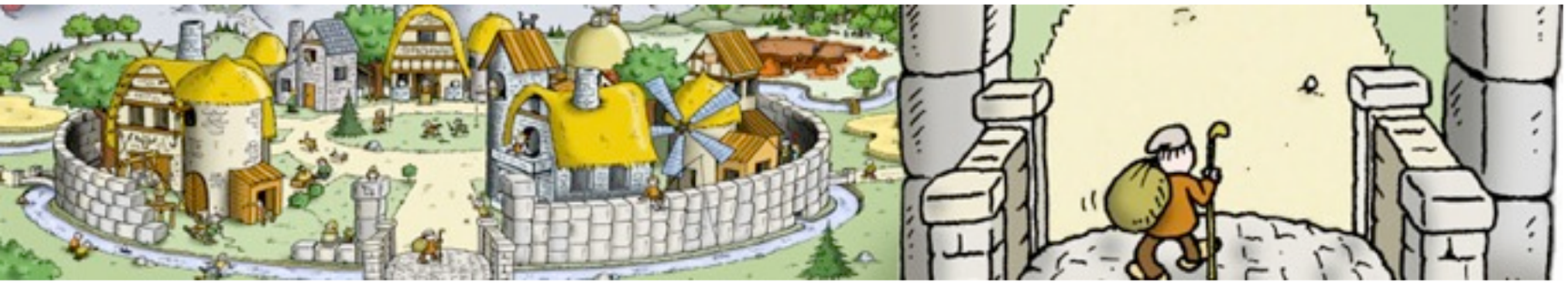
Social Trust



Can we predict strength of ties?

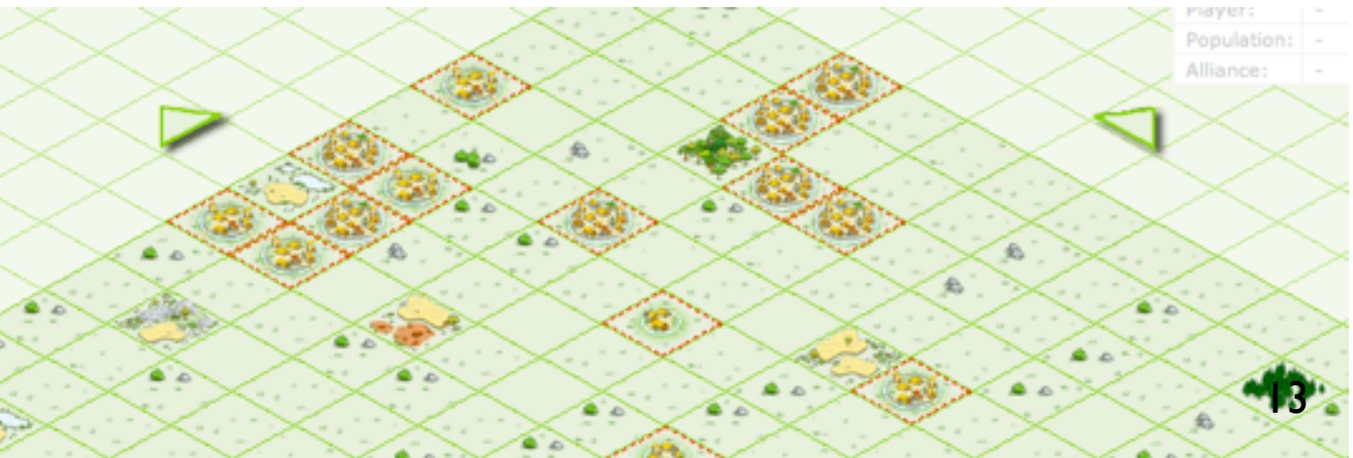
[Huang et al, SBP 13]

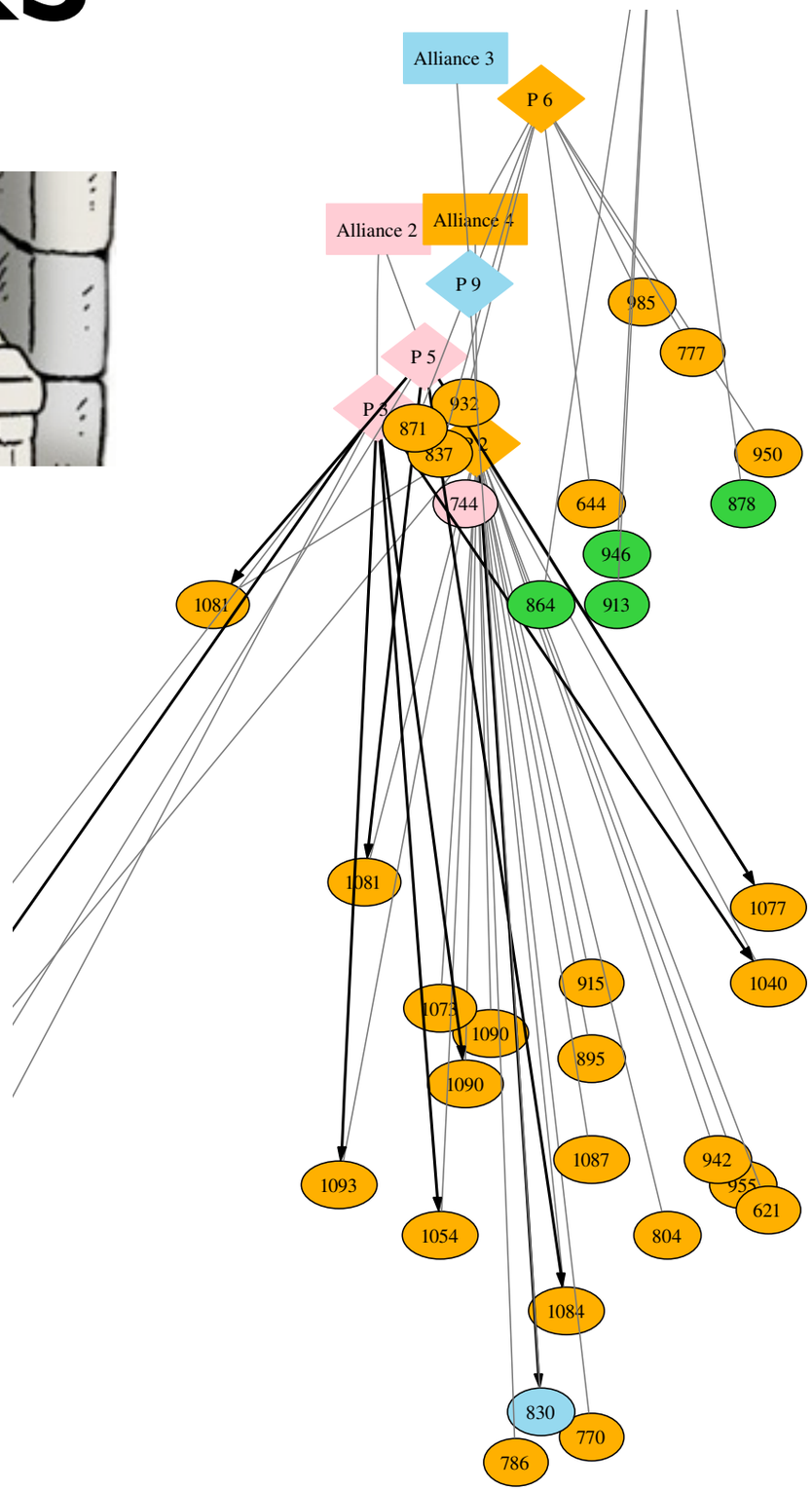
Dynamic networks



Travian: A massively multiplayer real-time strategy game

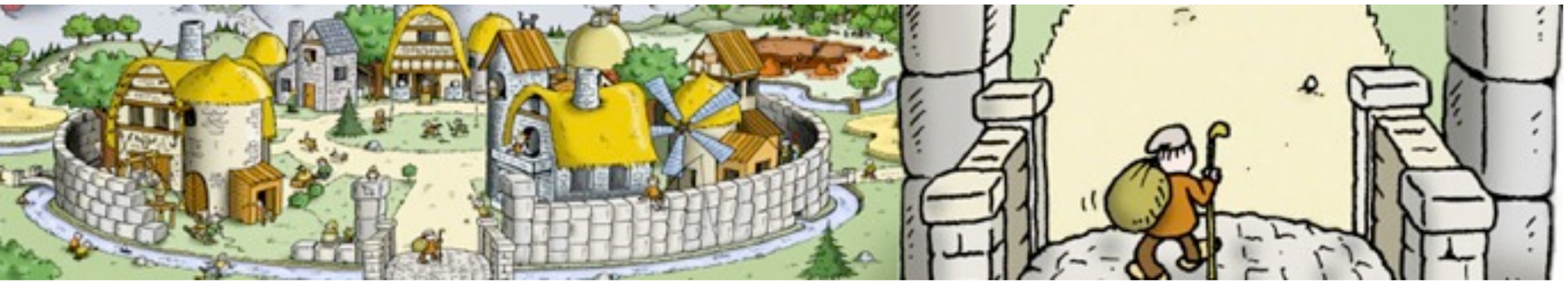
Can we build a model
of this world ?
Can we use it for playing
better ?





[Thon et al, MLJ 11]

Dynamic networks



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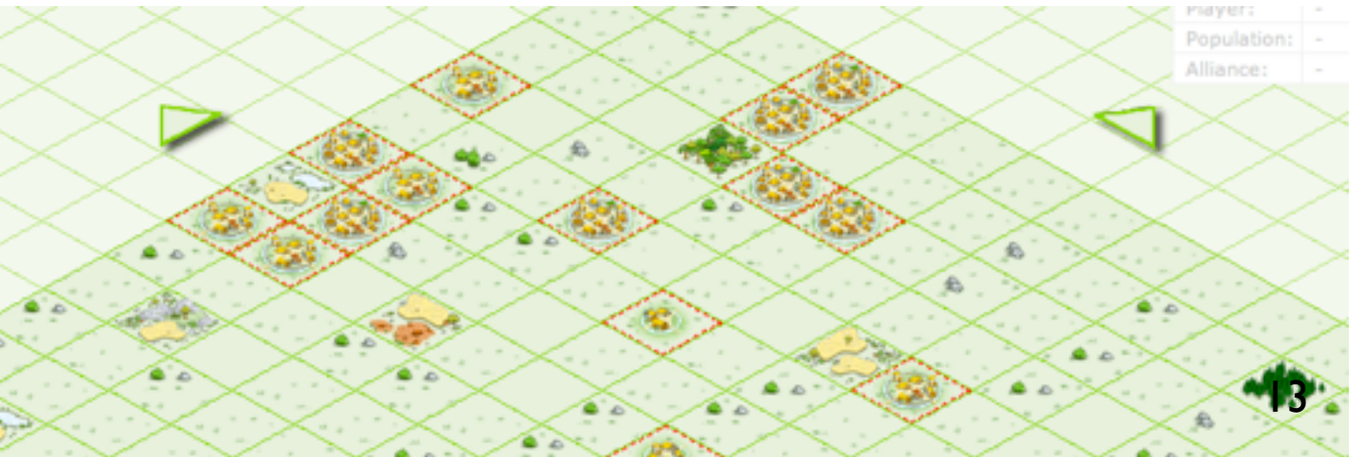
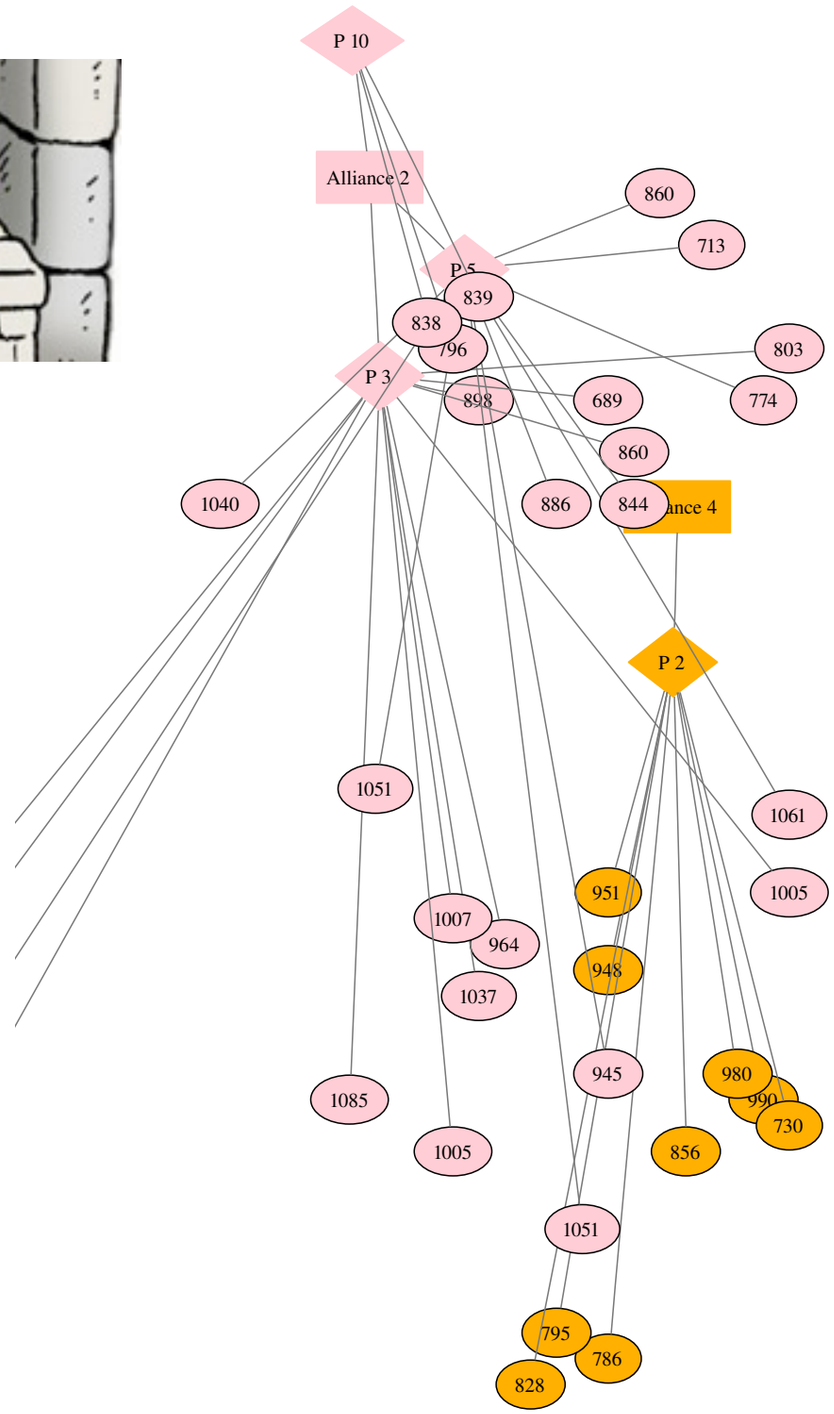
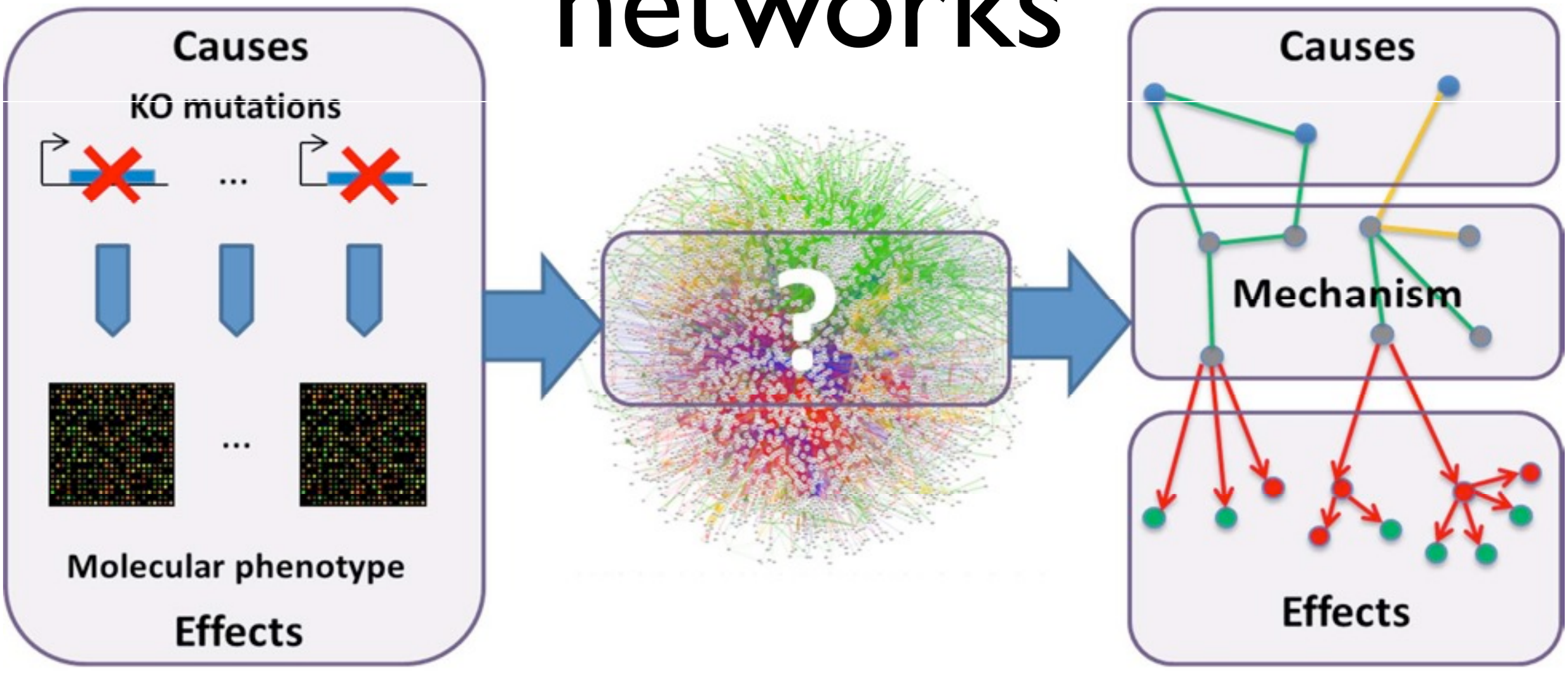




[Thon et al, MLJ 11]

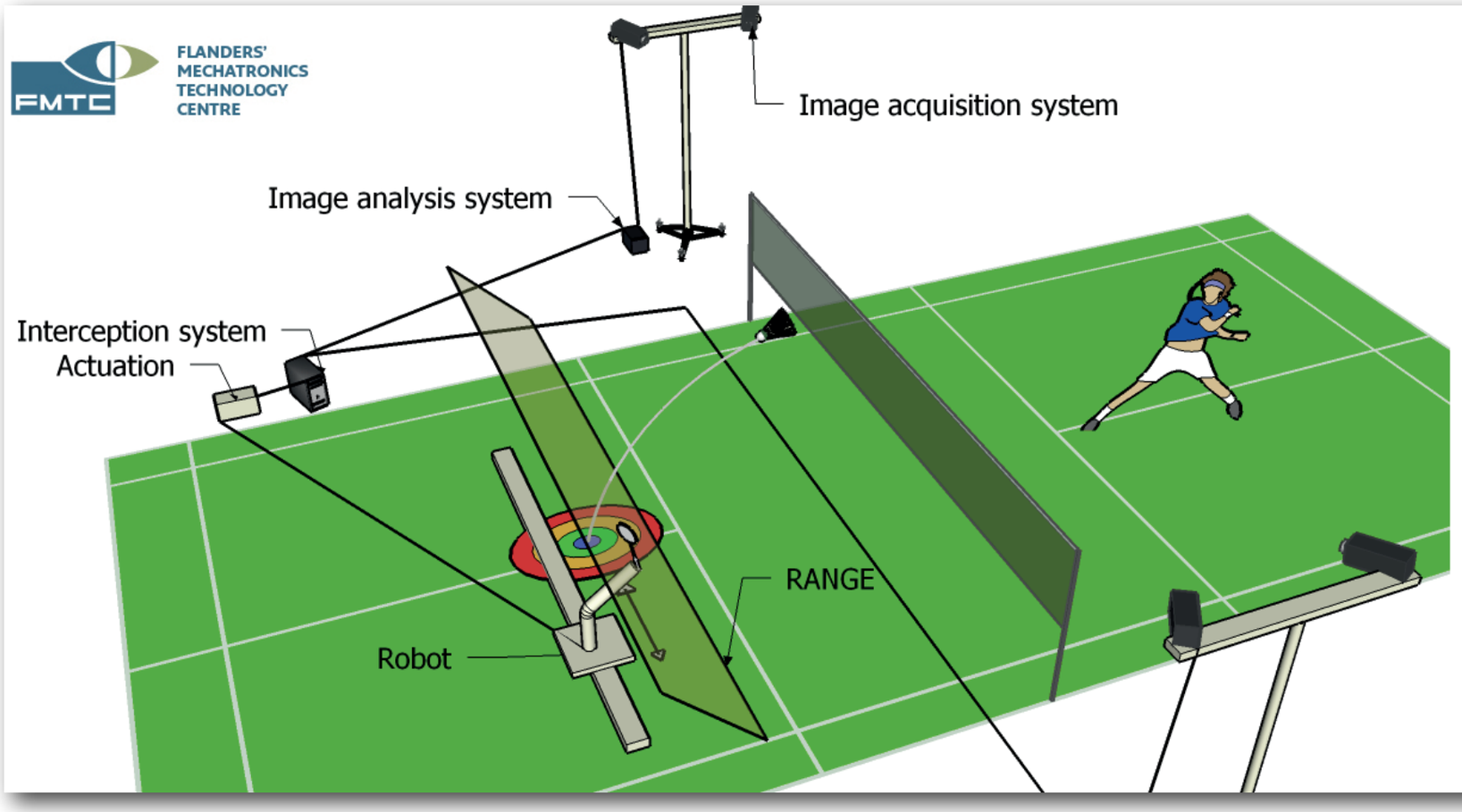
Molecular interaction networks

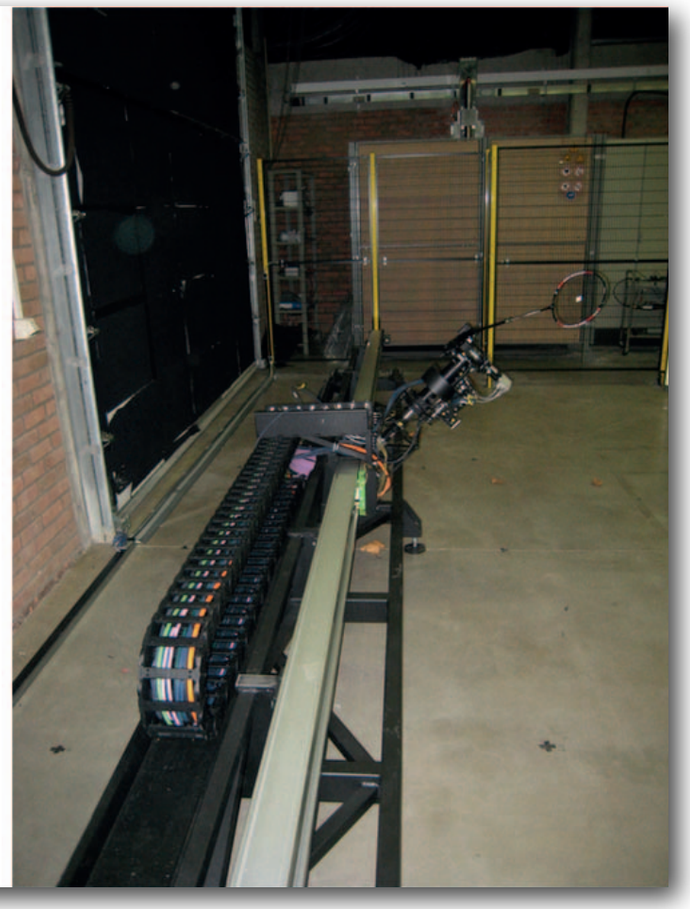


Can we find the mechanism connecting causes to effects?

[De Maeyer et al, Molecular Biosystems 13]

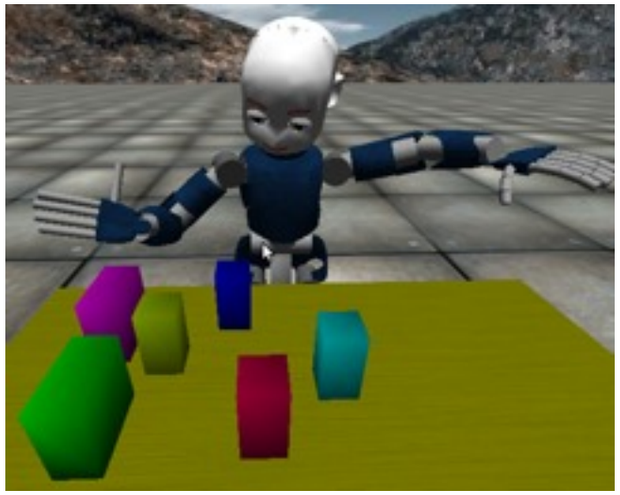
Diagnosing machine failures



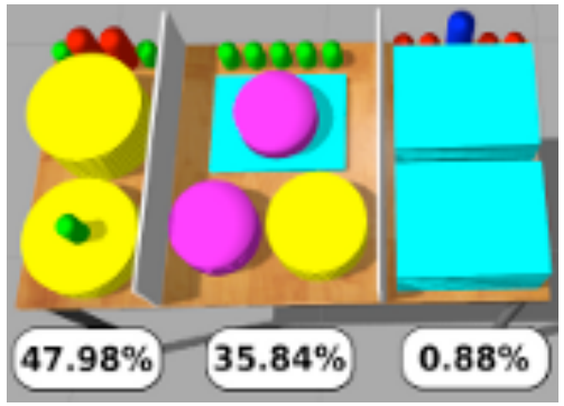


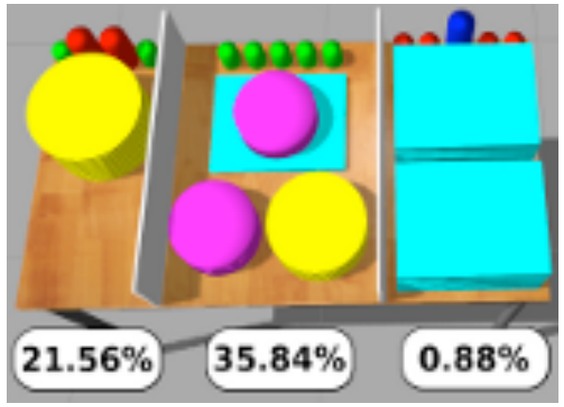
Can we build a model of the robot's working and use it to find causes of failures?

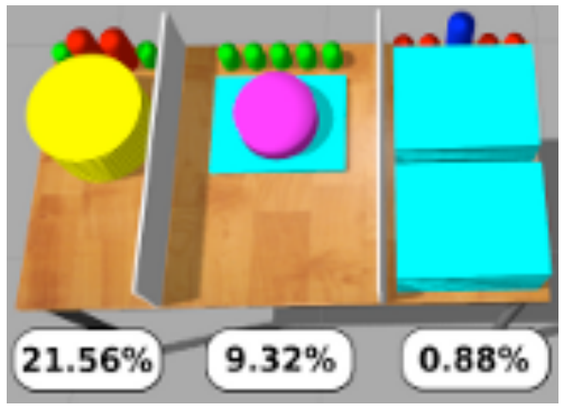
[Schramm, Meert and Driessens]



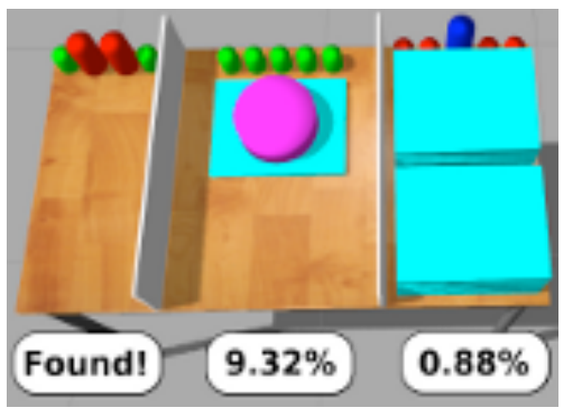
Robotics





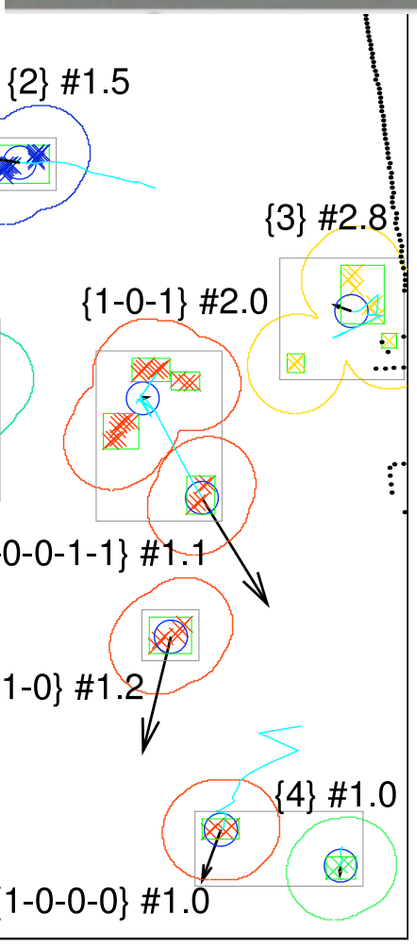


- How to achieve a specific configuration of objects on the shelf?
- Where's the orange mug?
- Where's something to serve soup in?



[Moldovan et al, ICRA 12, 14]

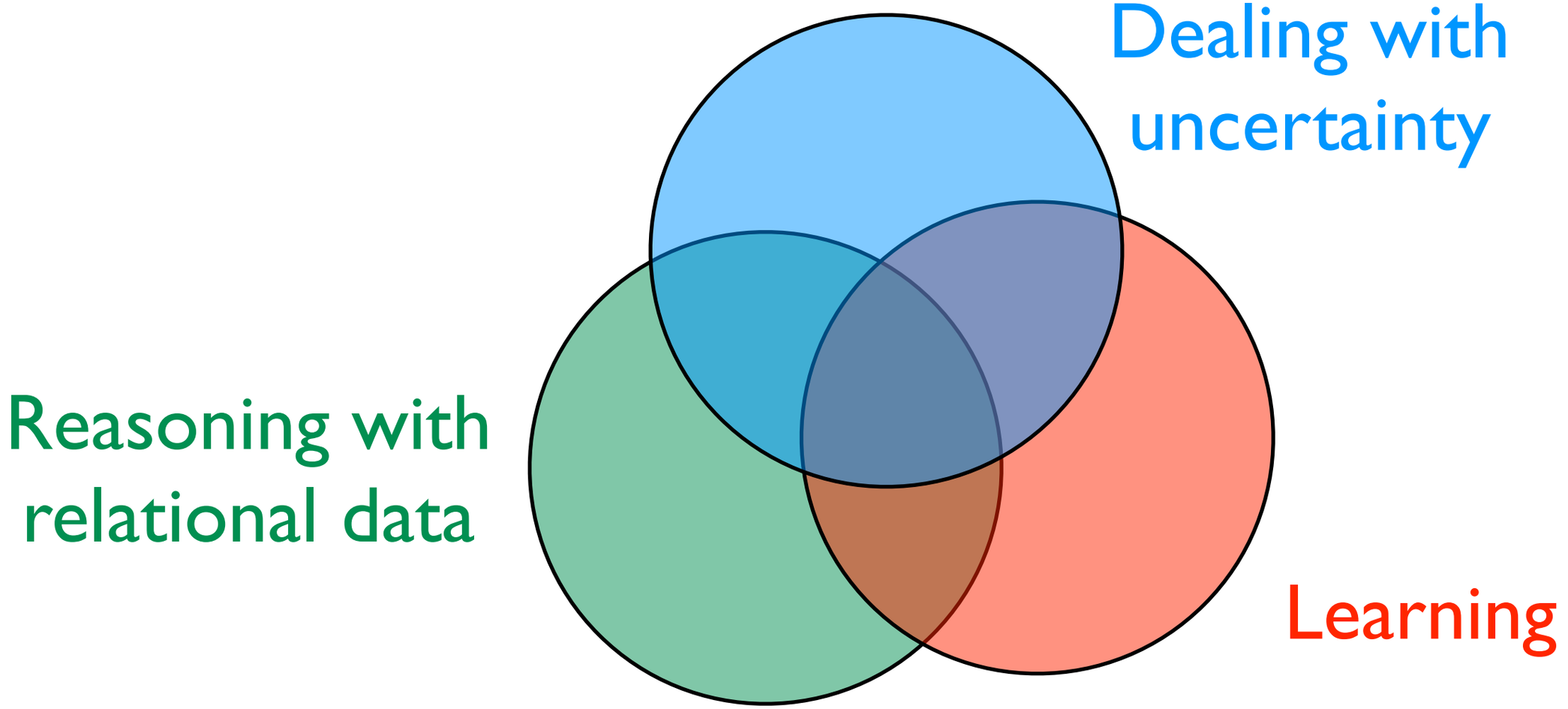
Analyzing Video Data



- Track people or objects over time? Even if temporarily hidden?
- Recognize activities?
- Infer object properties?

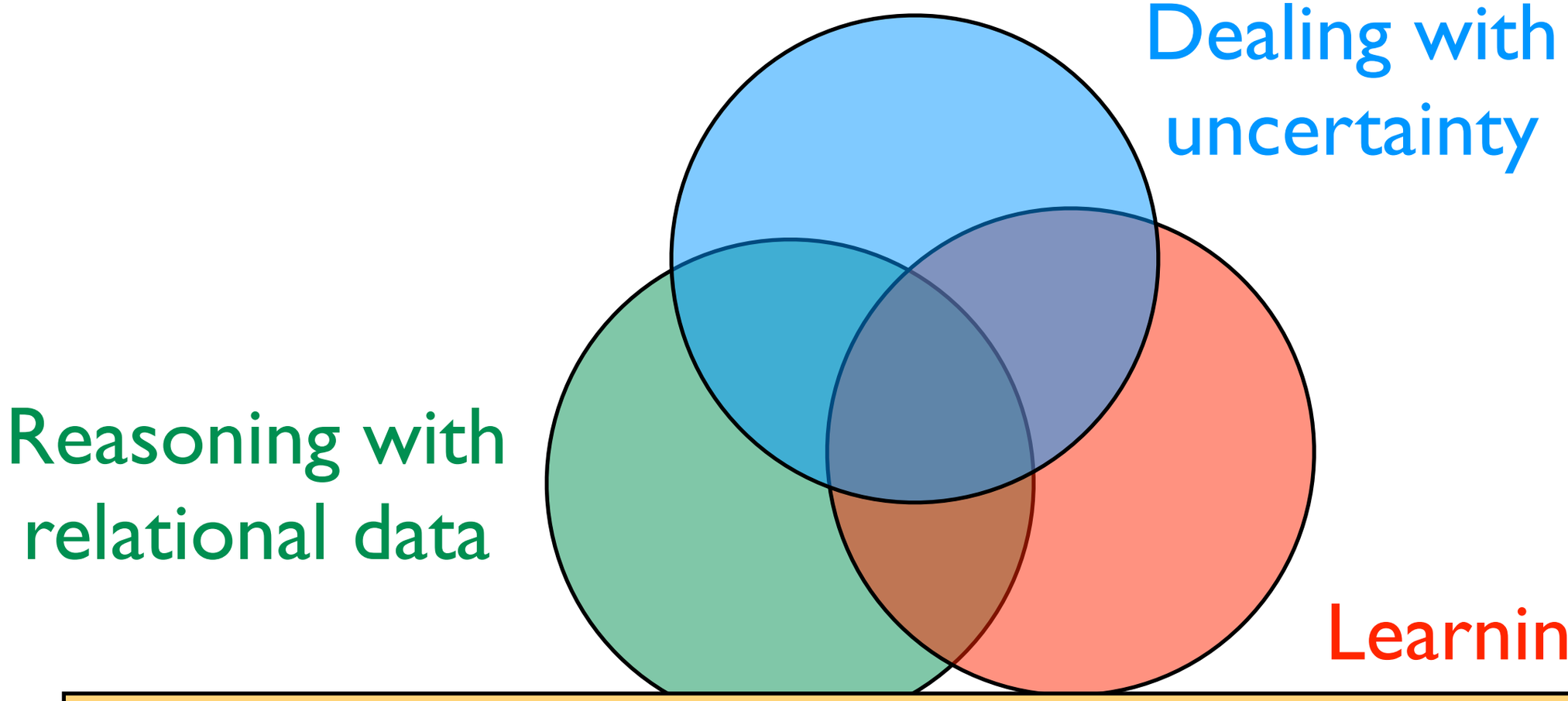
[Skarlatidis et al, TPLP 14;
Nitti et al, IROS 13, ICRA 14]

Common theme



Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

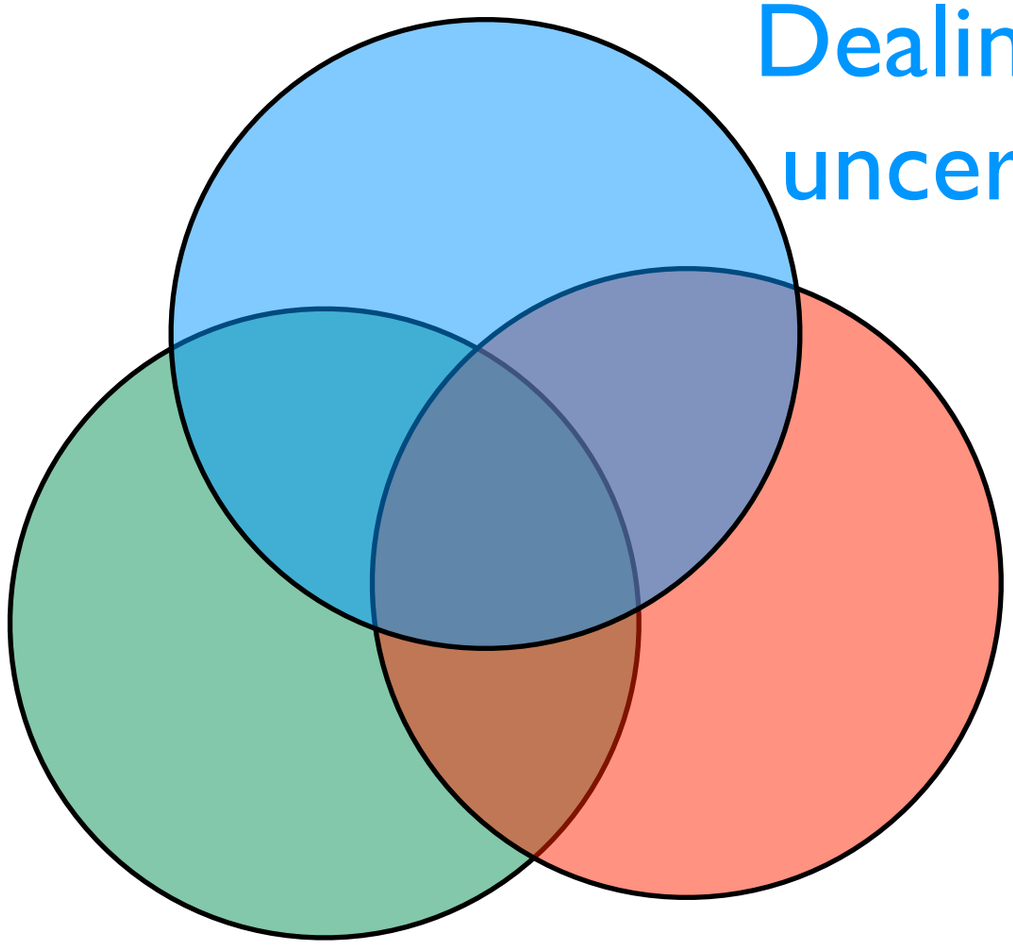
Common theme



- many different formalisms
- our focus: probabilistic logic programming
- won't cover lifted graphical models such as MLNs, PRMs, PSL, RBNs, ...

ProbLog

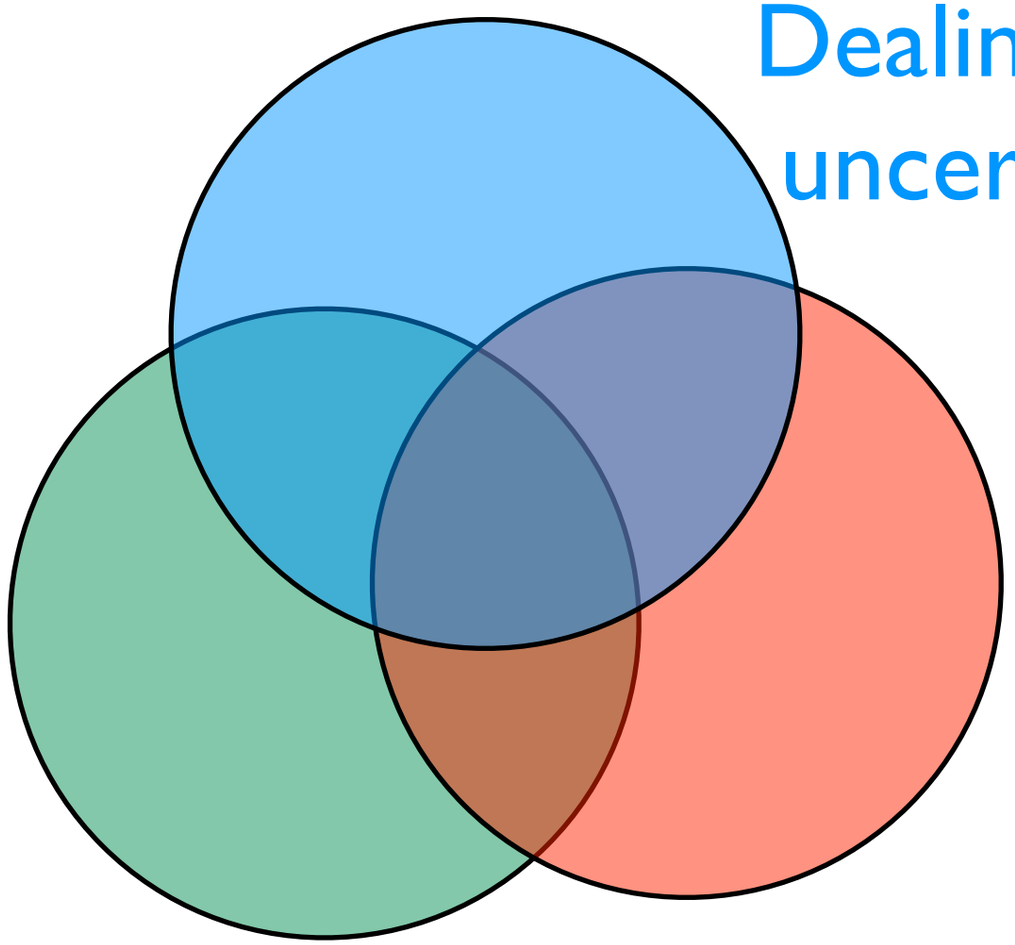
probabilistic Prolog



http://dtai.cs.kuleuven.be/problog/

ProbLog

probabilistic Prolog




```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X), smokes(Y).
```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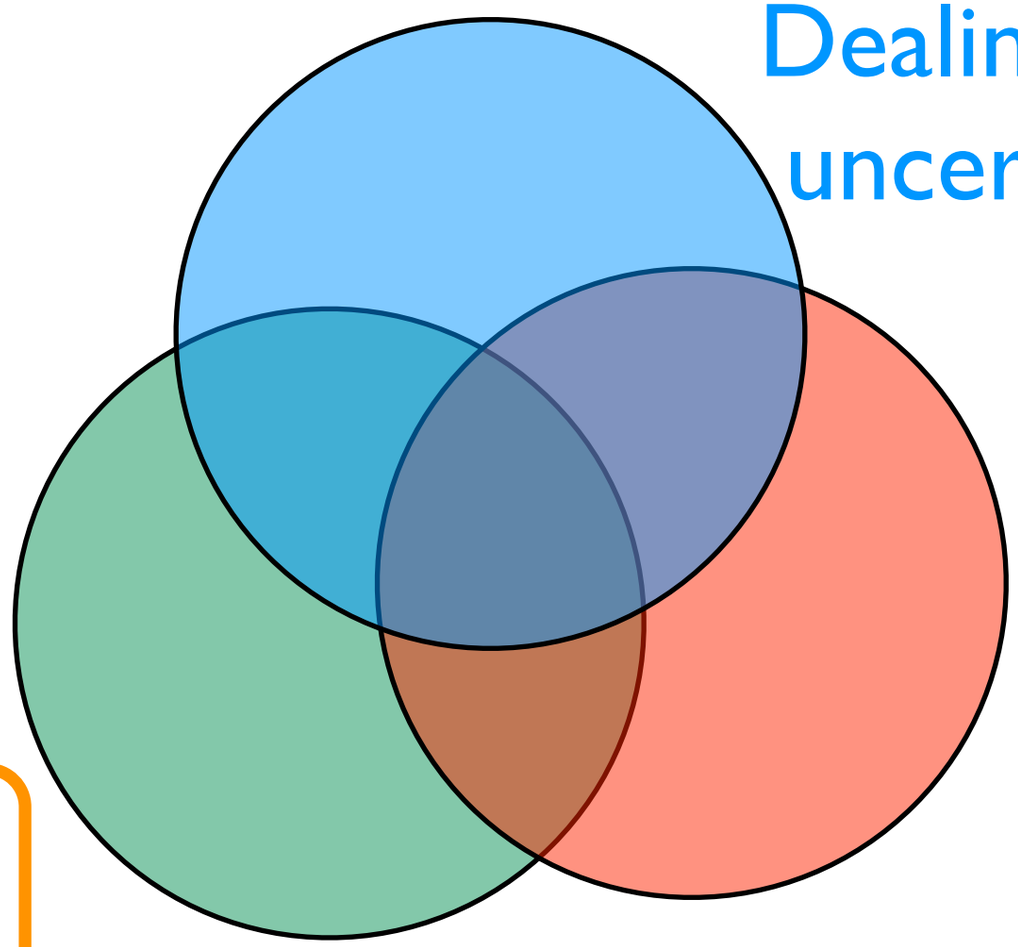
ProbLog

probabilistic Prolog




```
stress(ann).
influences(ann,bob).
influences(bob,carl).
```

one world

```
smokes(X) :- stress(X).
smokes(X) :-
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```


http://dtai.cs.kuleuven.be/problog/

ProbLog

probabilistic Prolog

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0.2::influences(bob,carl).
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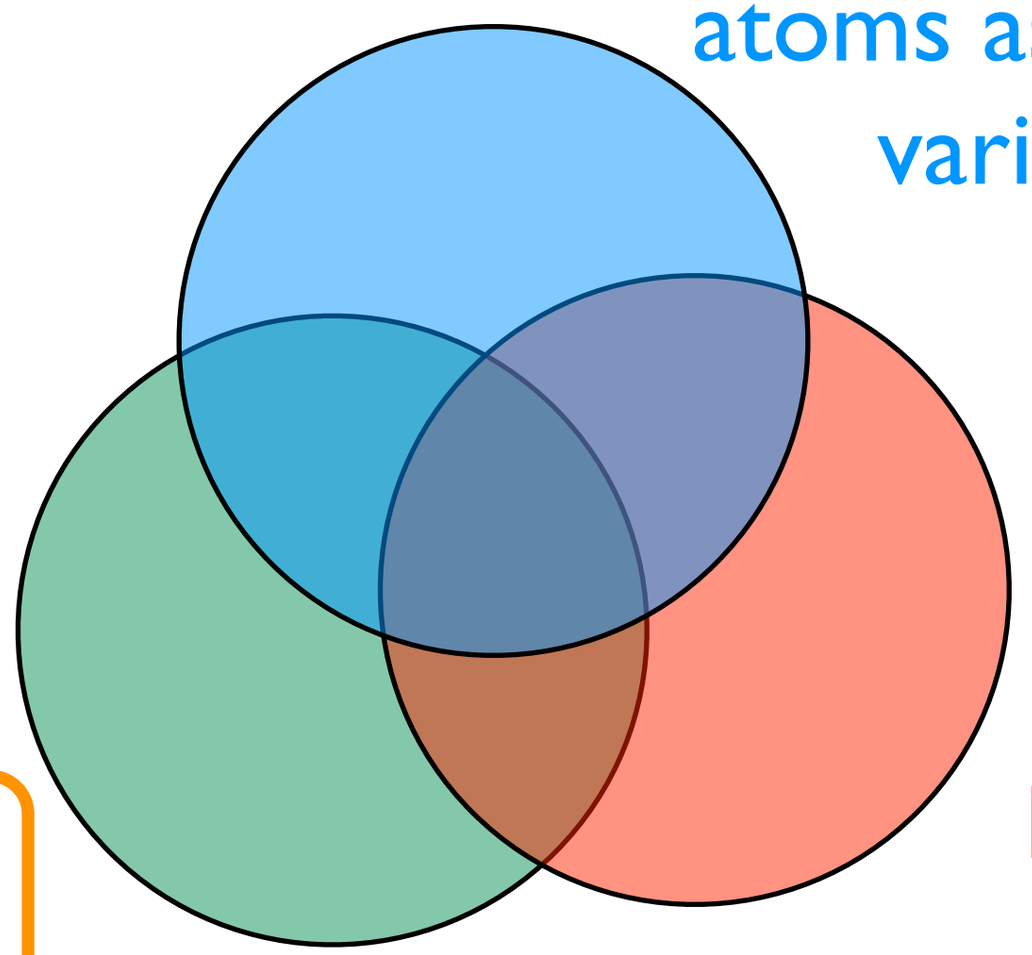
atoms as random variables



Prolog / logic programming

Learning

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several possible worlds

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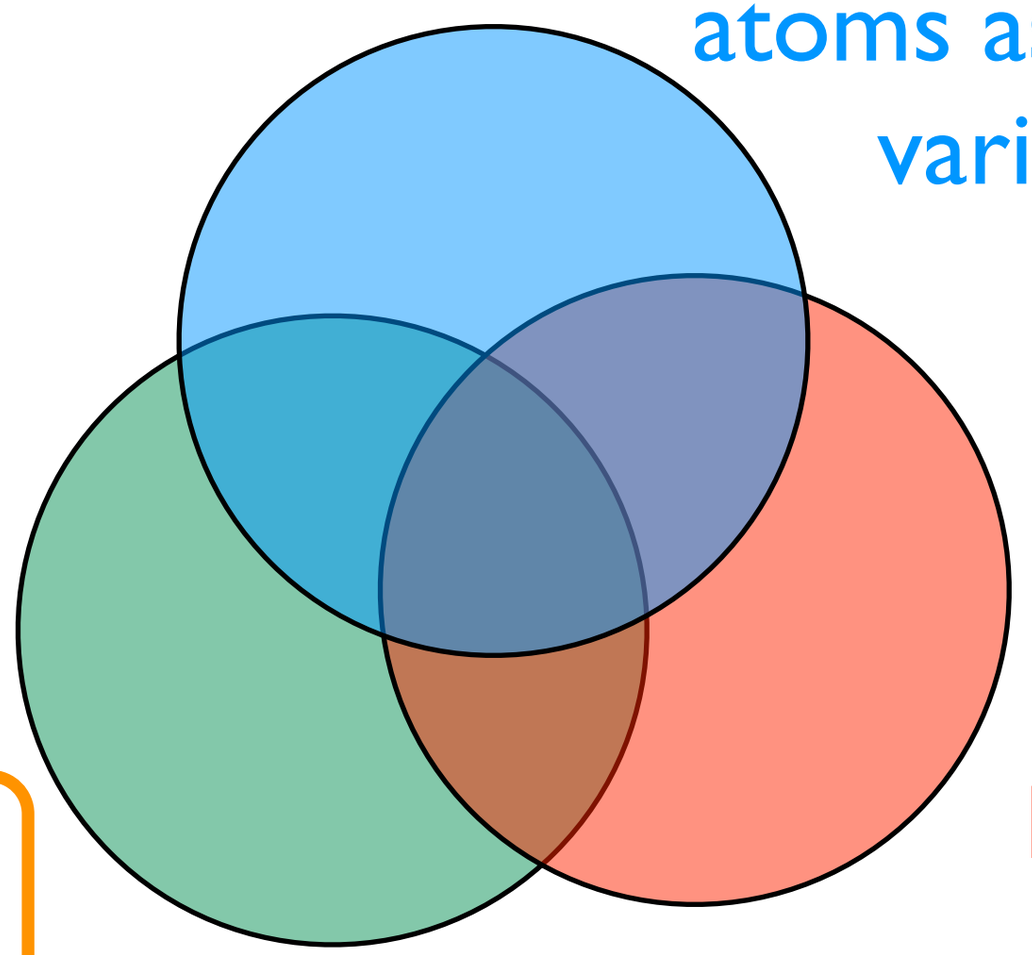

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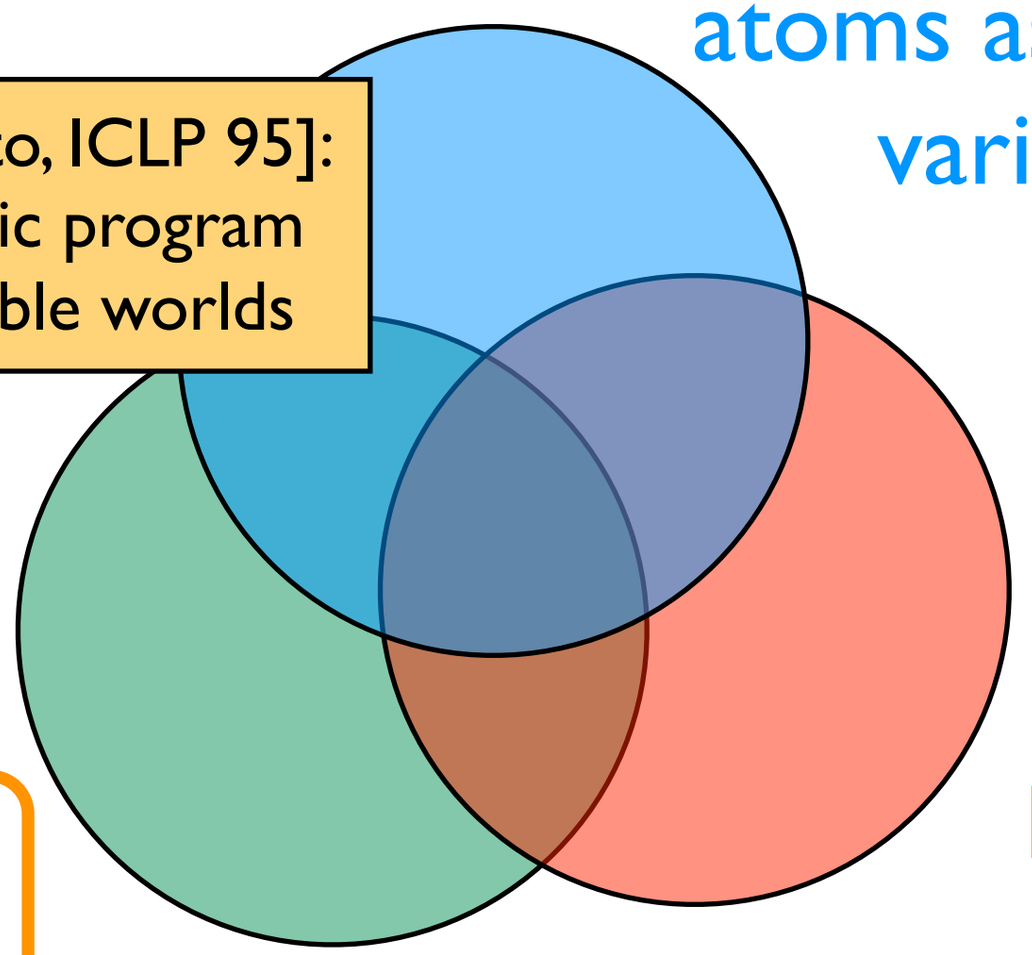
Distribution Semantics [Sato, ICLP 95]:
probabilistic choices + logic program
→ distribution over possible worlds



Prolog / logic programming

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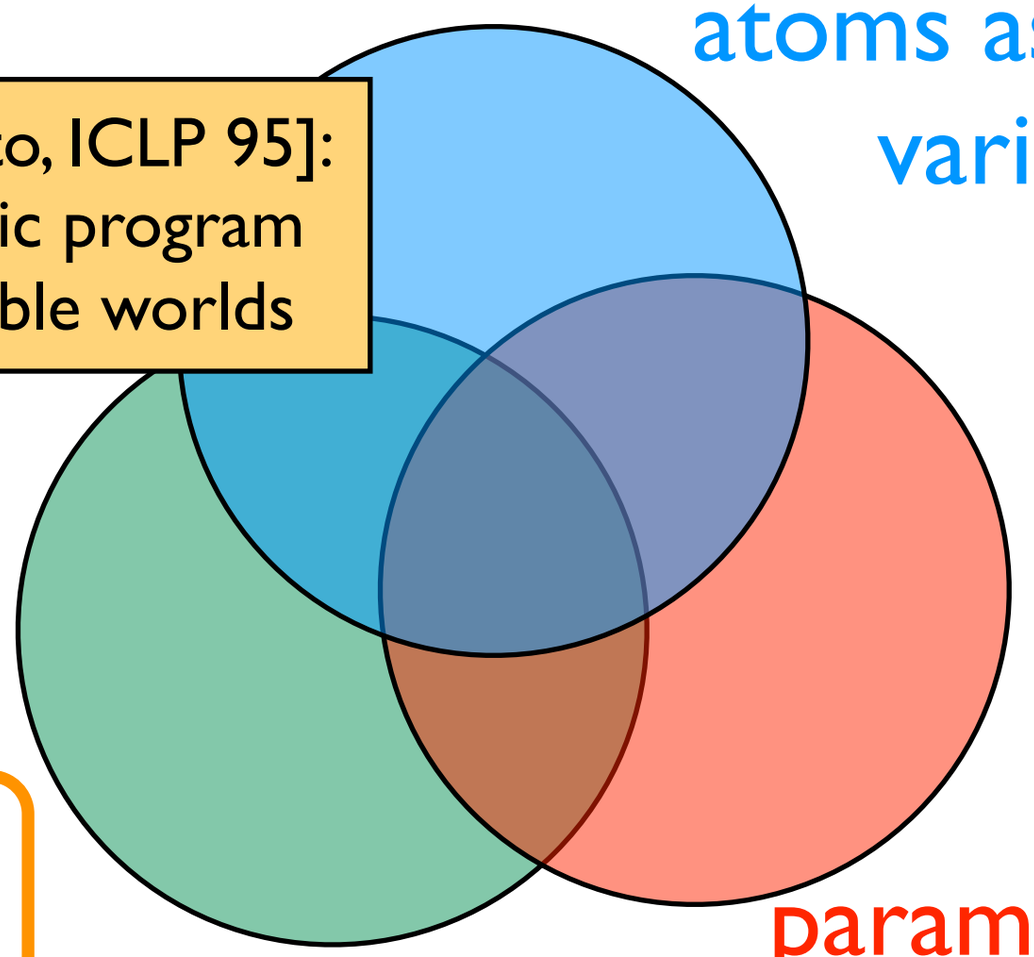

Prolog / logic programming

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parameter learning, adapted relational learning techniques

http://dtai.cs.kuleuven.be/problog/

Probabilistic Prologs: Two Views

- Distribution semantics:
  - probability distribution over interpretations
  - degree of belief
- Stochastic Logic Programs (SLPs):
  - probability distribution over query answers
  - like in probabilistic grammars

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Probabilistic Logic Programming

Distribution Semantics [Sato, ICLP 95]:
probabilistic choices + logic program
→ distribution over possible worlds

e.g., PRISM, ICL, ProbLog, LPADs, CP-logic, ...

multi-valued switches

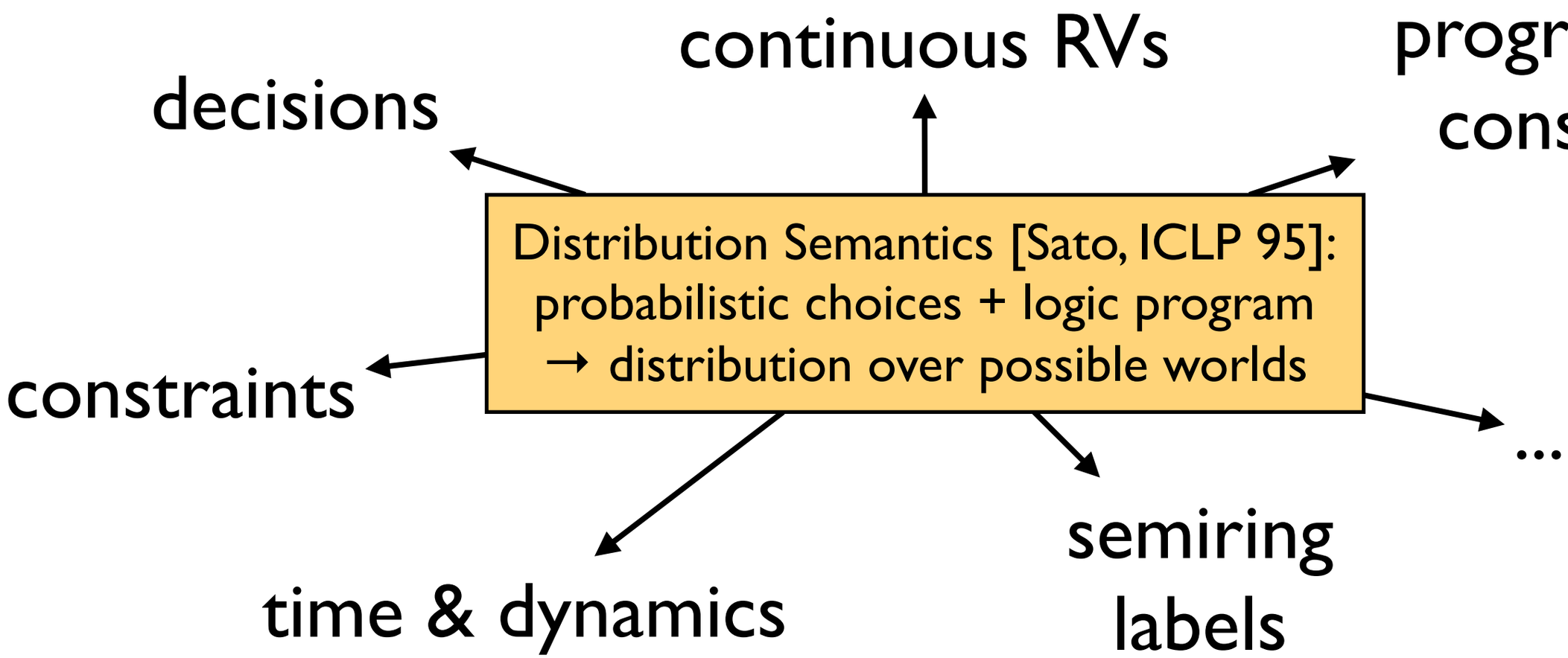
probabilistic alternatives

probabilistic facts

annotated disjunctions

causal-probabilistic laws

Extensions of basic PLP

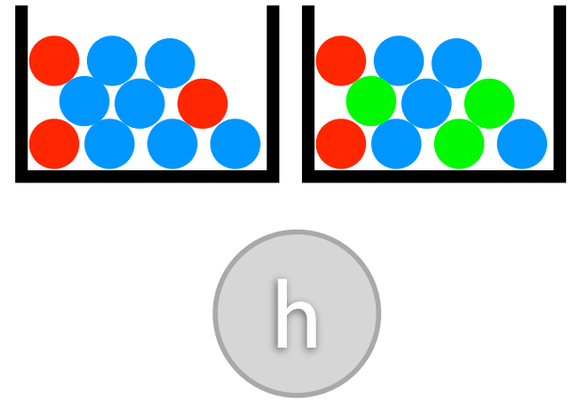


Roadmap

- Modeling (with detours to related work)
- Reasoning (and a bit of learning)
- Language extensions

ProbLog by example:

A bit of gambling



- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

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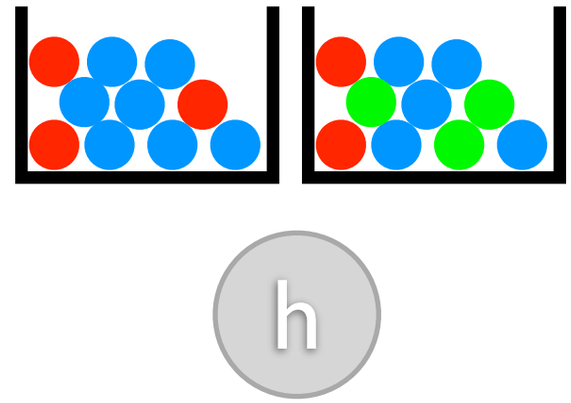
A bit of gambling



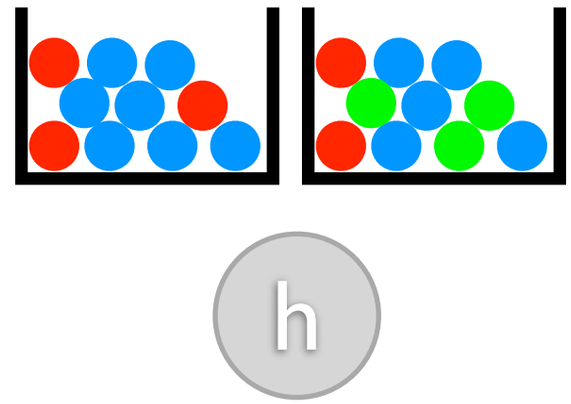
- toss (biased) coin & draw ball from each urn
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.
```

probabilistic fact: heads is true with probability 0.4 (and false with 0.6)

ProbLog by example:

A bit of gambling



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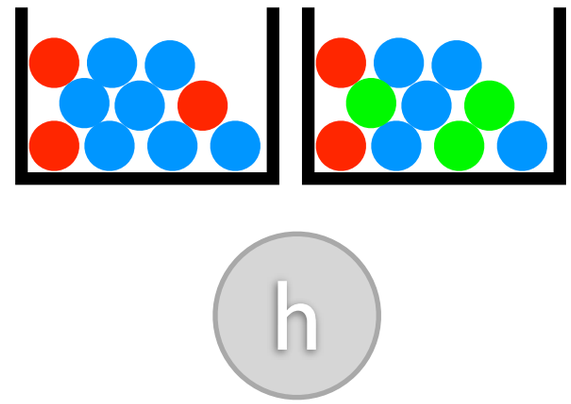
- win if (heads and a red ball) or (two balls of same color)

```
0.4 :: heads.
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
```

annotated disjunction: first ball is red with probability 0.3 and blue with 0.7

ProbLog by example:

A bit of gambling



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0.4 :: heads.

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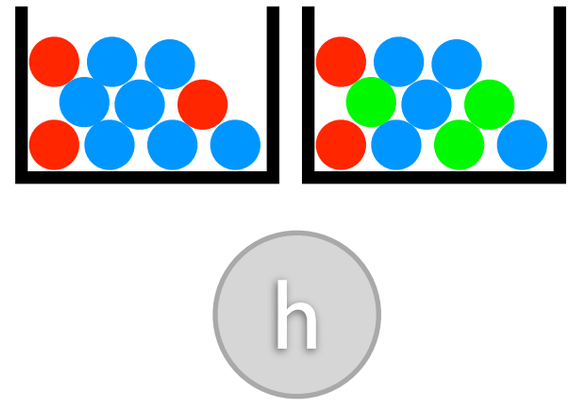
0.2 :: col(2,red); 0.3 :: col(2,green);
                   0.5 :: col(2,blue) <- true.
```

annotated disjunction: second ball is red with probability 0.2, green with 0.3, and blue with 0.5

ProbLog by example:

A bit of gambling



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```

logical rule encoding background knowledge

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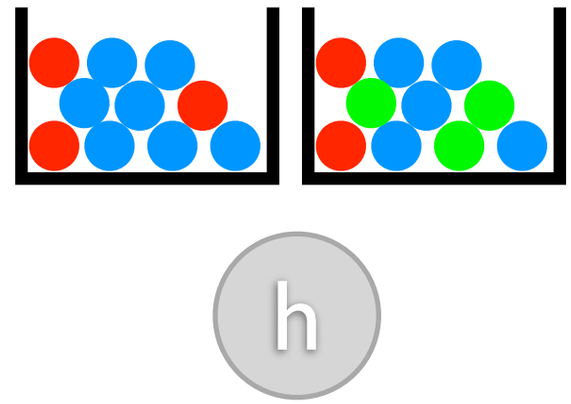

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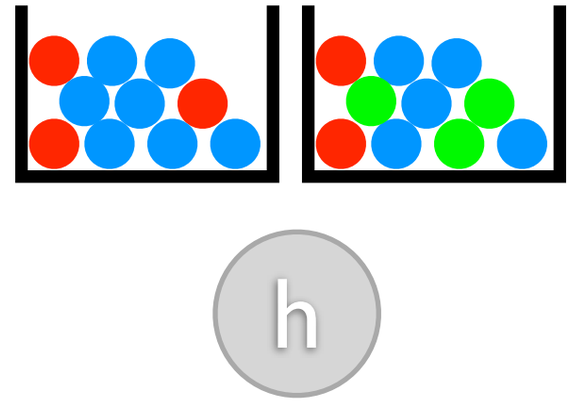

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```

probabilistic choices

```
win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

consequences

Questions

```
0.4 :: heads.

0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
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win :- heads, col(_,red).
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- Probability of **`win`**?
- Probability of **`win`** given **`col(2,green)`**?
- Most probable world where **`win`** is true?

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marginal probability

- Probability of `win`?

query

- Probability of `win` given `col(2,green)`?
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marginal probability

- Probability of **`win`**?

conditional probability

- Probability of **`win`** given **`col(2,green)`**?

evidence

- Most probable world where **`win`** is true?

Questions

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MPE inference

Possible Worlds

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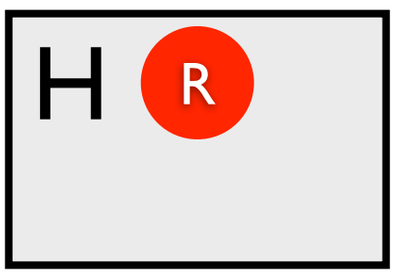
0.4 ×0.3



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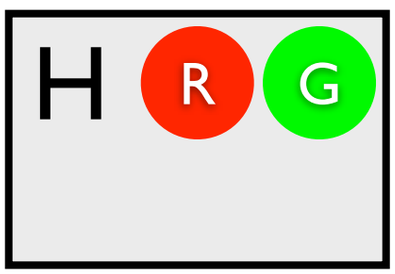

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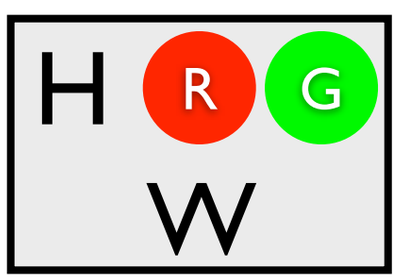

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$0.4 \times 0.3 \times 0.3$ $(1-0.4)$

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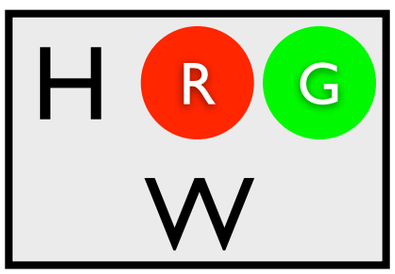

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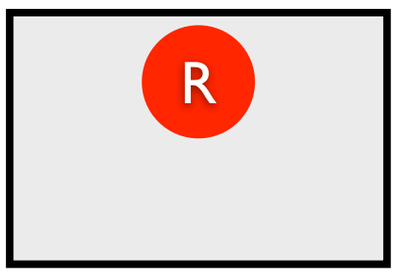

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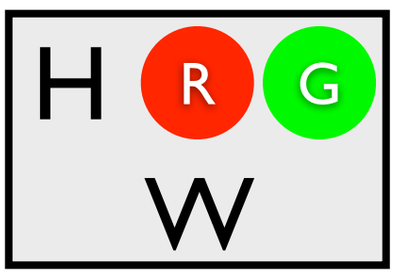

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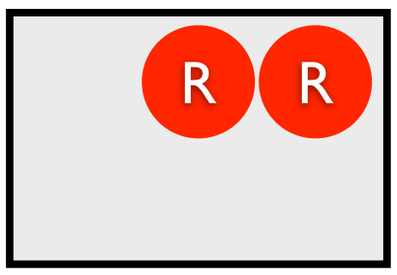

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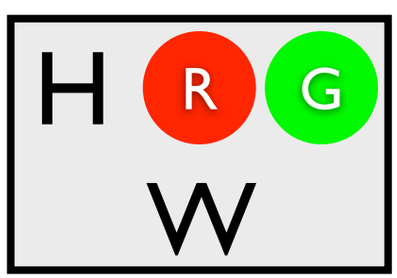

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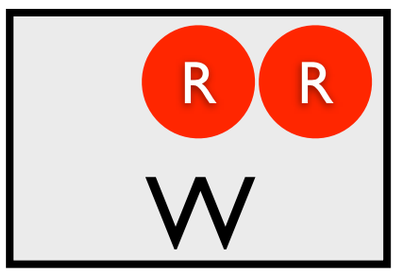

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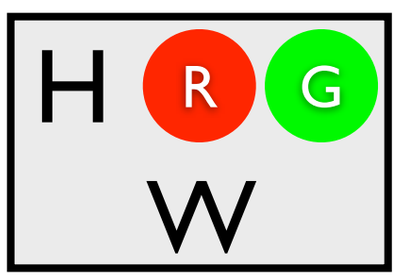

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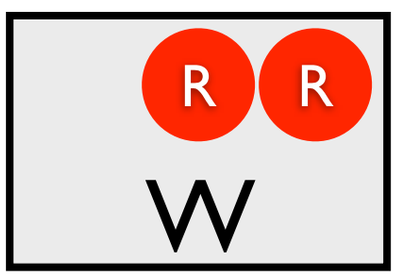

$(1-0.4)$

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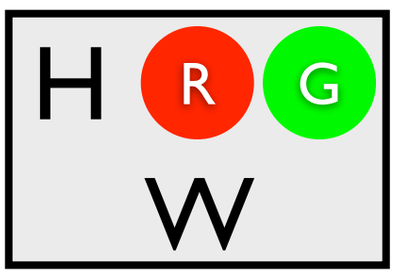

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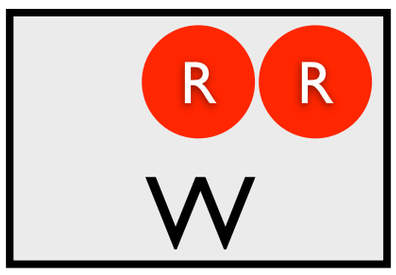

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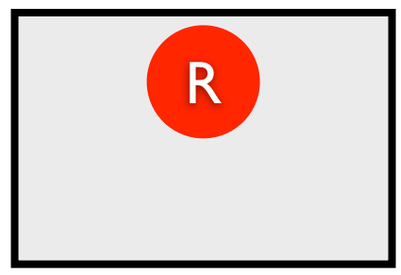

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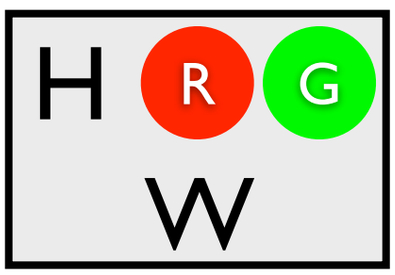
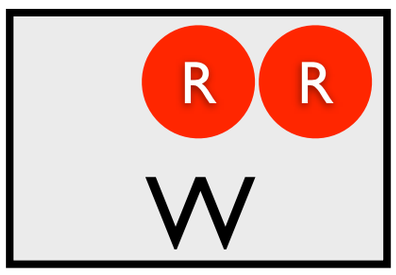
```

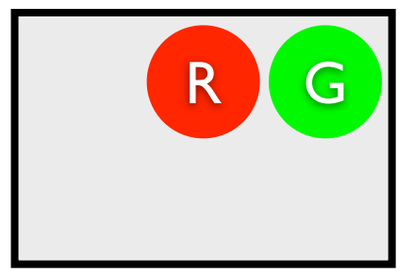
$0.4 \times 0.3 \times 0.3$



$(1-0.4) \times 0.3 \times 0.2$



$(1-0.4) \times 0.3 \times 0.3$



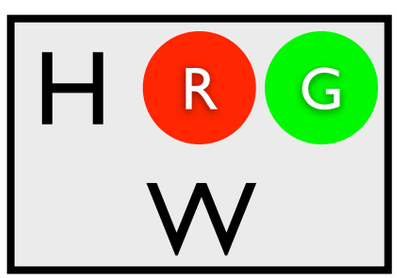
Possible Worlds

```
0.4 :: heads.

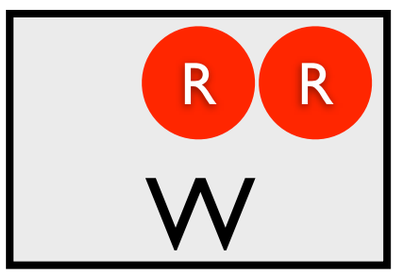
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue) <- true.

win :- heads, col(_,red).
win :- col(1,C), col(2,C).
```

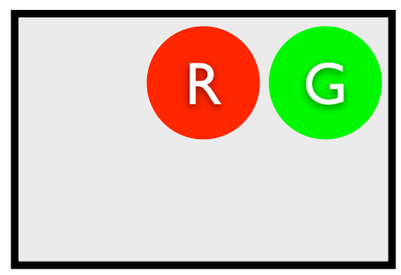
0.4 ×0.3 ×0.3



(1−0.4)×0.3 ×0.2



(1−0.4)×0.3 ×0.3



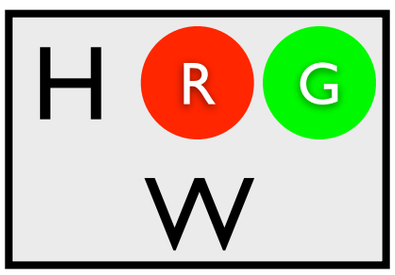
Possible Worlds

```
0.4 :: heads.

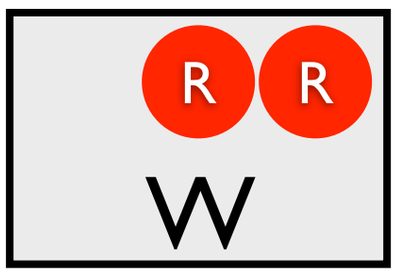
0.3 :: col(1,red); 0.7 :: col(1,blue) <- true.
0.2 :: col(2,red); 0.3 :: col(2,green); 0.5 :: col(2,blue) <- true.

win :- heads, col( ,red).
win :- col(1,C), col(2,C).
```

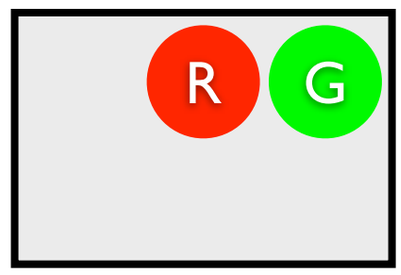
0.4 ×0.3 ×0.3



(1−0.4)×0.3 ×0.2

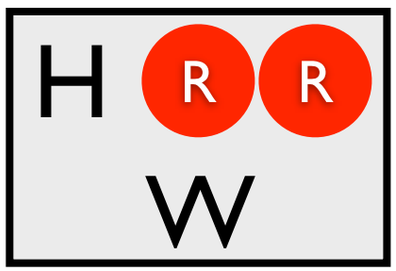


(1−0.4)×0.3 ×0.3

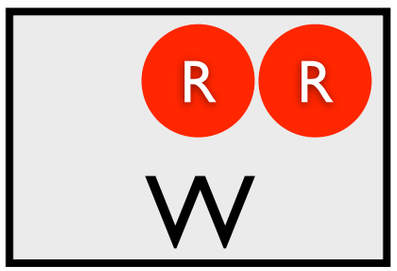


All Possible Worlds

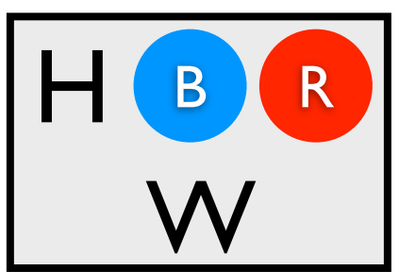
0.024



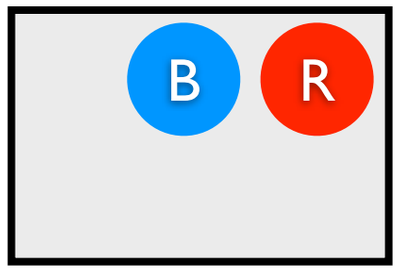
0.036



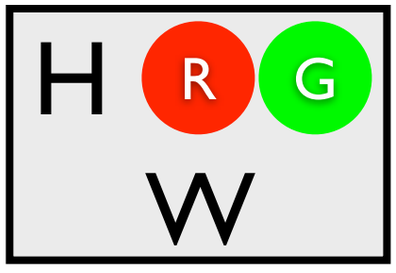
0.056



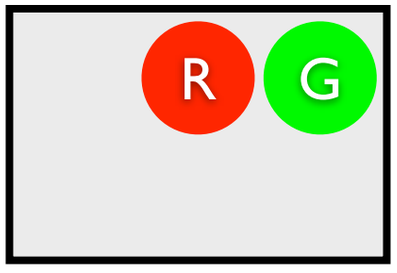
0.084



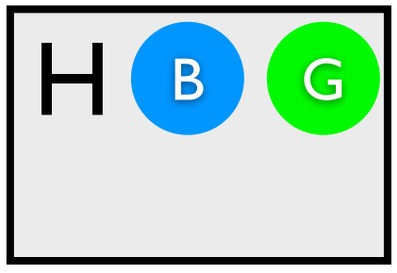
0.036



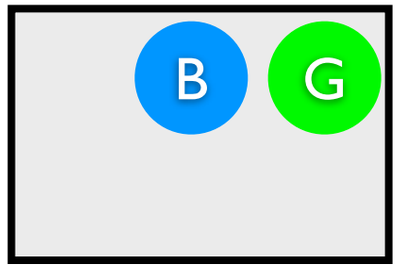
0.054



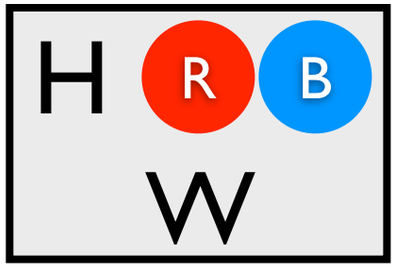
0.084



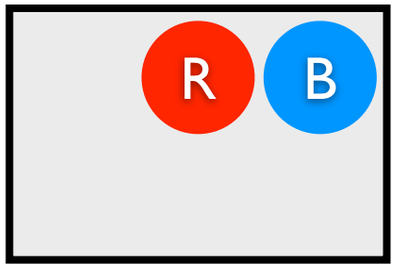
0.126



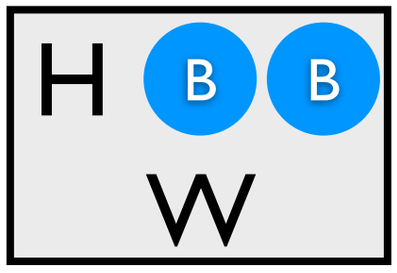
0.060



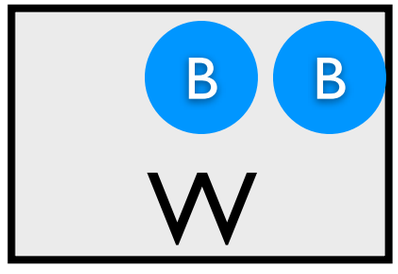
0.090



0.140

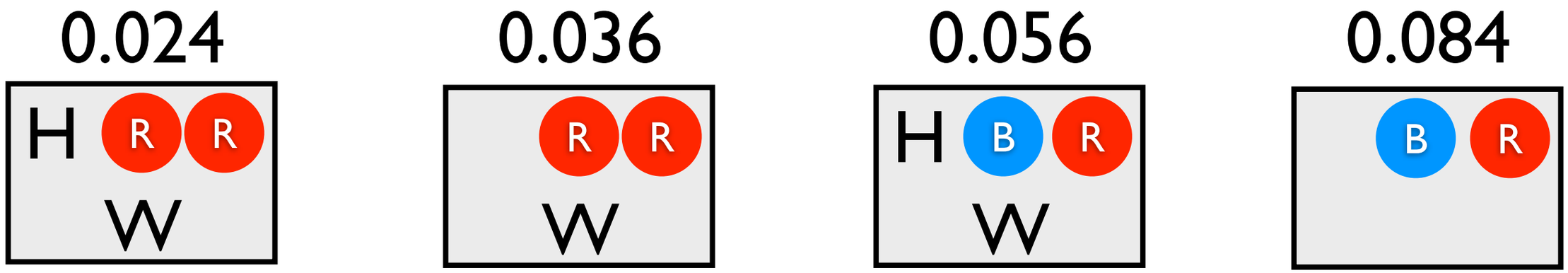


0.210



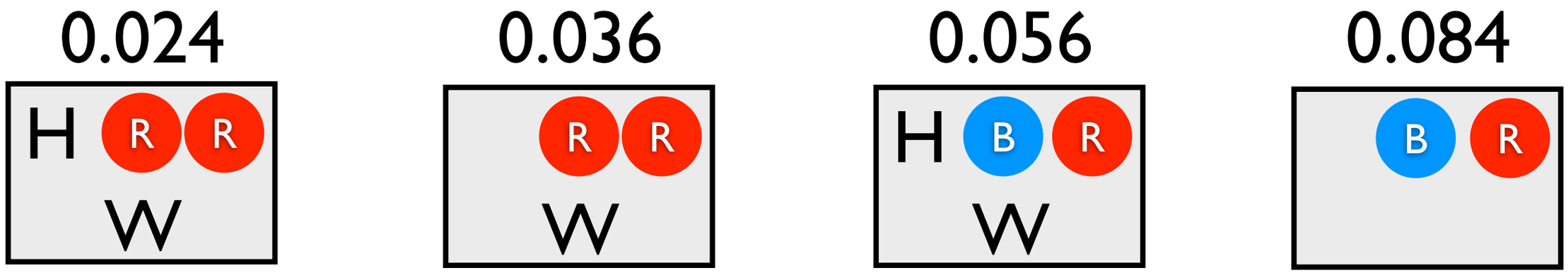
Most likely world where **win** is true?

MPE Inference



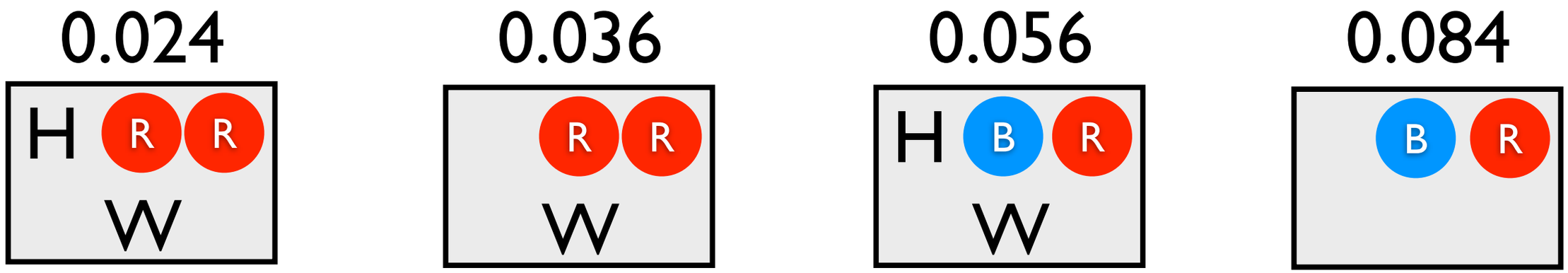
Most likely world where **win** is true?

MPE Inference



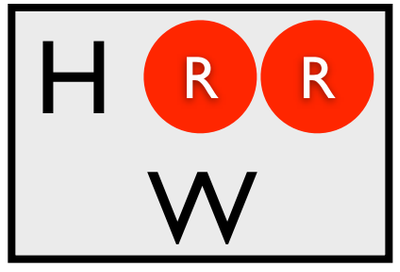
Most likely world where `col(2,blue)` is false?

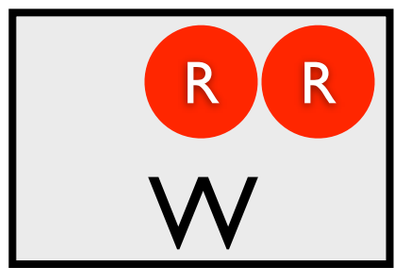
MPE Inference

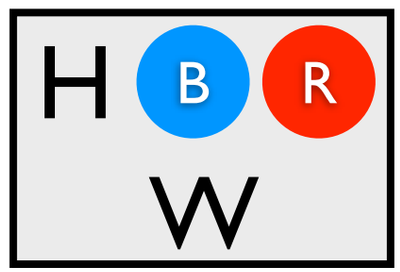


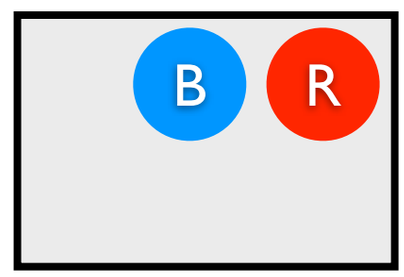
Most likely world where `col(2,blue)` is false?

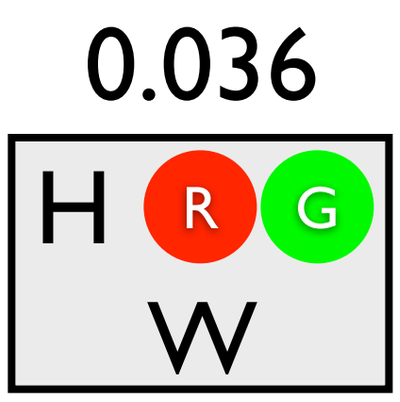
MPE Inference

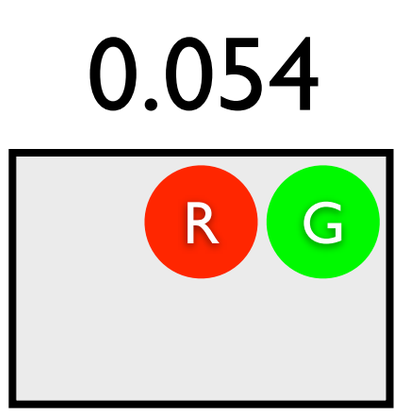


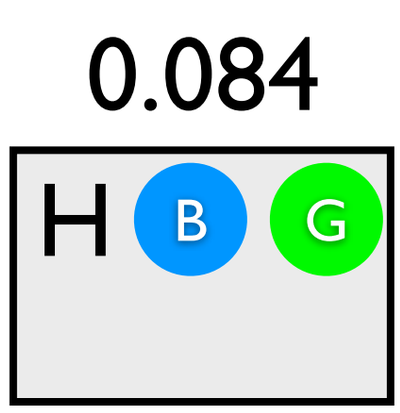


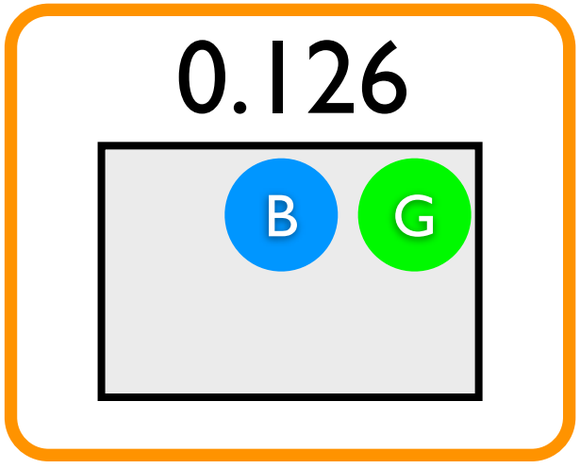


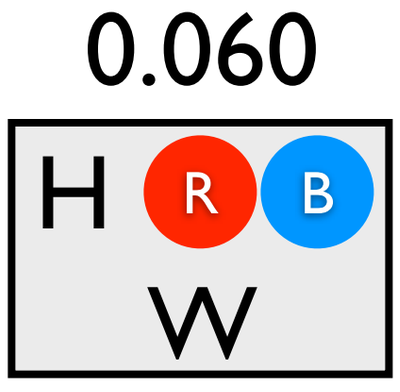


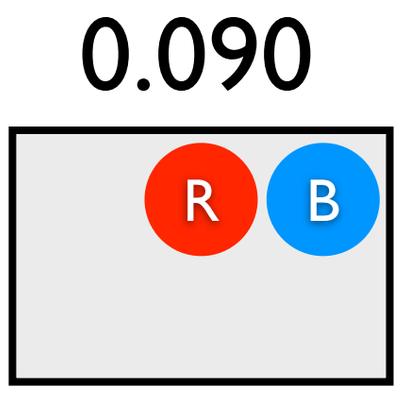


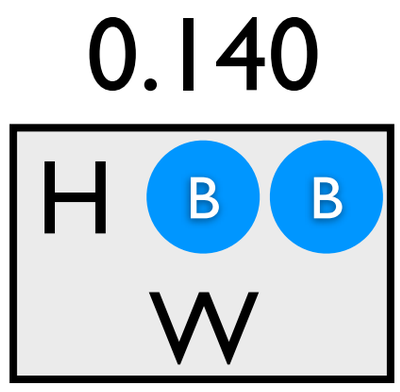


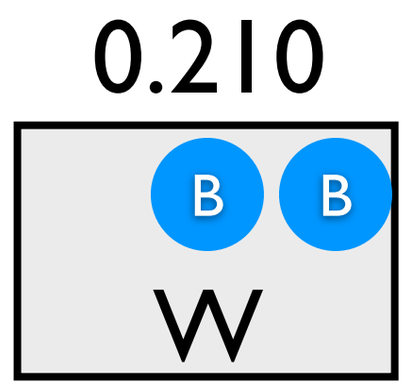






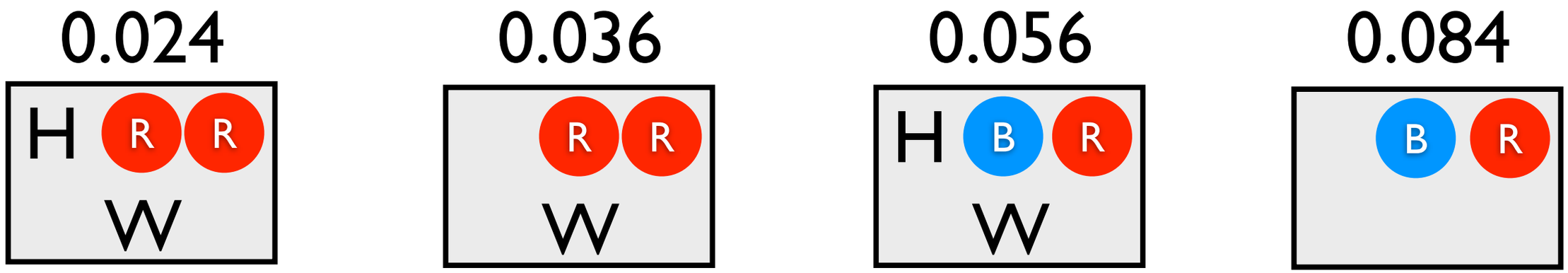






P(win)=?

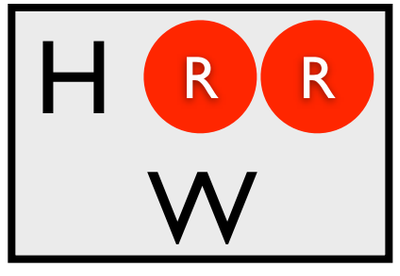
Marginal Probability



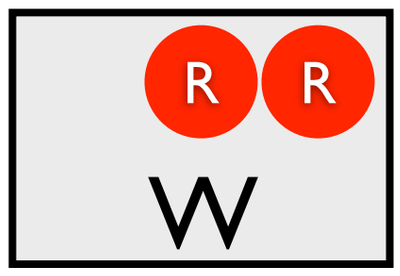
P(win)= Σ

Marginal Probability

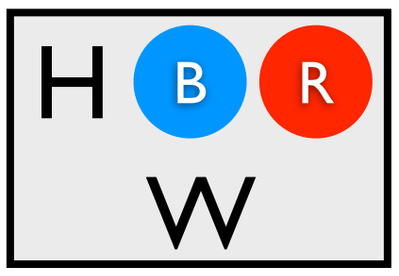
0.024



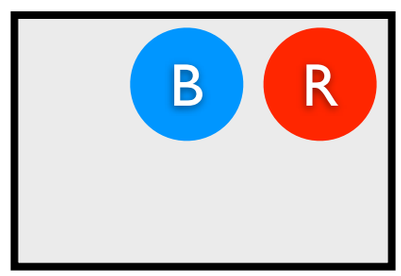
0.036



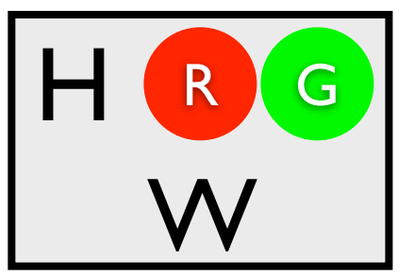
0.056



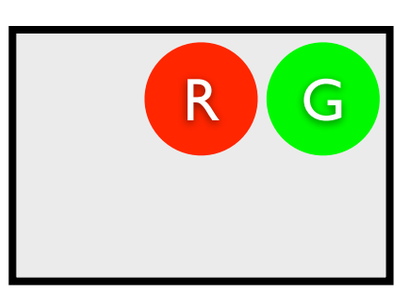
0.084



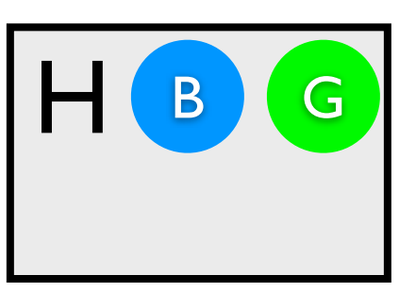
0.036



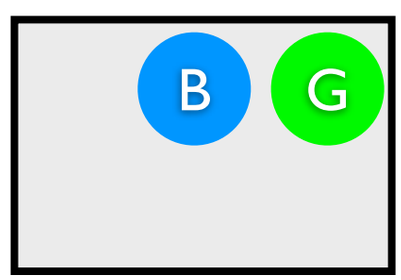
0.054



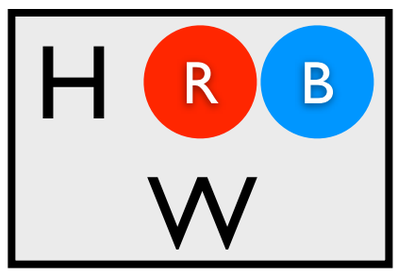
0.084



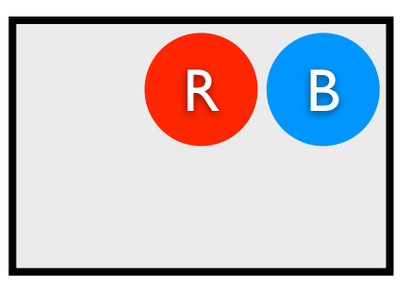
0.126



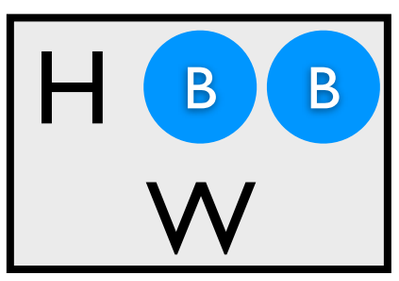
0.060



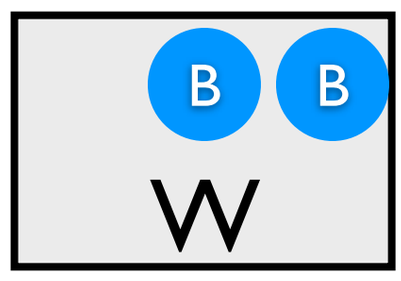
0.090



0.140



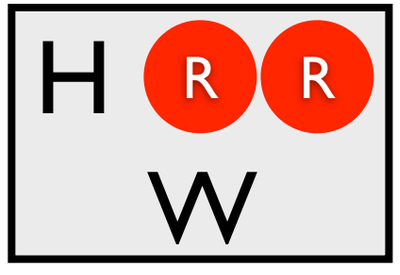
0.210



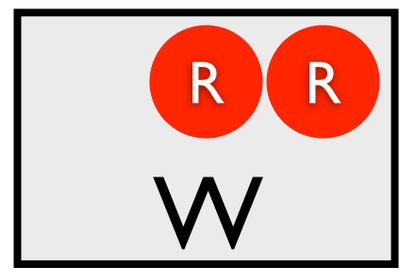
P(win)= Σ =0.562

Marginal Probability

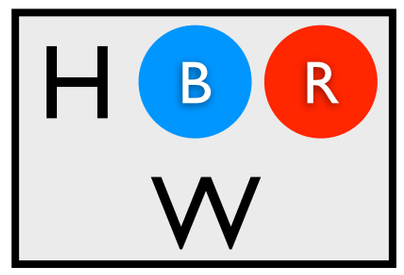
0.024



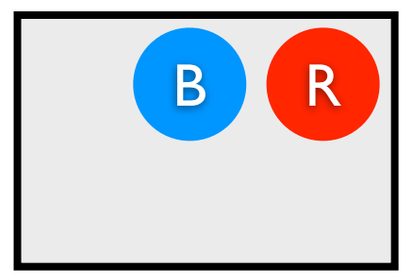
0.036



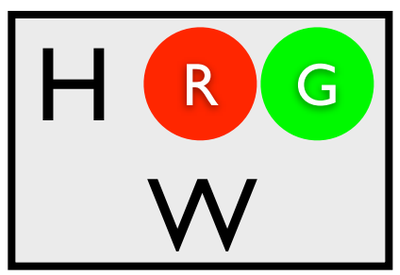
0.056



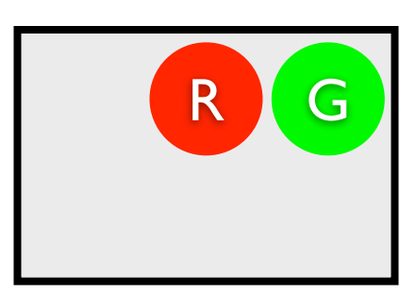
0.084



0.036



0.054



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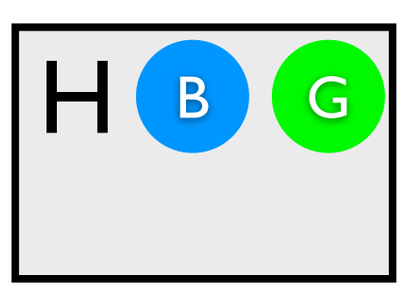
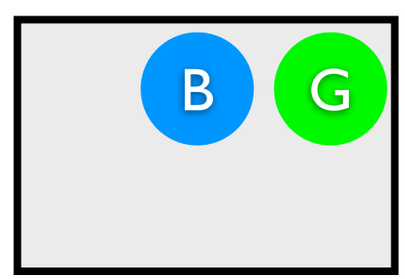
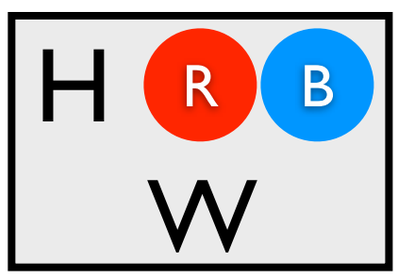


0.126



0.060



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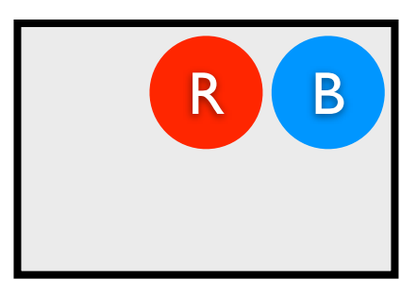
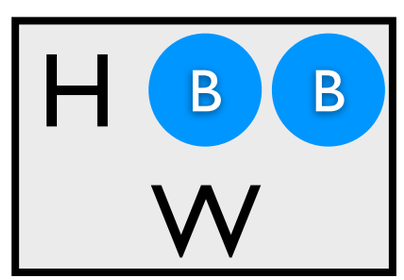
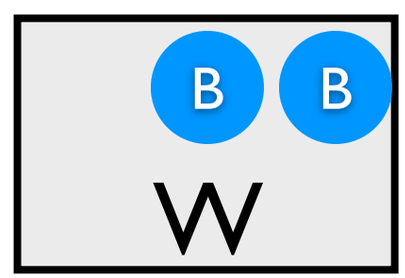


0.140



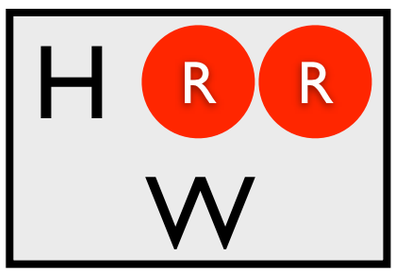
0.210



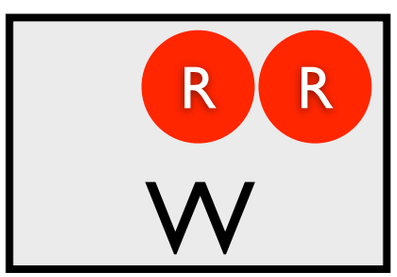
P(win|col(2,green))= ?

Conditional Probability

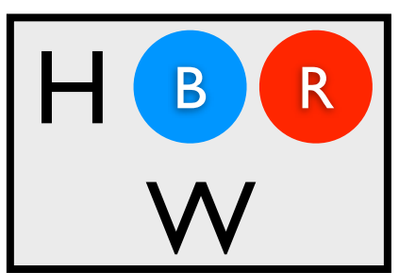
0.024



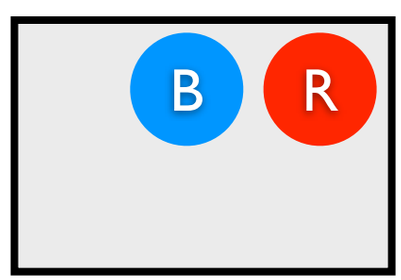
0.036



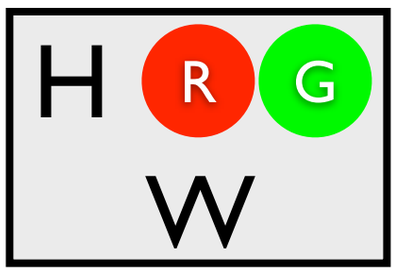
0.056



0.084



0.036



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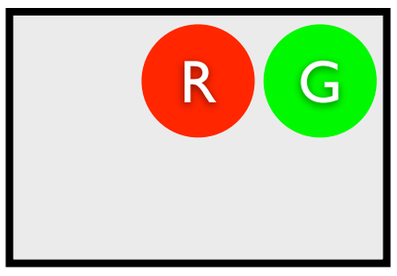
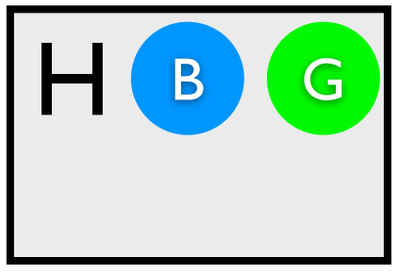
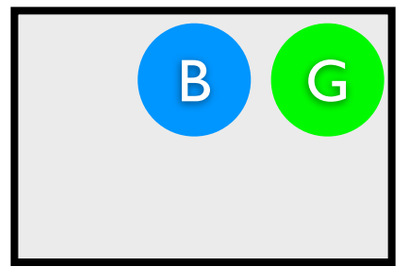


0.084



0.126



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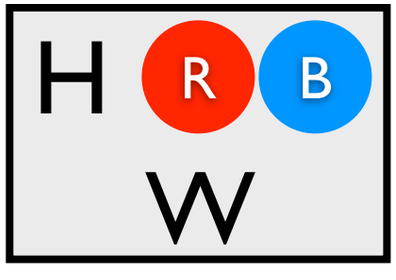
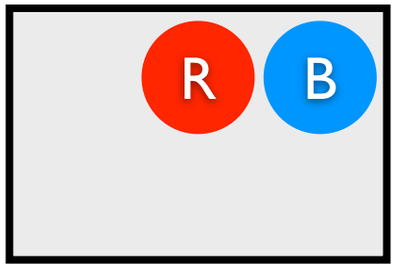
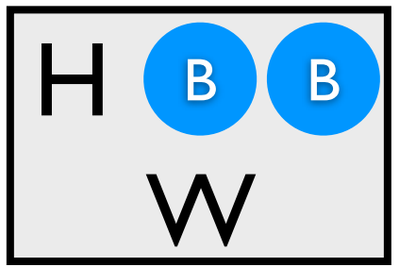


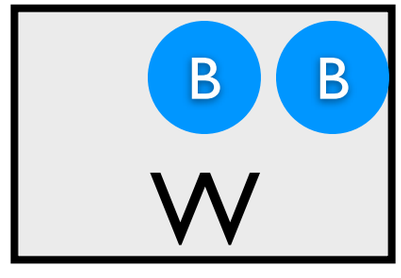
0.090



0.140



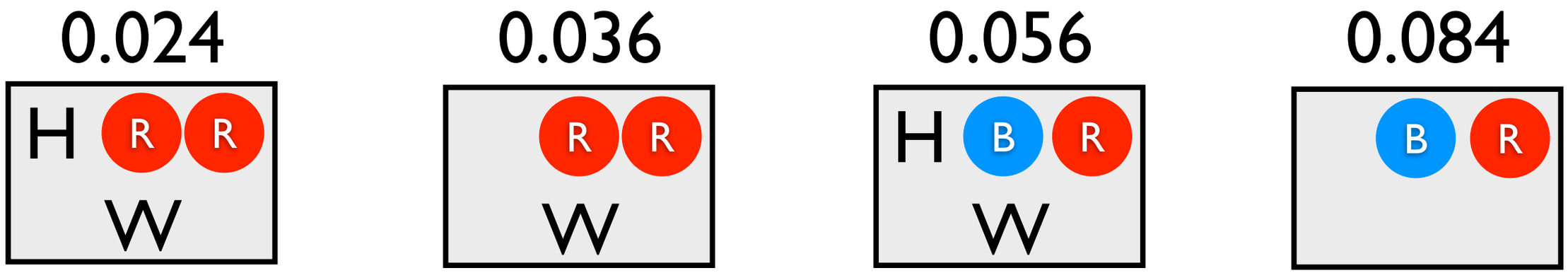
0.210



$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

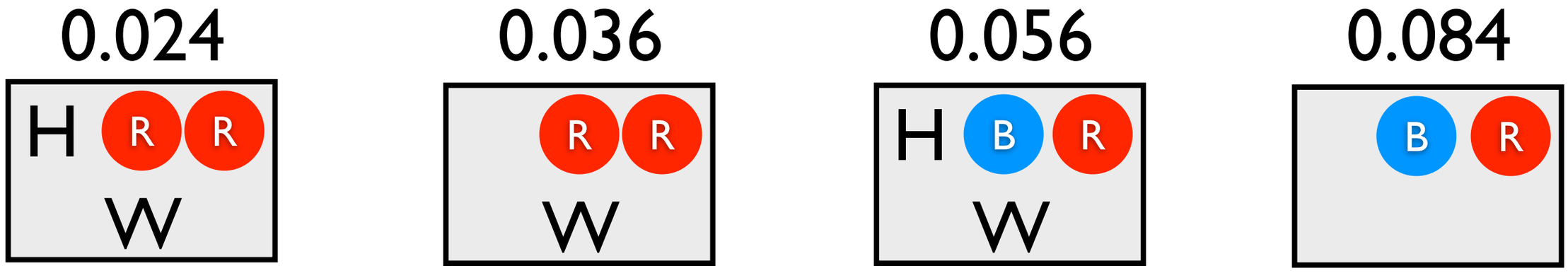
Conditional Probability



$$P(win|col(2,green)) = \Sigma/\Sigma$$

$$= P(win \wedge col(2,green))/P(col(2,green))$$

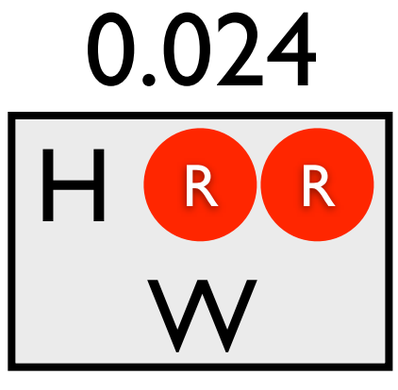
Conditional Probability

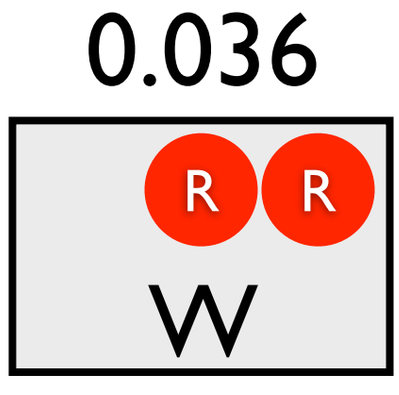


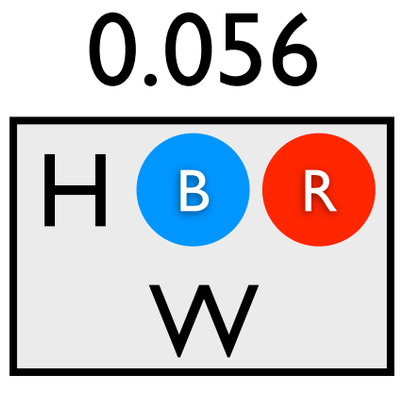
P(win|col(2,green))= ∑/∑

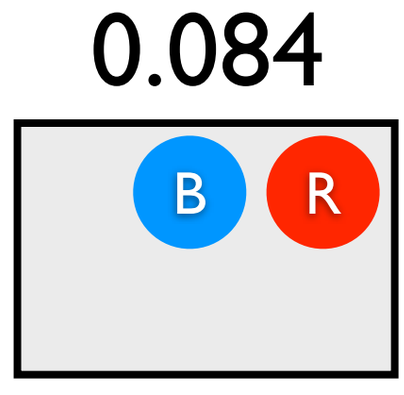
=0.036/0.3=0.12

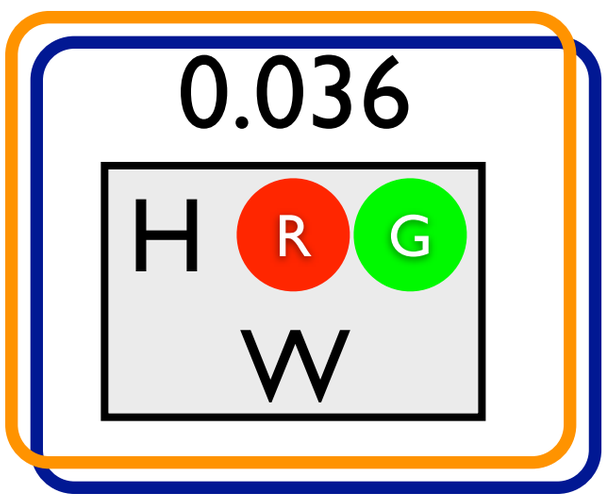
Conditional Probability

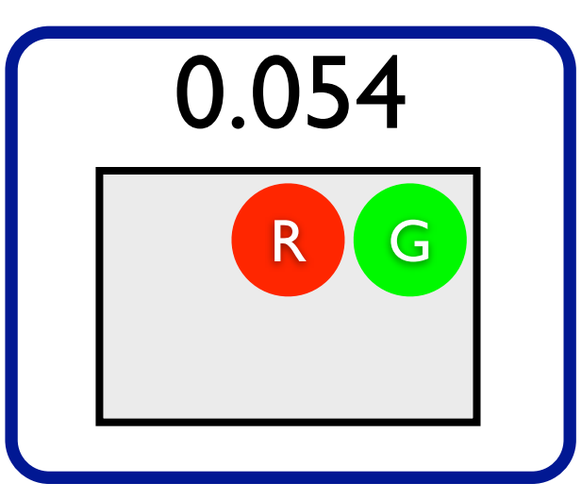


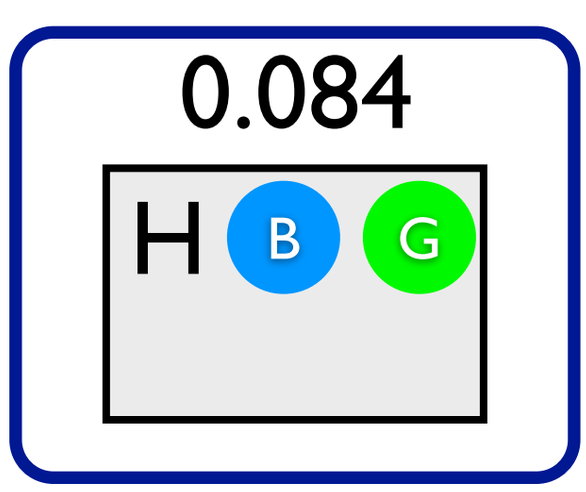


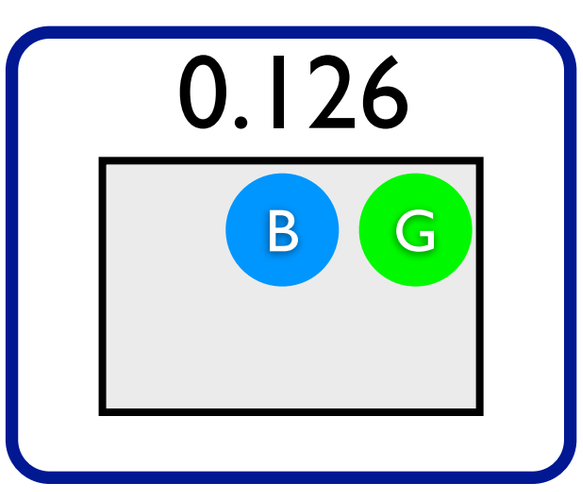


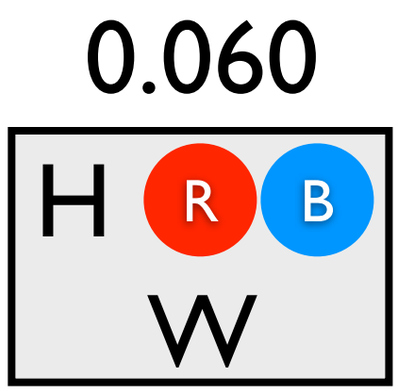


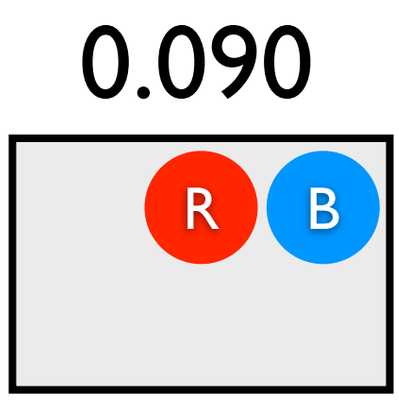


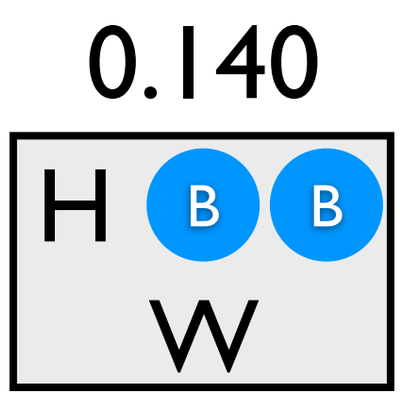


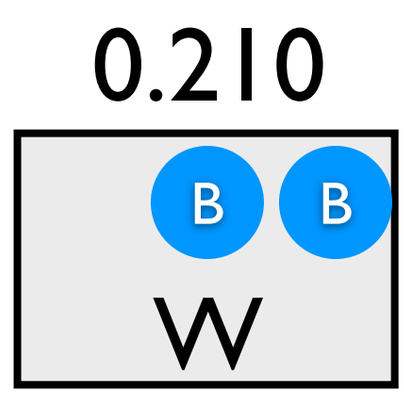












Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

sum over possible worlds where Q is true

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

Distribution Semantics

(with probabilistic facts)

[Sato, ICLP 95]

query

sum over possible worlds where Q is true

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

subset of probabilistic facts

Prolog rules

probability of possible world

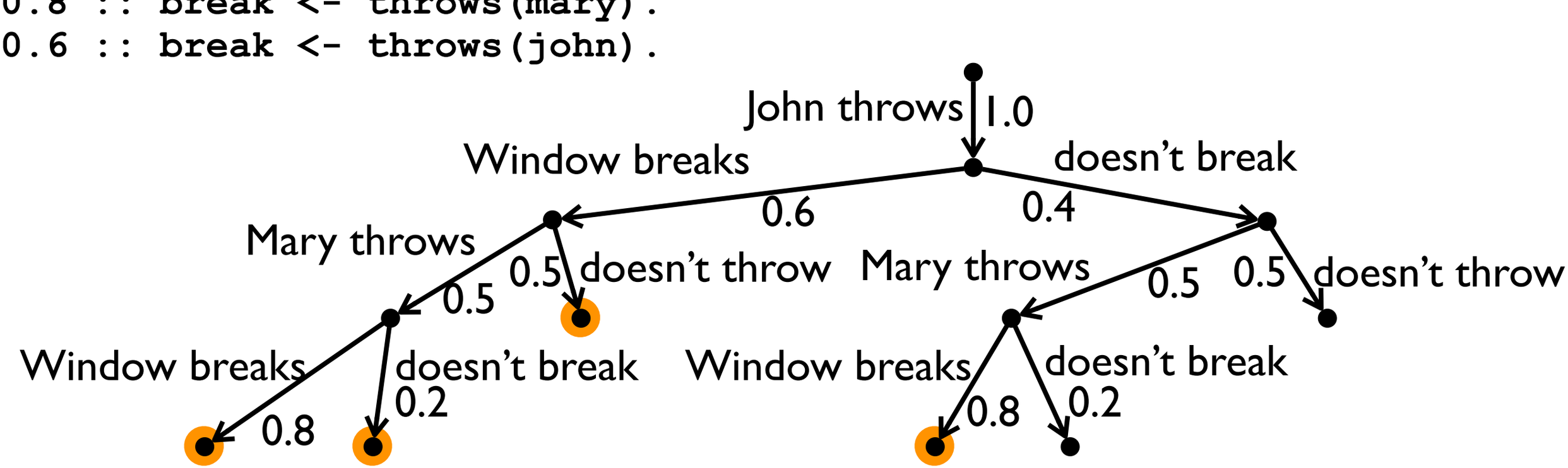
[Vennekens et al, ICLP 04]

Alternative view: CP-Logic

```
throws(john).
0.5::throws(mary).

0.8 :: break <- throws(mary).
0.6 :: break <- throws(john).
```

probabilistic causal laws



P(break)=0.6×0.5×0.8+0.6×0.5×0.2+0.6×0.5+0.4×0.5×0.8

Answering Questions

Given:

program

queries

evidence

?

Find:

marginal probabilities

conditional probabilities

MPE state

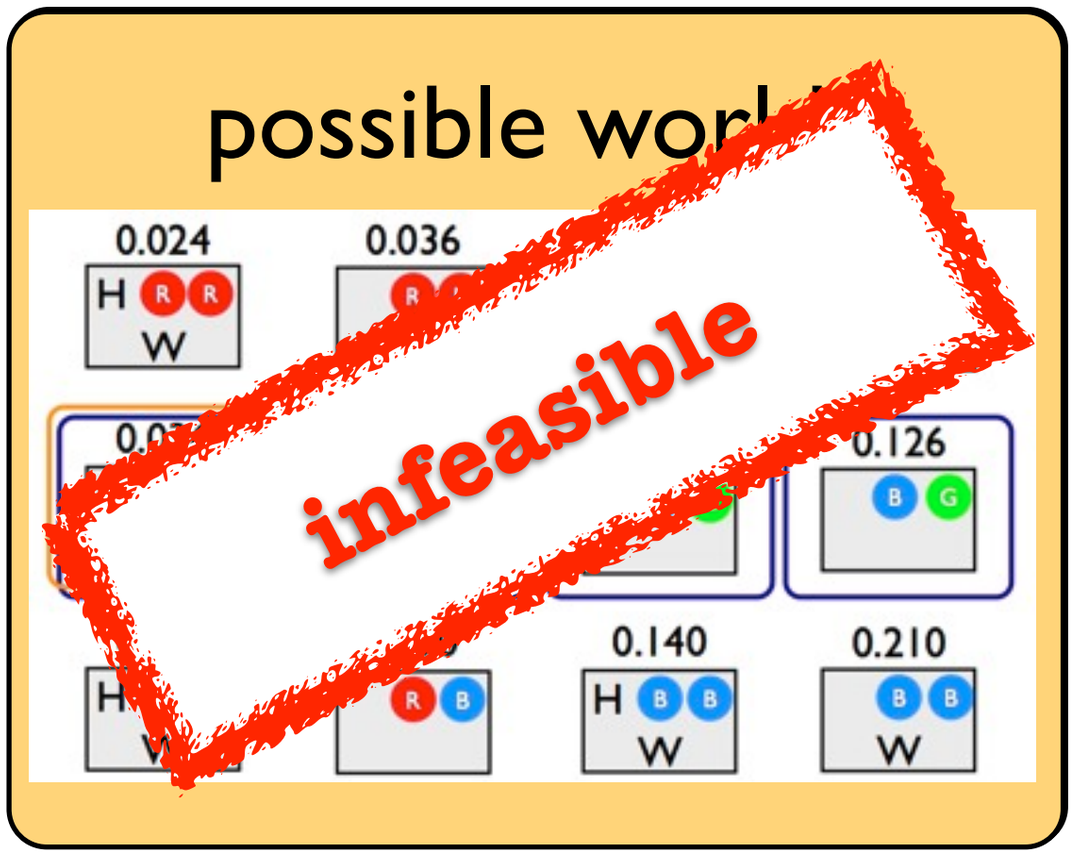
Answering Questions

Given:

program

queries

evidence



Find:

marginal probabilities

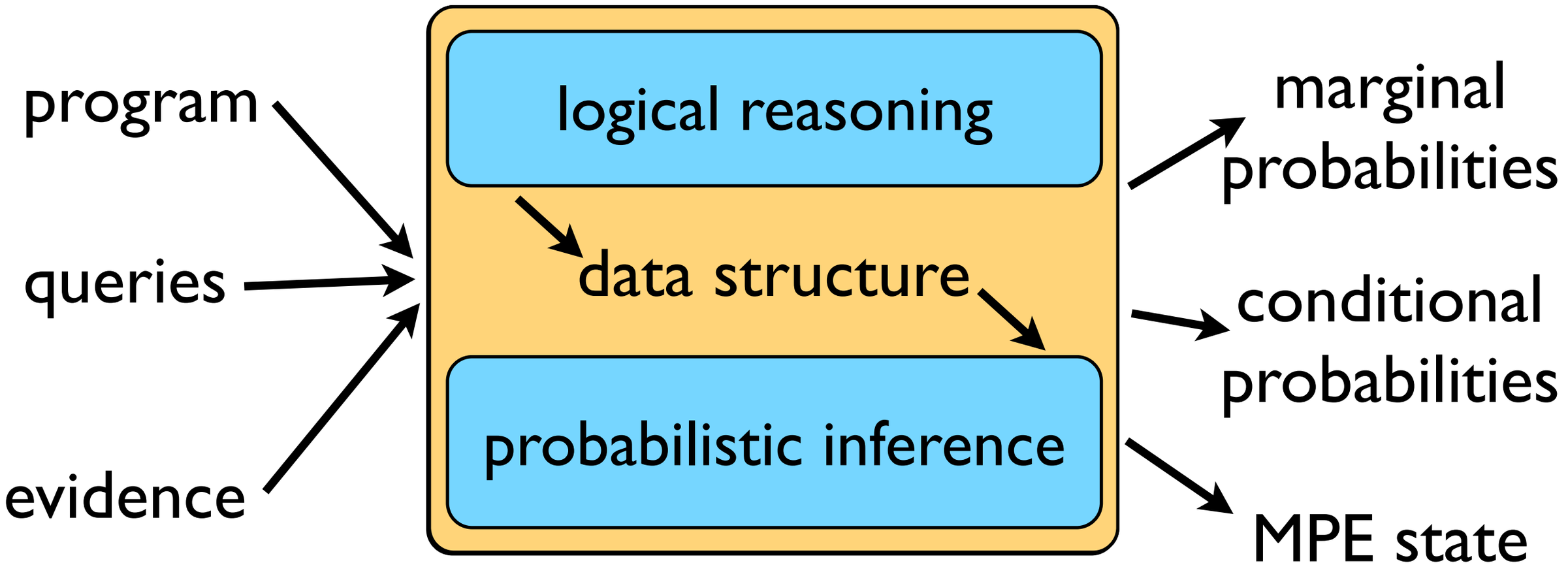
conditional probabilities

MPE state

Answering Questions

Given:

Find:

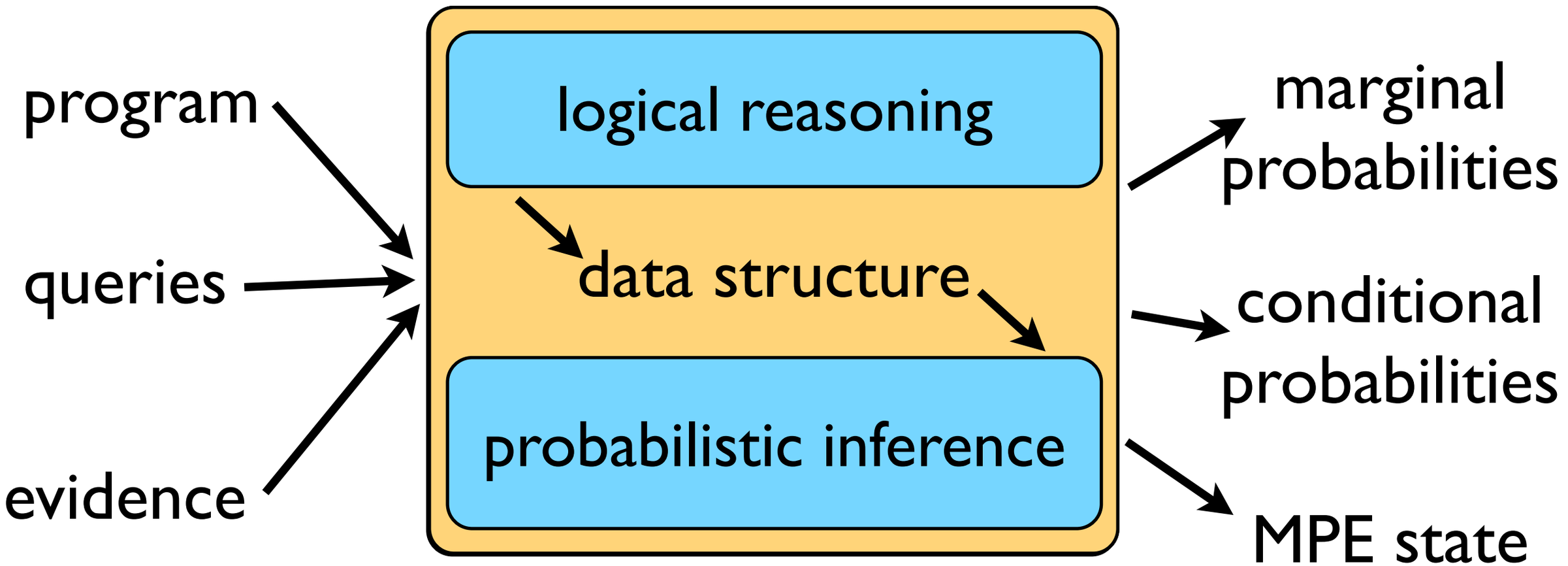


Answering Questions

more on this later

Given:

Find:



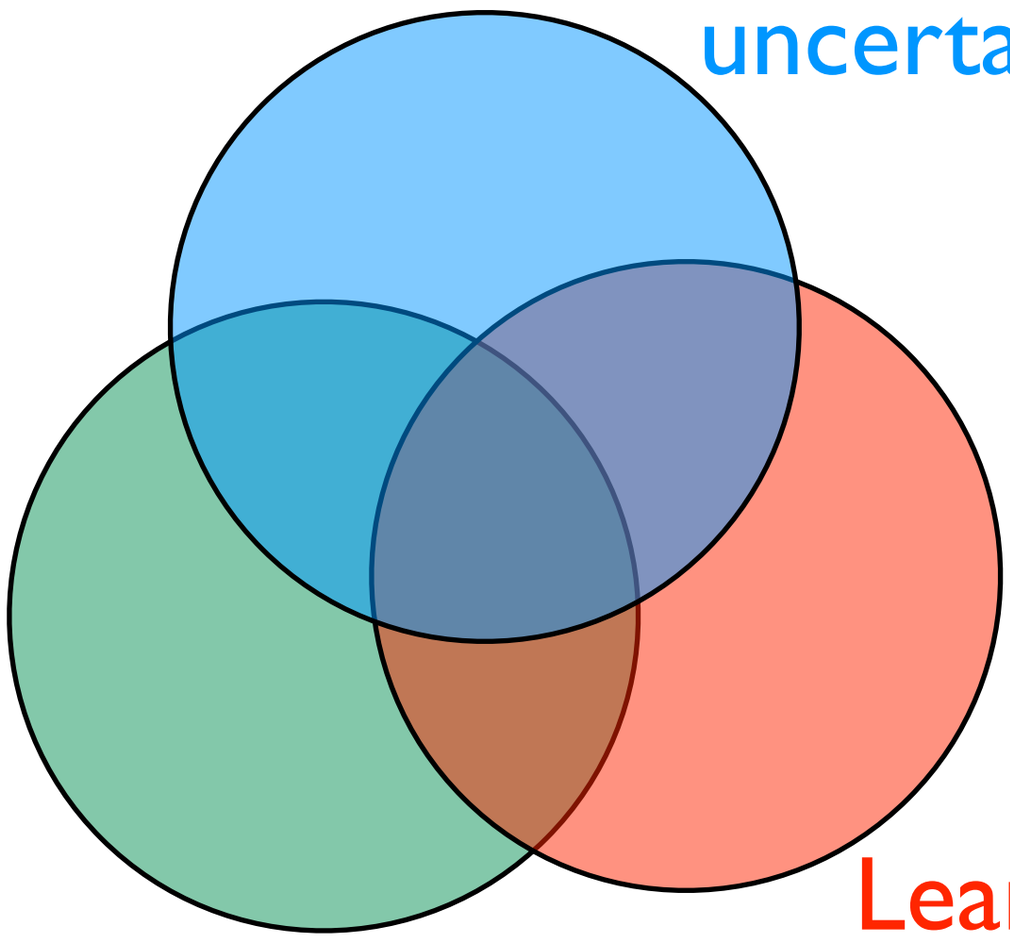
[Suciu et al 2011]

Probabilistic Databases

Dealing with uncertainty

Reasoning with relational data

Learning



[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

bornIn

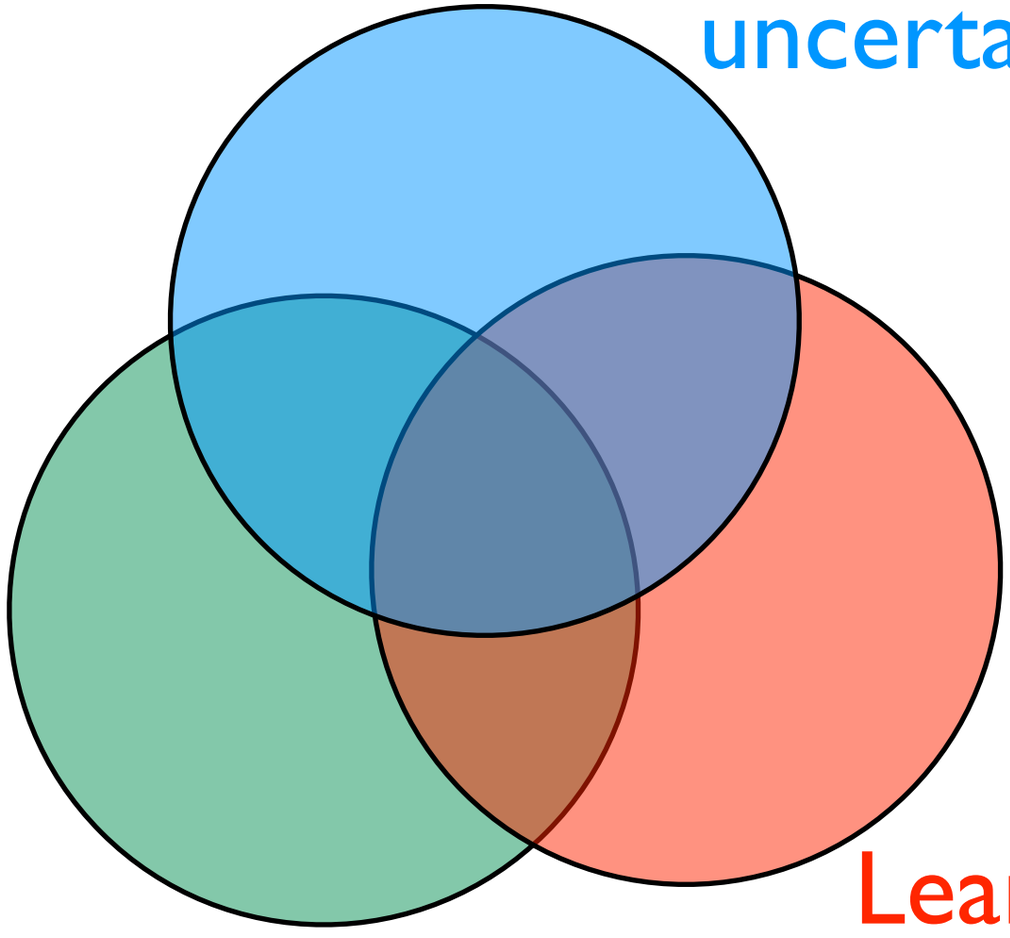
person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database

Dealing with uncertainty



Learning

[Suciu et al 2011]

Probabilistic Databases

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

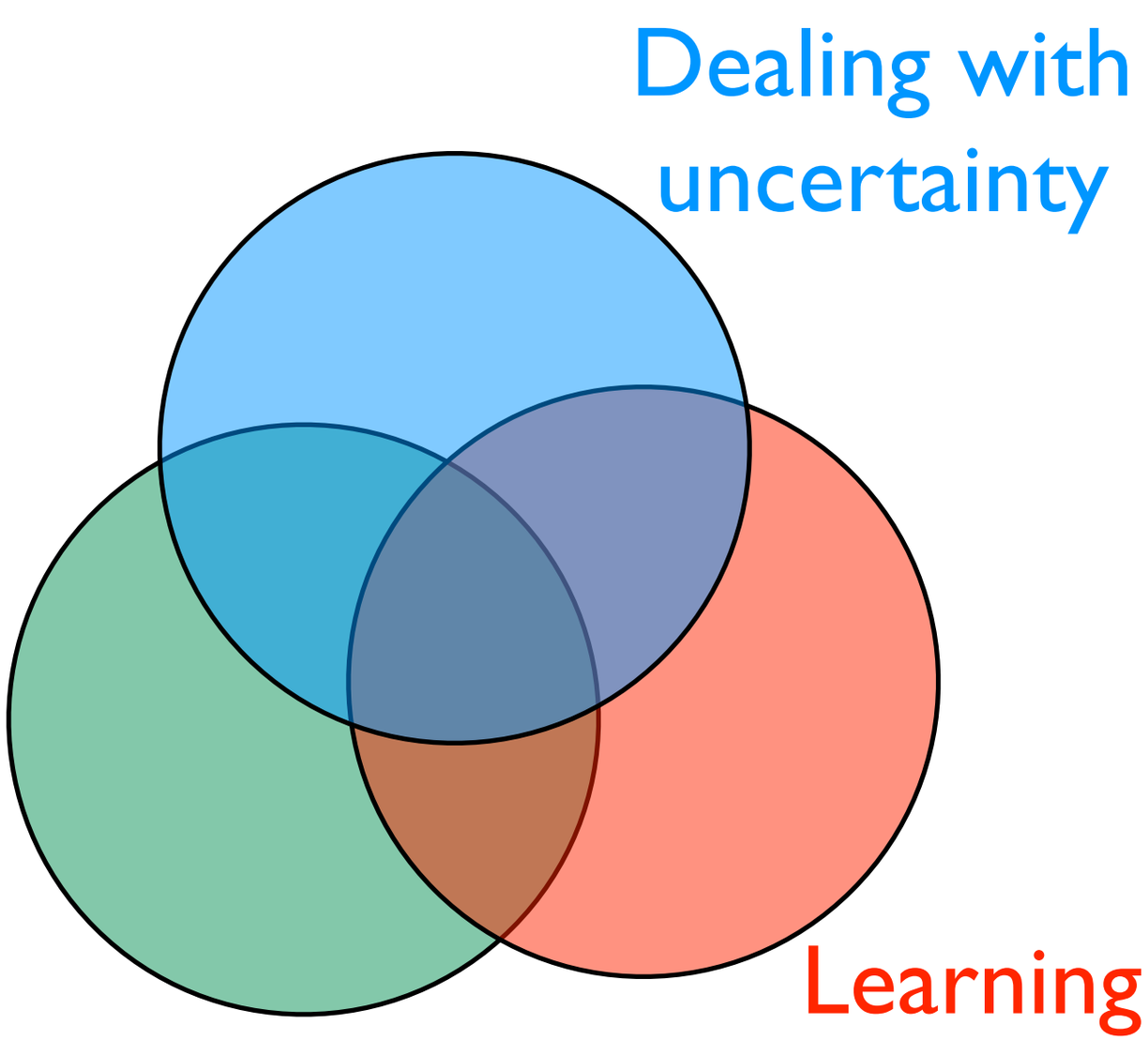
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



[Suciu et al 2011]

Probabilistic Databases

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random variables

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

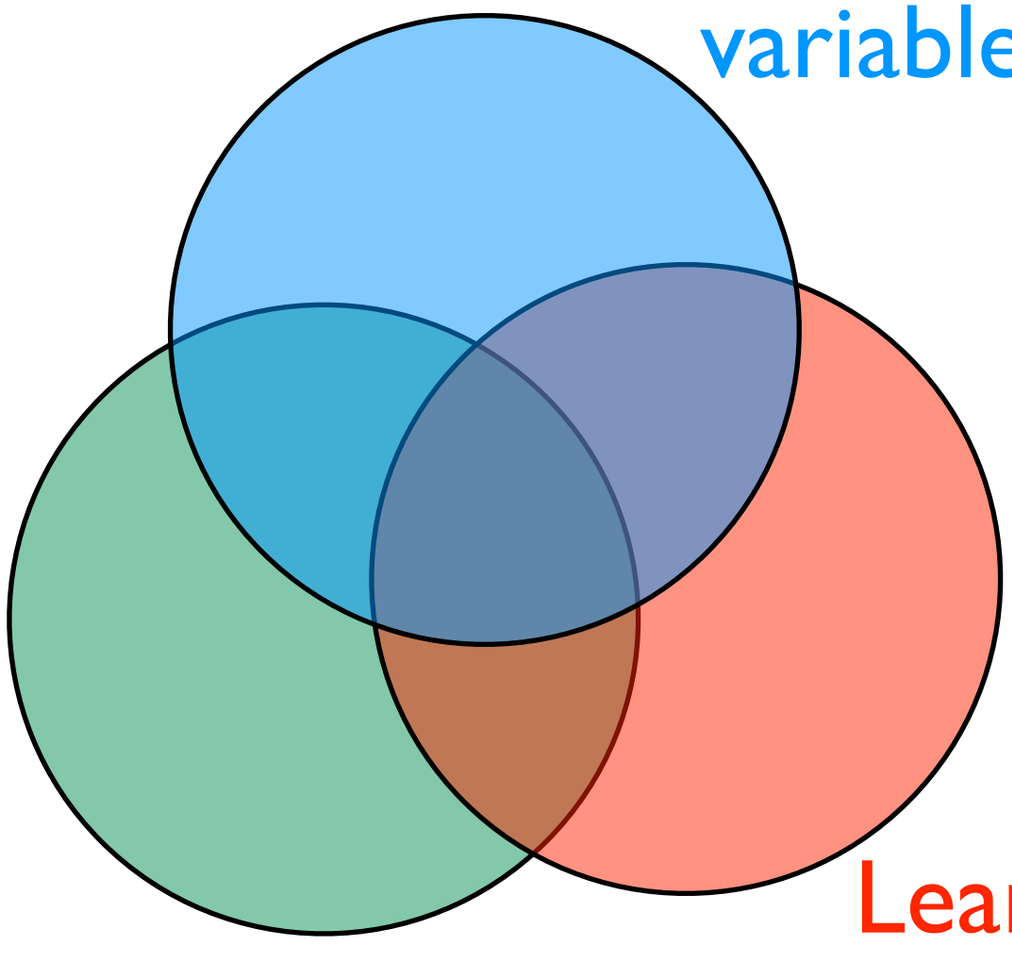
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Learning

[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
eve	new york	0.90
tom	paris	0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random

```
select x.person, y.country
from bornIn x, cityIn y
where x.city=y.city
```

one world

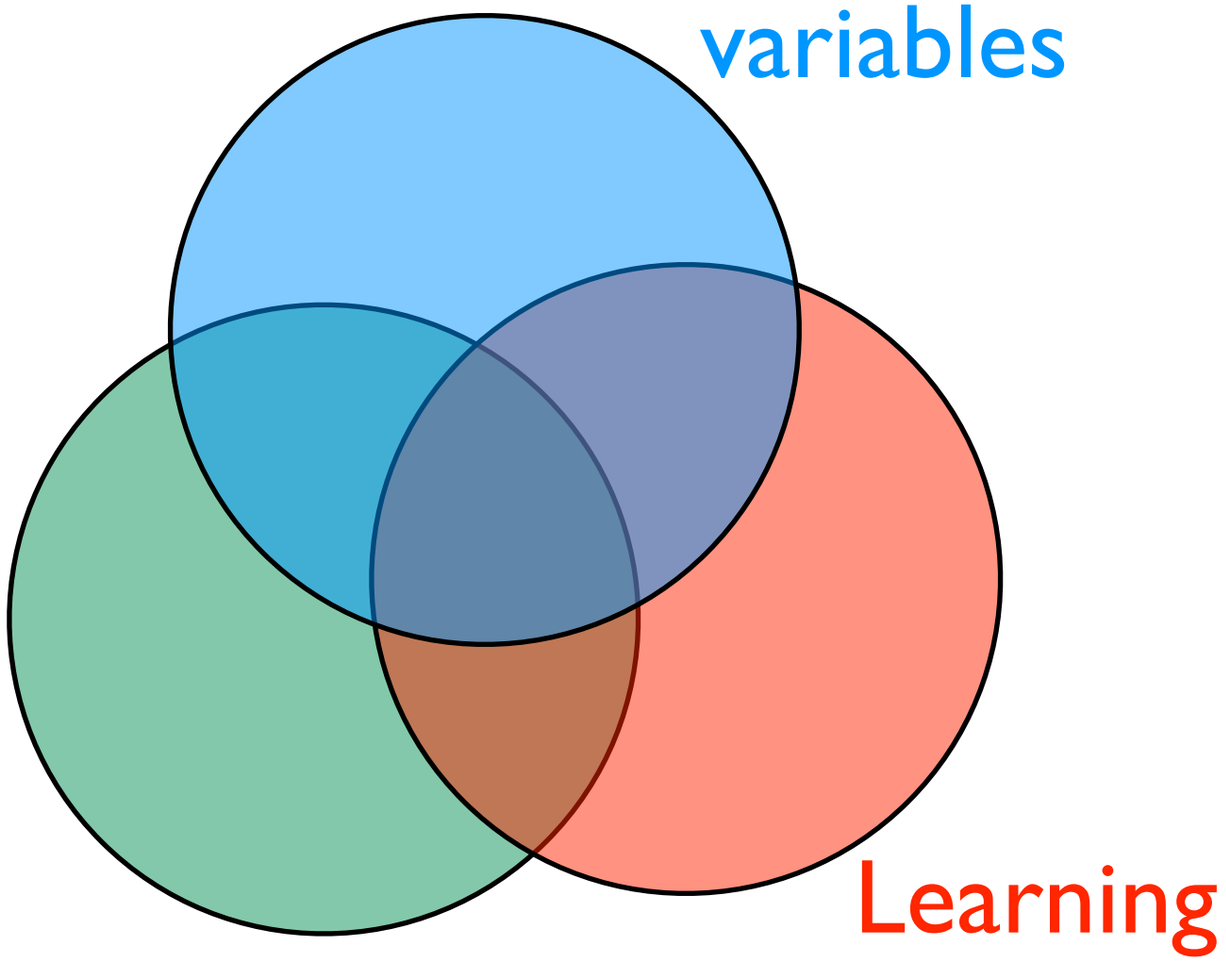
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



[Suciu et al 2011]

Probabilistic Databases

several possible worlds

bornIn

person	city	P
ann	london	0.87
bob	york	0.95
		0.90
		0.56

cityIn

city	country	P
london	uk	0.99
york	uk	0.75
paris	usa	0.40

tuples as random

probabilistic tables + database queries
→ distribution over possible worlds

```
select
from bornIn x, cityIn y
where x.city=y.city
```

one world

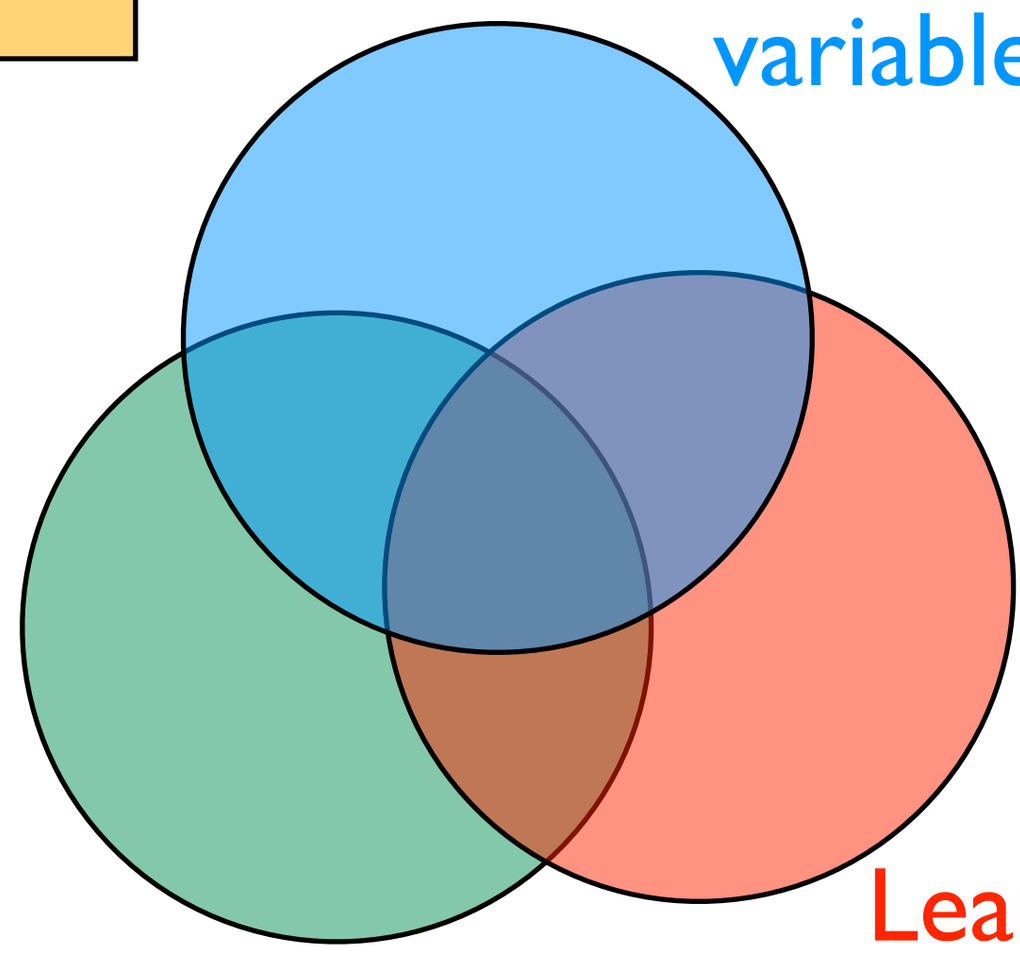
bornIn

person	city
ann	london
bob	york
eve	new york
tom	paris

cityIn

city	country
london	uk
york	uk
paris	usa

relational database



Example: Information Extraction

Recently-Learned Facts twitter Refresh

instance	iteration	date learned	confidence
kelly_andrews is a female	826	29-mar-2014	98.7
investment_next_year is an economic sector	829	10-apr-2014	95.3
shibenik is a geopolitical entity that is an organization	829	10-apr-2014	97.2
quality_web_design_work is a character trait	826	29-mar-2014	91.0
mercedes_benz_cls_by_carlsson is an automobile manufacturer	829	10-apr-2014	95.2
social_work is an academic program at the university rutgers_university	827	02-apr-2014	93.8
dante wrote the book the_divine_comedy	826	29-mar-2014	93.8
willie_aames was born in the city los_angeles	831	16-apr-2014	100.0
kitt_peak is a mountain in the state or province arizona	831	16-apr-2014	96.9
greenwich is a park in the city london	831	16-apr-2014	100.0

instances for many different relations

degree of certainty

NELL: http://rtw.ml.cmu.edu/rtw/

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

[Example from Suciu et al 2011]

Querying: relational database

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microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```


[Example from Suciu et al 2011]

Querying: relational database

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Company	Product
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Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: relational database

ProducesProduct

Company	Product
sony	walkman
microsoft	mac_os_x
ibm	personal_computer
microsoft	mac_os
adobe	adobe_indesign
adobe	adobe_dreamweaver
...	...

HeadquarteredIn

Company	City
microsoft	redmond
ibm	san_jose
emirates_airlines	dubai
honda	torrance
horizon	seattle
egyptair	cairo
adobe	san_jose
...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company
personal_computer	ibm
adobe_indesign	adobe
adobe_dreamweaver	adobe

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

same query -

probabilities handled implicitly

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.96x0.99=0.95

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

0.9x0.93=0.83

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

0.87x0.93=0.80

[Example from Suciu et al 2011]

Querying: probabilistic db

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
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...	...	...

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microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```

answer tuples ranked by probability

Product	Company	P
personal_computer	ibm	0.95
adobe_indesign	adobe	0.83
adobe_dreamweaver	adobe	0.80

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

HeadquarteredIn

Company	City	P
microsoft	redmond	1.00
ibm	san_jose	0.99
emirates_airlines	dubai	0.93
honda	torrance	0.93
horizon	seattle	0.93
egyptair	cairo	0.93
adobe	san_jose	0.93
...	...	...

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and
y.City='san_jose'
```


[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

ProducesProduct

Company	Product	P
sony	walkman	0.96
microsoft	mac_os_x	0.96
ibm	personal_computer	0.96
microsoft	mac_os	0.9
adobe	adobe_indesign	0.9
adobe	adobe_dreamweaver	0.87
...	...	...

```
0.96::producesProduct(sony,walkman).
0.96::producesProduct(microsoft,mac_os_x).
0.96::producesProduct(ibm,personal_computer).
0.9::producesProduct(microsoft,mac_os).
0.9::producesProduct(adobe,adobe_indesign).
0.87::producesProduct(adobe,adobe_dreamweaver).
...
```


[Example from Suciu et al 2011]

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with tuple-level uncertainty in ProbLog?

```
select x.Product, x.Company
from ProducesProduct x, HeadquarteredIn y
where x.Company=y.Company and y.City='san_jose'
```

```
result(Product,Company) :-
    producesProduct(Company,Product),
    headquarteredIn(Company,san_jose).
query(result(_,_)).
```

PDB with tuple-level uncertainty in ProbLog?

```
0.96::producesProduct(sony,walkman).
0.96::producesProduct(microsoft,mac_os_x).
0.96::producesProduct(ibm,personal_computer).
0.9::producesProduct(microsoft,mac_os).
0.9::producesProduct(adobe,adobe_indesign).
0.87::producesProduct(adobe,adobe_dreamweaver).
...
1.00::headquarteredIn(microsoft,redmond).
0.99::headquarteredIn(ibm,san_jose).
0.93::headquarteredIn(emirates_airlines,dubai).
0.93::headquarteredIn(honda,torrance).
0.93::headquarteredIn(horizon,seattle).
0.93::headquarteredIn(egyptair,cairo).
0.93::headquarteredIn(adobe,san_jose).
...
      result(Product,Company) :- producesProduct(Company,Product),
                                 headquarteredIn(Company,san_jose).

      query(result(_,_)).
```

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

PDB with attribute-level uncertainty in ProbLog?

color

item	color	P
mug	green blue	0.65 0.35
plate	pink red purple	0.23 0.14 0.63

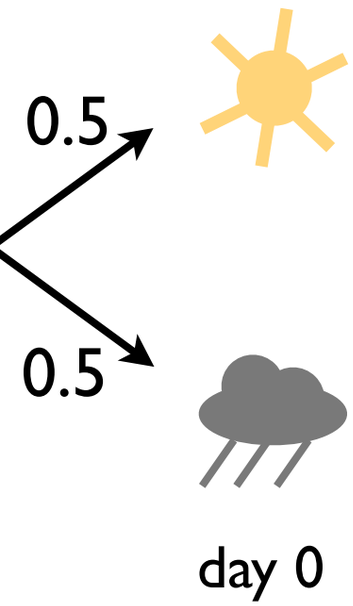
```
0.65::color(mug,green); 0.35::color(mug,blue) <- true.
0.23::color(plate,pink); 0.14::color(plate,red);
                                0.63::color(plate,purple) <- true.
```

ProbLog by example:

Rain or sun?

ProbLog by example:

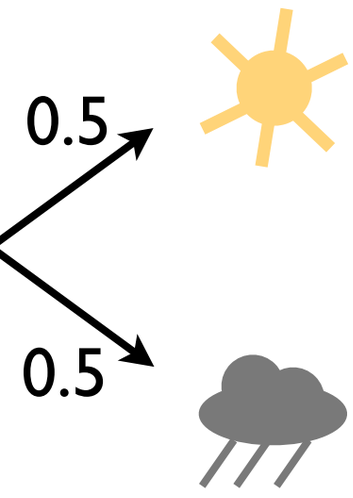
Rain or sun?



day 0

ProbLog by example:

Rain or sun?

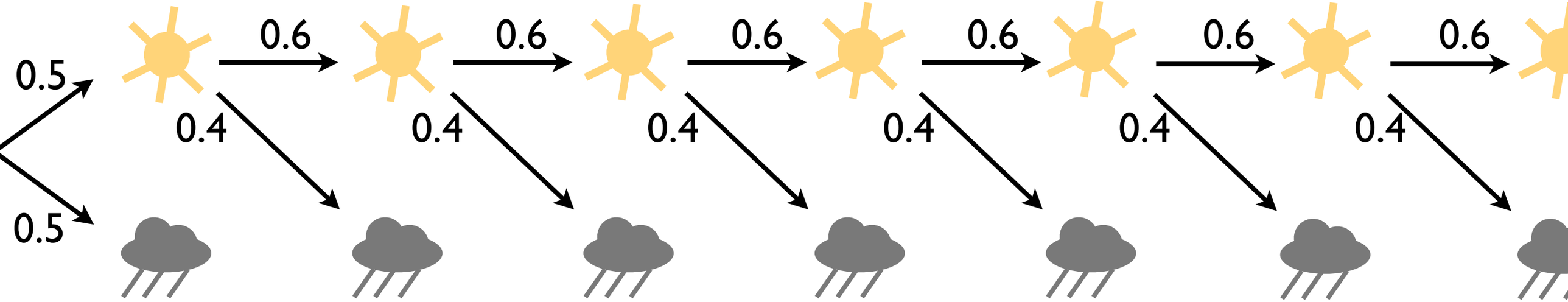


day 0

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

ProbLog by example:

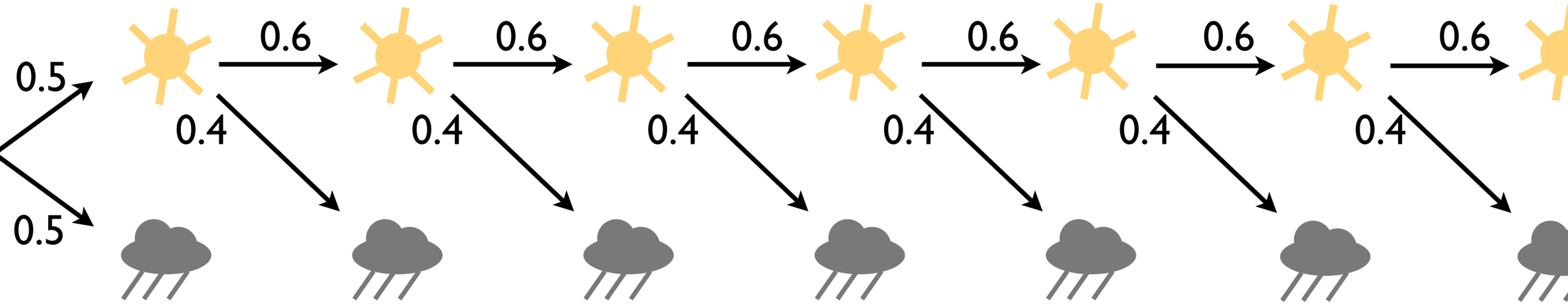
Rain or sun?




```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

ProbLog by example:

Rain or sun?

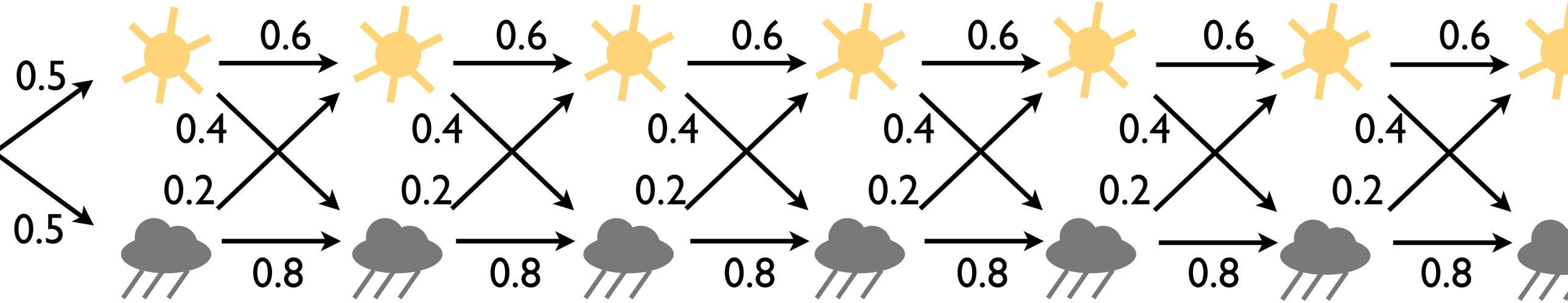



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(sun,Tprev).
```

ProbLog by example:

Rain or sun?

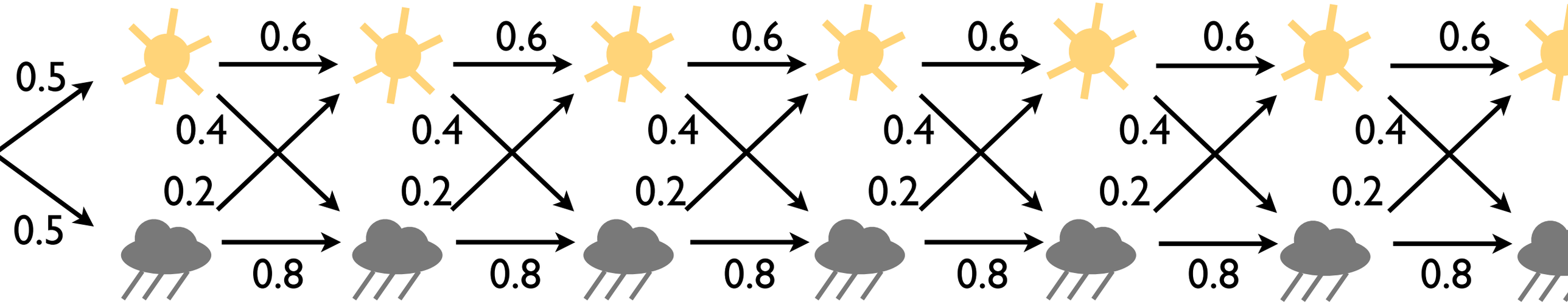



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
```

ProbLog by example:

Rain or sun?

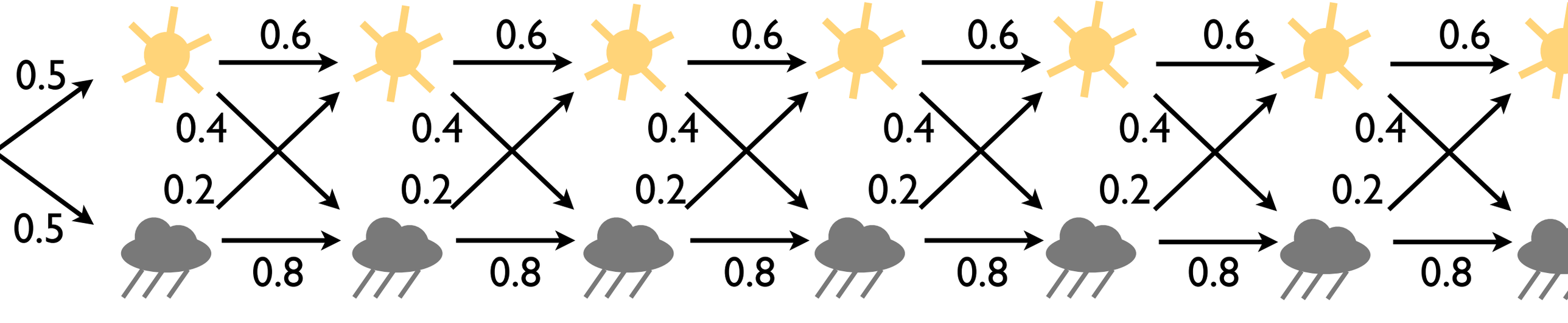



```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(rain,Tprev).
```

ProbLog by example:

Rain or sun?



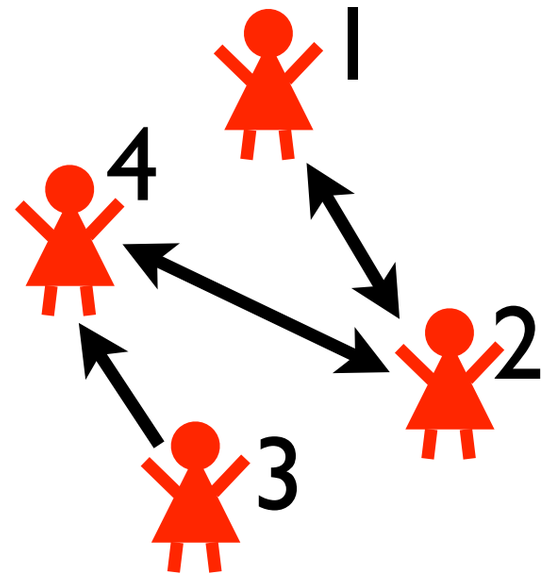

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                  <- T>0, Tprev is T-1, weather(rain,Tprev).
```

infinite possible worlds! BUT: finitely many partial worlds suffice to answer any given ground query

ProbLog by example:

Friends & smokers

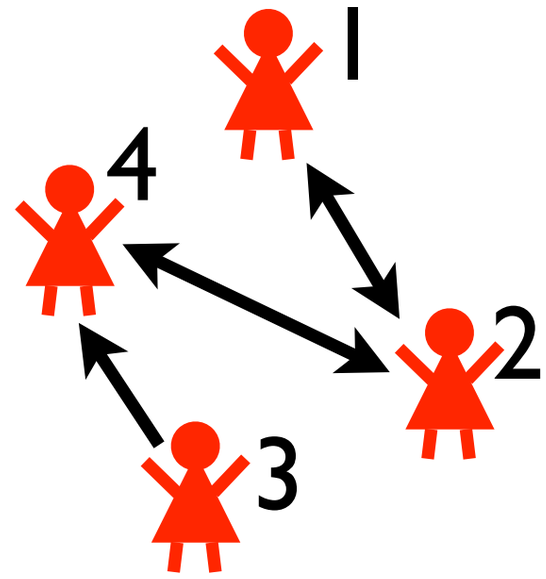



```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers



typed probabilistic facts

= a probabilistic fact for each grounding

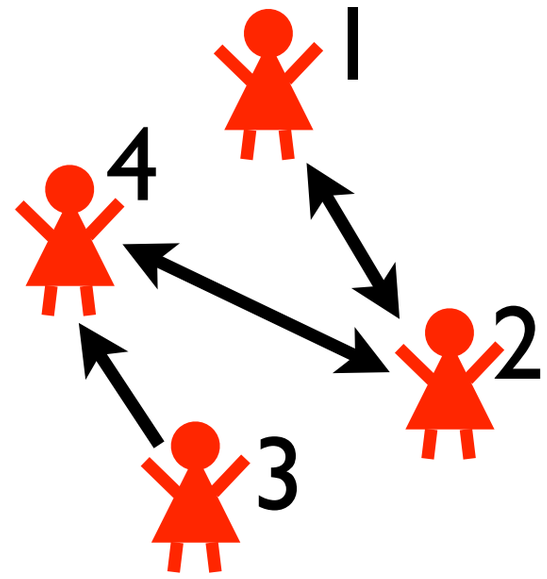
```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
                person(X), person(Y).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers



typed probabilistic facts

= a probabilistic fact for each grounding

```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).
```

```
0.3::stress(1).
0.3::stress(2).
0.3::stress(3).
0.3::stress(4).
```

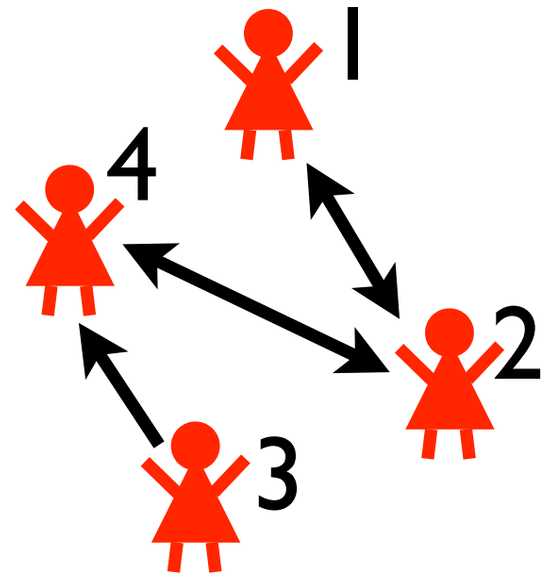
```
0.2::influences(1,1).
0.2::influences(1,2).
0.2::influences(1,3).
0.2::influences(1,4).
0.2::influences(2,1).
...
0.2::influences(4,2).
0.2::influences(4,3).
0.2::influences(4,4).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).

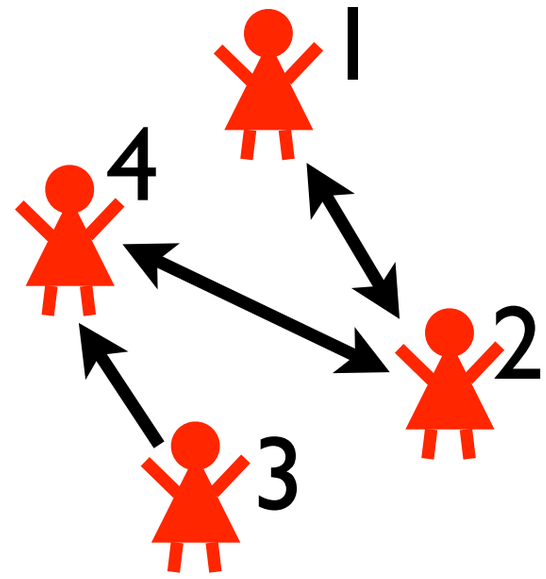
smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), influences(Y,X), smokes(Y).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
0.2::influences(X,Y):-
              person(X), person(Y).

smokes(X)  :- stress(X).
smokes(X)  :-
     friend(X,Y), influences(Y,X), smokes(Y).

0.4::asthma(X) <- smokes(X).
```

```
person(1).
person(2).
person(3).
person(4).

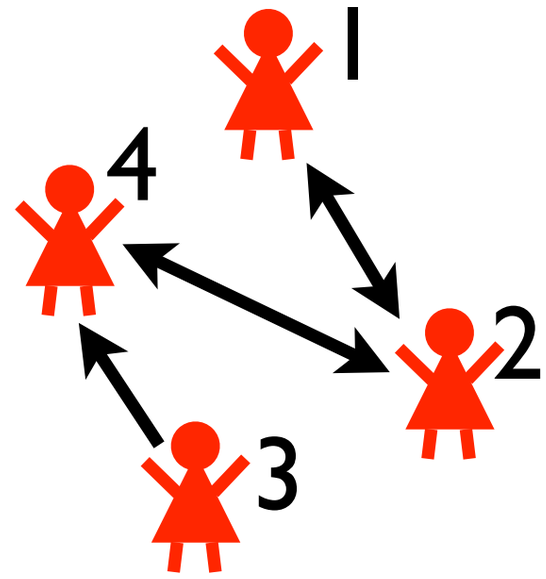
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

annotated disjunction with implicit head atom:

with probability 0.6, nothing happens

ProbLog by example:

Friends & smokers




```
0.3::stress(X):- person(X).
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              person(X), person(Y).

smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), influences(Y,X), smokes(Y).

0.4::asthma(X) <- smokes(X).
```

```
person(1).
person(2).
person(3).
person(4).

friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight), P is 1.0/Weight.
```

flexible probability:
computed from the weight of the item

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

```
1/6::pack(skis).
1/4::pack(boots).
1/3::pack(helmet).
1/2::pack(gloves).
```

```
P::pack(Item) :- weight(Item,Weight), P is 1.0/Weight.
```

flexible probability:
computed from the weight of the item

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).
```

list of all items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

pack first item, decrease limit by its weight, and continue with rest of items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

do **not** pack first item,
continue with rest of items

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

no items left: did we add too much?

ProbLog by example:

Limited Luggage

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :- weight(Item,Weight),  P is 1.0/Weight.

excess(Limit) :- excess([skis,boots,helmet,gloves],Limit).

excess([],Limit) :- Limit<0.
excess([I|R],Limit) :-
   pack(I), weight(I,W), L is Limit-W, excess(R,L).
excess([I|R],Limit) :-
   \+pack(I), excess(R,Limit).
```

Summary: ProbLog Syntax

- input database: ground facts

```
person(bob).
```

- probabilistic facts

```
0.5::stress(bob).
```

- typed probabilistic facts (body deterministic)

```
0.5::stress(X) :- person(X).
```

- flexible probabilities

```
P::pack(Item) :- weight(Item,W),
                 P is 1.0/W.
```

- annotated disjunctions

```
0.4::asthma(X) <- smokes(X).
```

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.
```

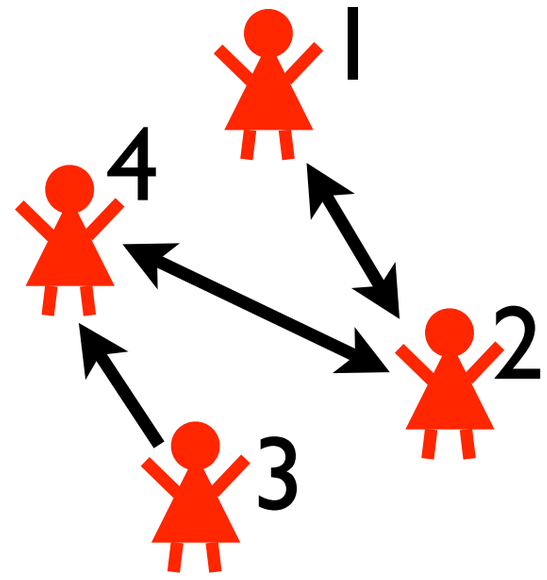
- Prolog clauses

```
smokes(X) :- influences(Y,X), smokes(Y).
```

```
excess([I|R],Limit) :- \+pack(I), excess(R,Limit).
```

ProbLog example:

Friends & smokers



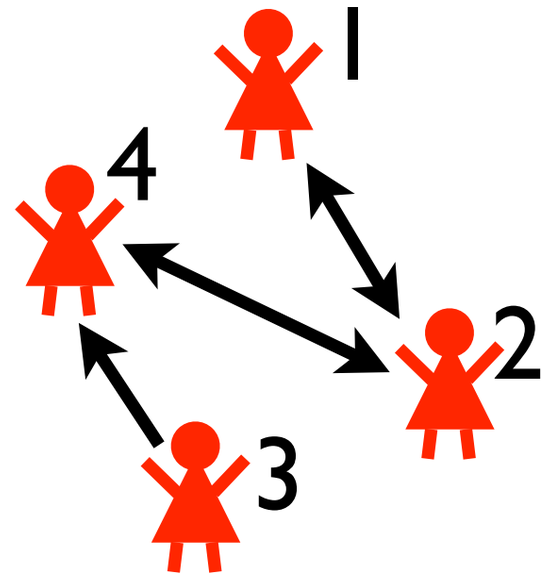

```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), smokes(Y).
```

ProbLog example:

Friends & smokers

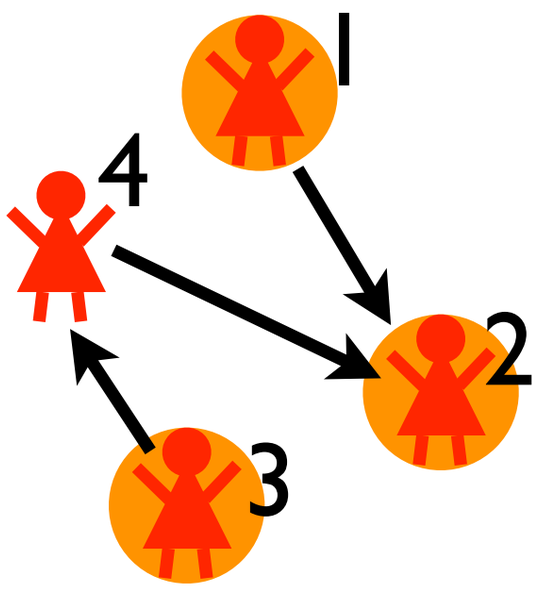



```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), smokes(Y).
```

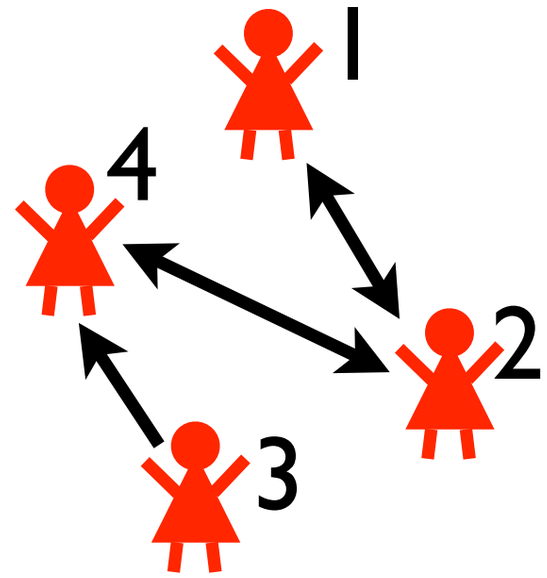
example possible world




```
friend(2,1).
friend(3,4).
friend(2,4).
stress(1).
stress(2).
stress(3).
smokes(1).
smokes(2).
smokes(3).
```

ProbLog example:

Friends & smokers

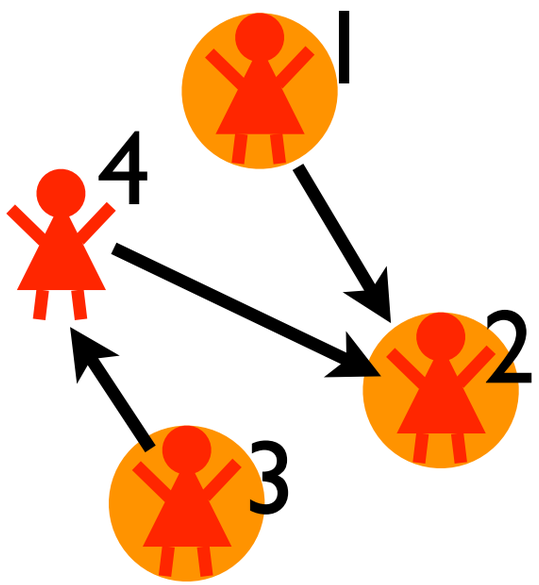



```
0.5::stress(1).
0.1::stress(2).
0.8::stress(3).
0.3::stress(4).

0.9::friend(1,2).
0.8::friend(2,1).
0.3::friend(2,4).
0.7::friend(3,4).
0.1::friend(4,2).

smokes(X) :- stress(X).
smokes(X) :-
     friend(X,Y), smokes(Y).
```

example possible world

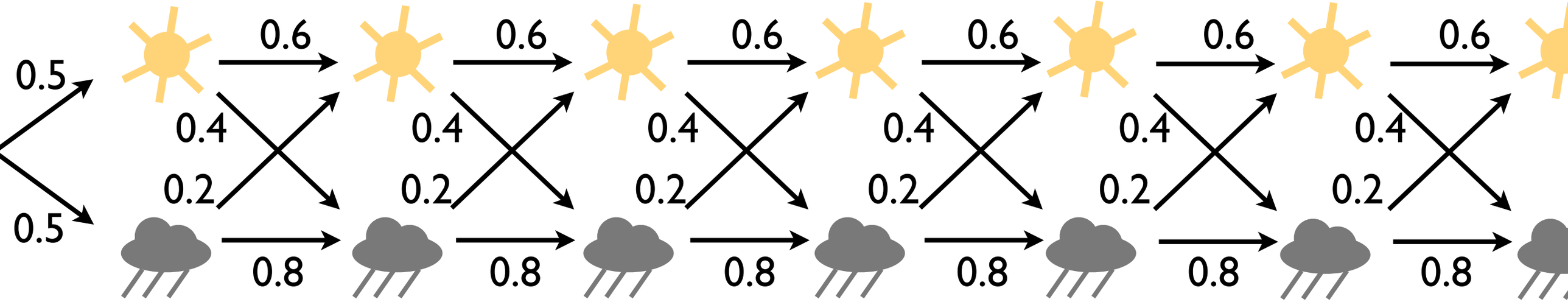



```
friend(2,1).
friend(3,4).
friend(2,4).
stress(1).
stress(2).
stress(3).
smokes(1).
smokes(2).
smokes(3).
```

- several instances of `smokes(X)` in same world
- `smokes(2)` : multiple derivations in same world
- distribution over worlds not (always) a distribution over computations / answers

ProbLog by example:

Rain or sun?




```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

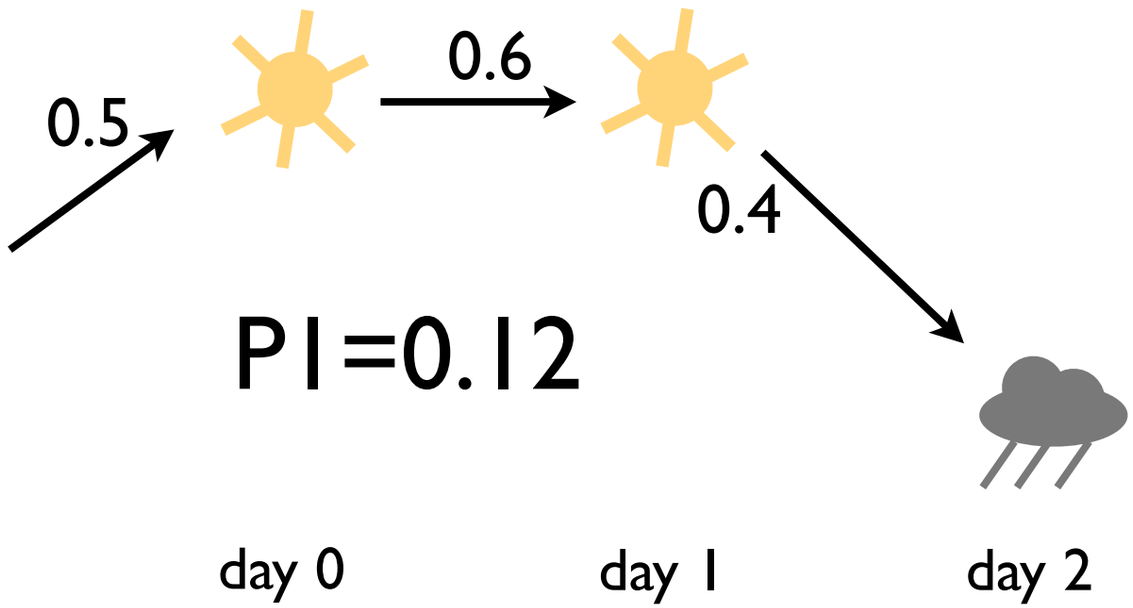
0.6::weather(sun,T) ; 0.4::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(sun,Tprev).
0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(rain,Tprev).
```

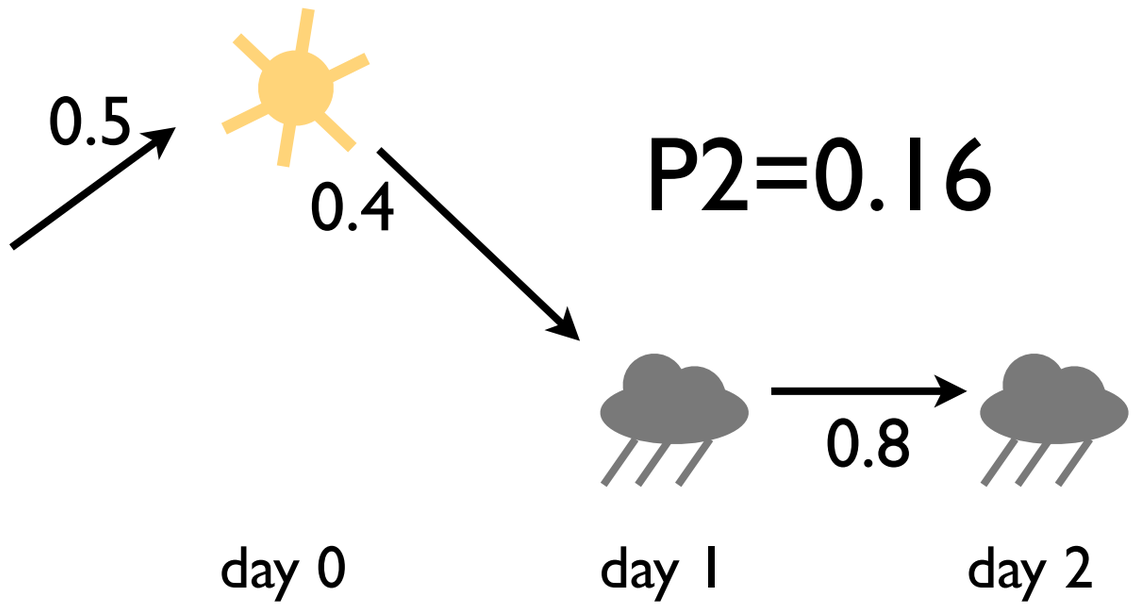
≤1 proof for a ground query per possible world → distribution over worlds is distribution over derivations!

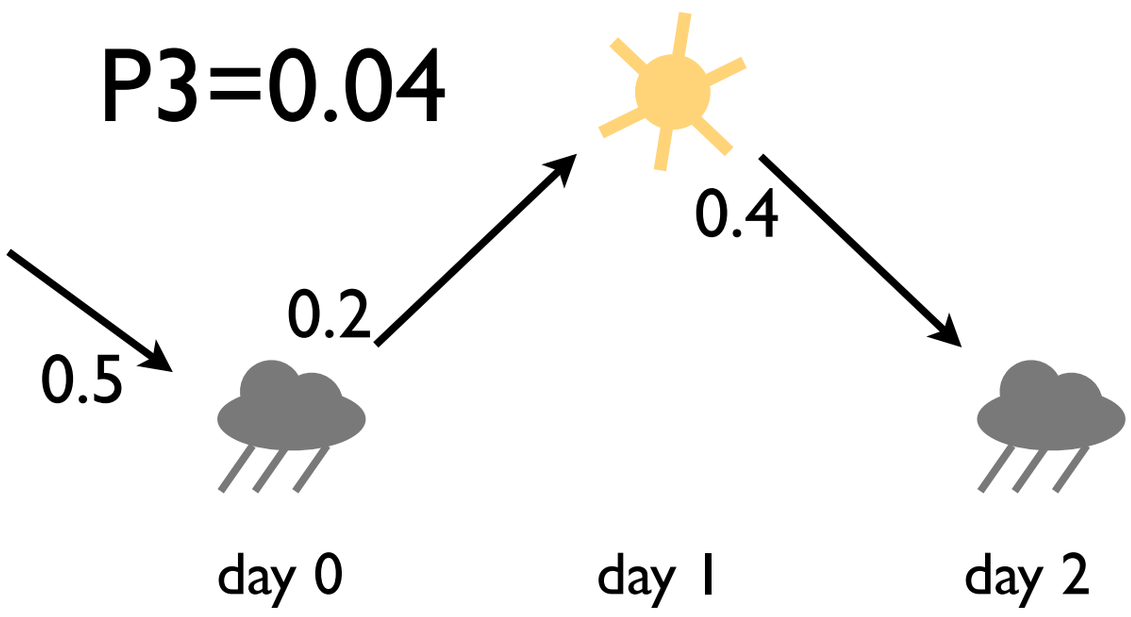
Possible worlds

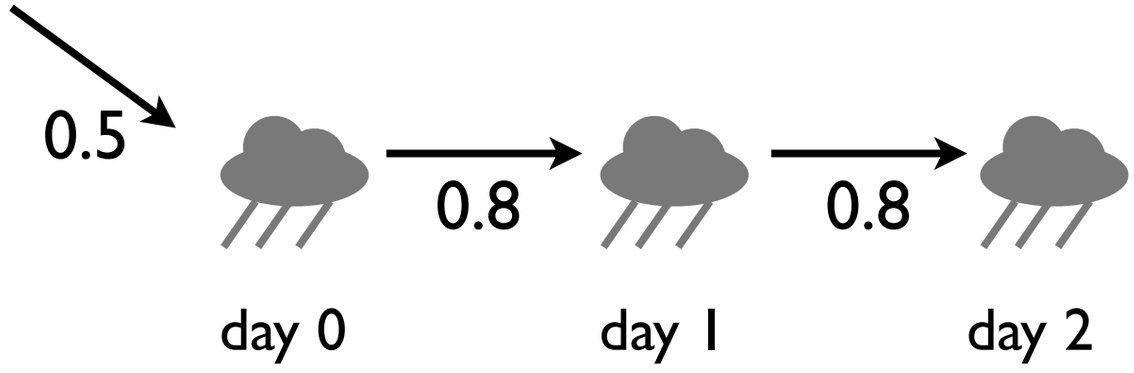
```
?- weather(rain,2).
```

P=P1+P2+P3+P4









Mutually Exclusive Rules:

no two rules apply simultaneously

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

day 0: either sun or rain

```
0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
                <- T>0, Tprev is T-1, weather(sun,Tprev).

0.2::weather(sun,T) ; 0.8::weather(rain,T)
                <- T>0, Tprev is T-1, weather(rain,Tprev).
```

Mutually Exclusive Rules:

no two rules apply simultaneously

first rule for day 0, others for later days

day 0: either sun or rain

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0.5::weather(sun,0) ; 0.5::weather(rain,0) <- true.

0.6::weather(sun,T) ; 0.4::weather(rain,T)
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0.2::weather(sun,T) ; 0.8::weather(rain,T)
                    <- T>0, Tprev is T-1, weather(rain,Tprev).
```

rules for T>0 cover mutually exclusive cases on previous day

[Sato and Kameya, JAIR 01]

PRISM

- Another probabilistic Prolog based on the distribution semantics
- Mutual exclusiveness assumption
  - allows for efficient inference by dynamic programming, cf. probabilistic grammars
  - but excludes certain models, e.g., smokers
- http://sato-www.cs.titech.ac.jp/prism/

PRISM

- "multi-valued random switches" = annotated disjunctions with body *true*
- stochastic memoization: switch gives fresh result on each call
- Prolog rules
- limited support for negation (compiling away)

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
   Now < T,
   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
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values(init,[sun,rain]).
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:- set_sw(init,[0.5,0.5]).
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```

probability distributions

```
weather(W,Time) :-
  Time >= 0,
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w(T,T,W,W).
w(Now,T,WNow,WT) :-
   Now < T,
   msw(tr(WNow),WNext),
   Next is Now+1,
   w(Next,T,WNext,WT).
```

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

probability distributions

```
weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **W0** to random value of **init**

Weather in PRISM

```
values(init,[sun,rain]).
values(tr(_),[sun,rain])
```

random variables and their values

```
:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
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probability distributions

```
weather(W,Time) :-
   Time >= 0,
   msw(init,W0),
   w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

set **`W0`** to random value of **`init`**

set **`WNext`** to random value of **`tr(WNow)`**, using **fresh** value on every call

Weather in PRISM / ProbLog

```
values(init,[sun,rain]).
values(tr(_),[sun,rain]).

:- set_sw(init,[0.5,0.5]).
:- set_sw(tr(sun),[0.6,0.4]).
:- set_sw(tr(rain),[0.2,0.8]).
```

```
weather(W,Time) :-
  Time >= 0,
  msw(init,W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    msw(tr(WNow),WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

```
0.5::init(sun) ;
  0.5::init(rain) <- true.
0.6::tr(T,sun,sun) ;
  0.4::tr(T,sun,rain) <- true.
0.2::tr(T,rain,sun) ;
  0.8::tr(T,rain,rain) <- true.
```

```
weather(W,Time) :-
  Time >= 0,
  init(W0),
  w(0,Time,W0,W).

w(T,T,W,W).
w(Now,T,WNow,WT) :-
    Now < T,
    tr(Now,WNow,WNext),
    Next is Now+1,
    w(Next,T,WNext,WT).
```

ProbLog needs to explicitly use different facts at each call

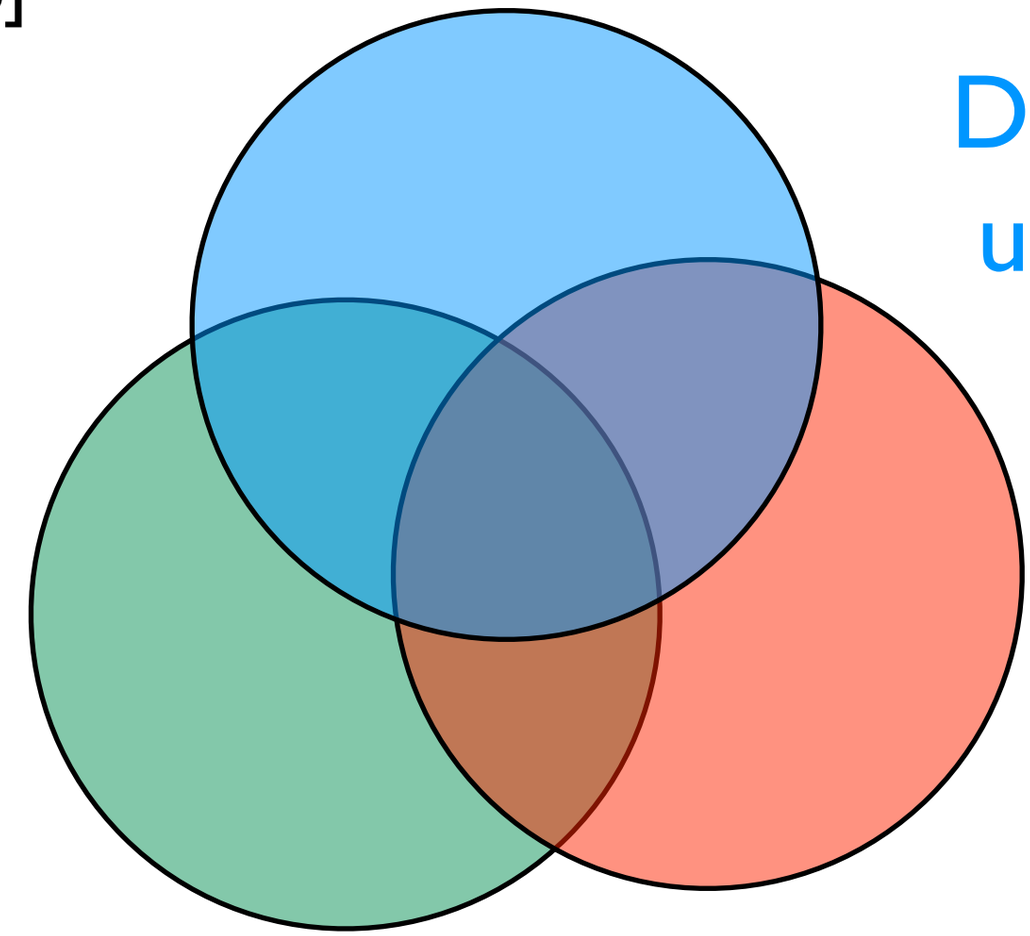
Probabilistic Programming Languages outside LP

- IBAL [Pfeffer 01]
- Figaro [Pfeffer 09]
- Church [Goodman et al 08]
- BLOG [Milch et al 05]
- and many more appearing recently

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

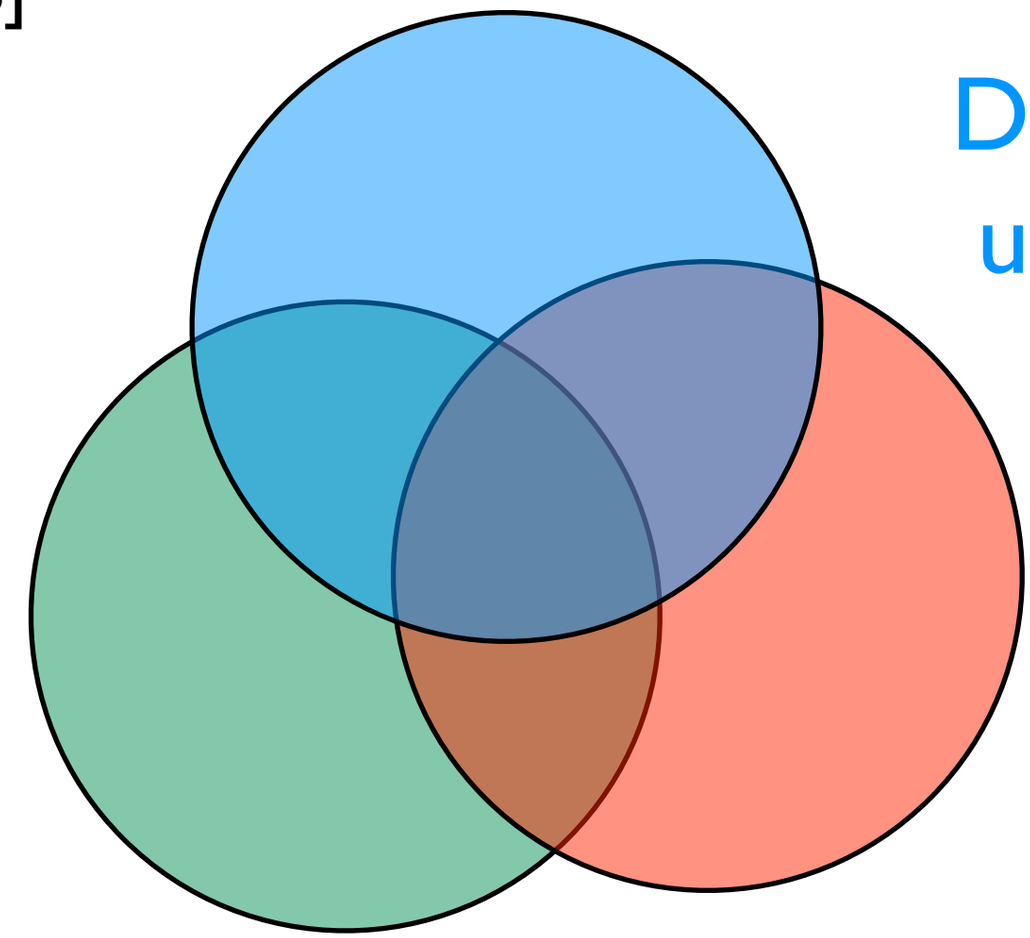
Reasoning with relational data

Learning

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

functional programming

Learning

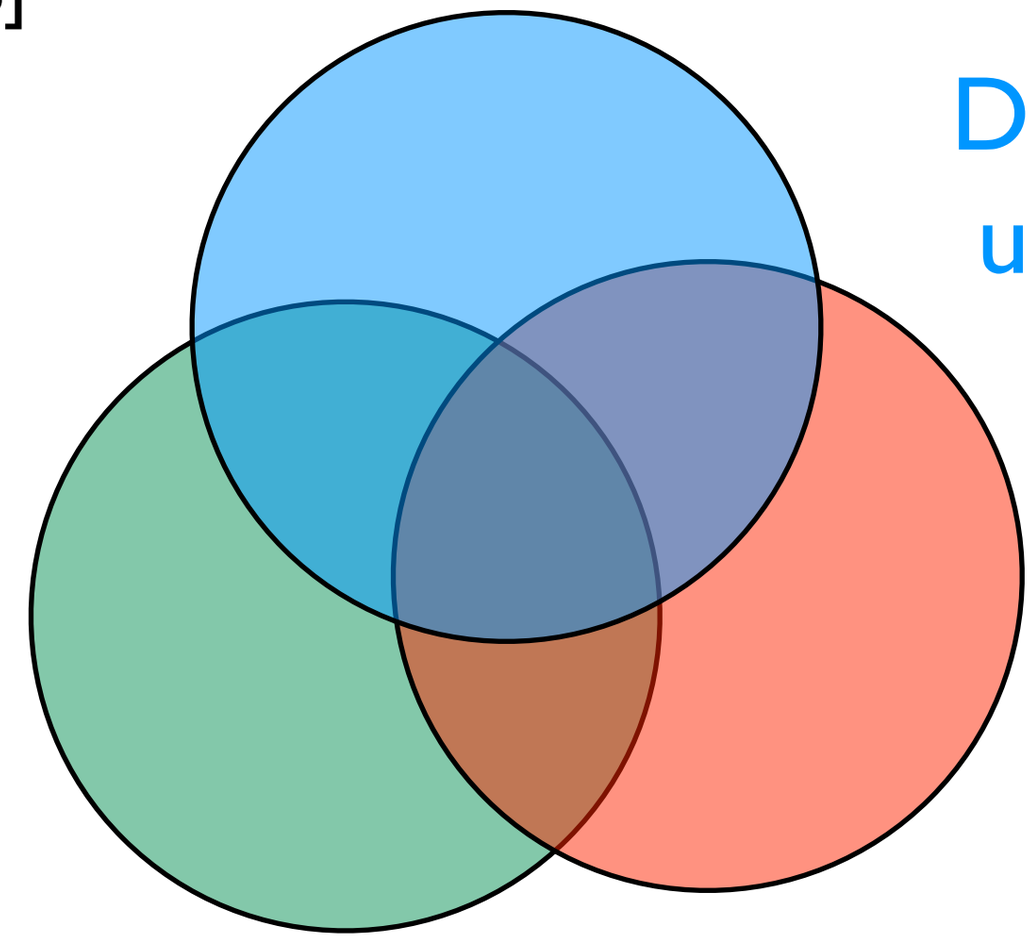
```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church

probabilistic functional programming

[Goodman et al, UAI 08]



Dealing with uncertainty

functional programming

Learning

one execution

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

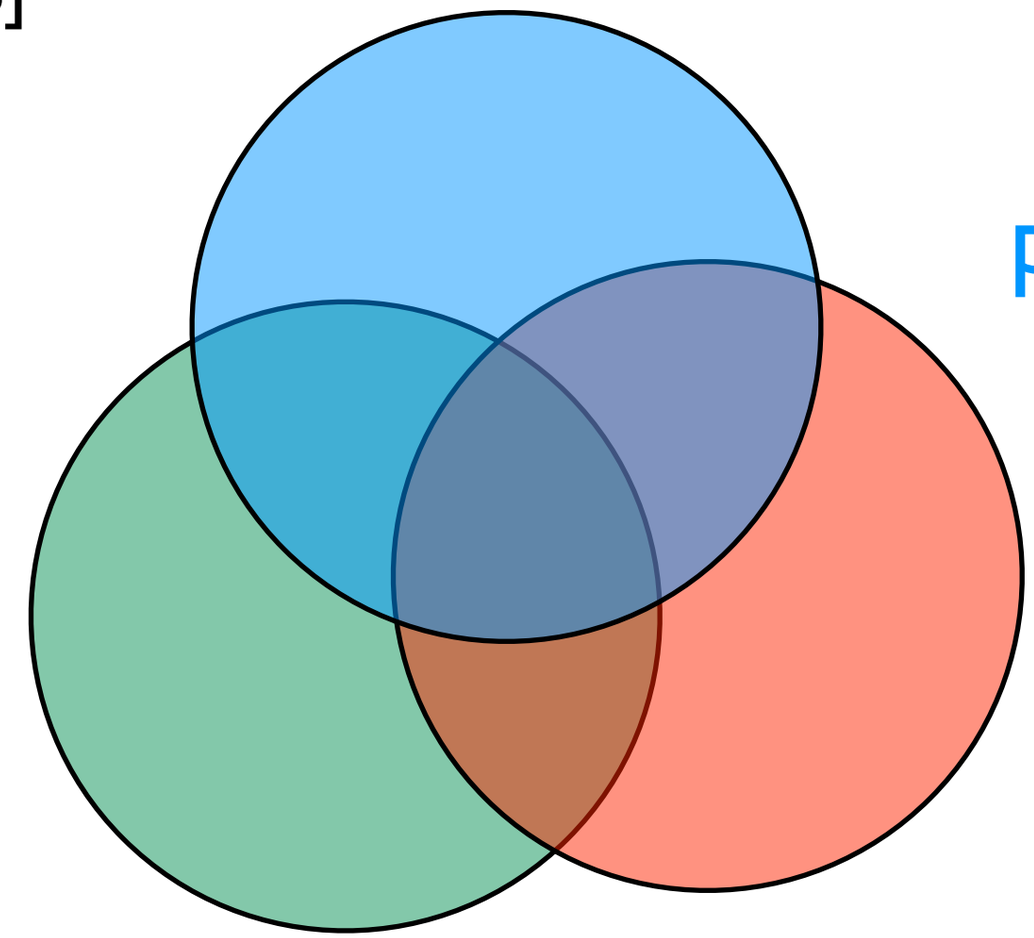
Church

probabilistic functional programming

[Goodman et al, UAI 08]

```
(define randplus5
 (lambda (x) (if (flip 0.6)
                 (+ x 5)
                 x)))

(map randplus5 '(1 2 3))
```



random primitives

functional programming

Learning

one execution

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church

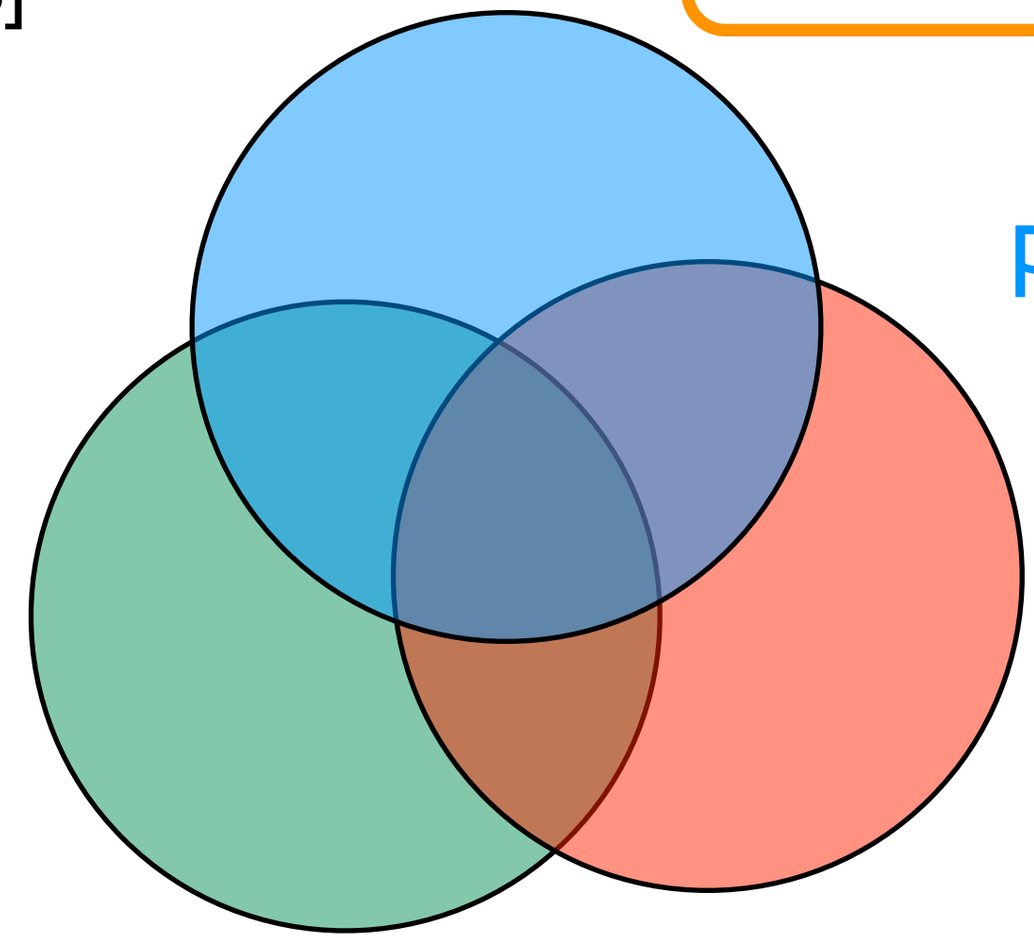
probabilistic functional programming

[Goodman et al, UAI 08]

several possible executions

```
(define randplus5
 (lambda (x) (if (flip 0.6)
                 (+ x 5)
                 x)))

(map randplus5 '(1 2 3))
```



random primitives

functional programming

Learning

one execution

```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church

probabilistic functional programming

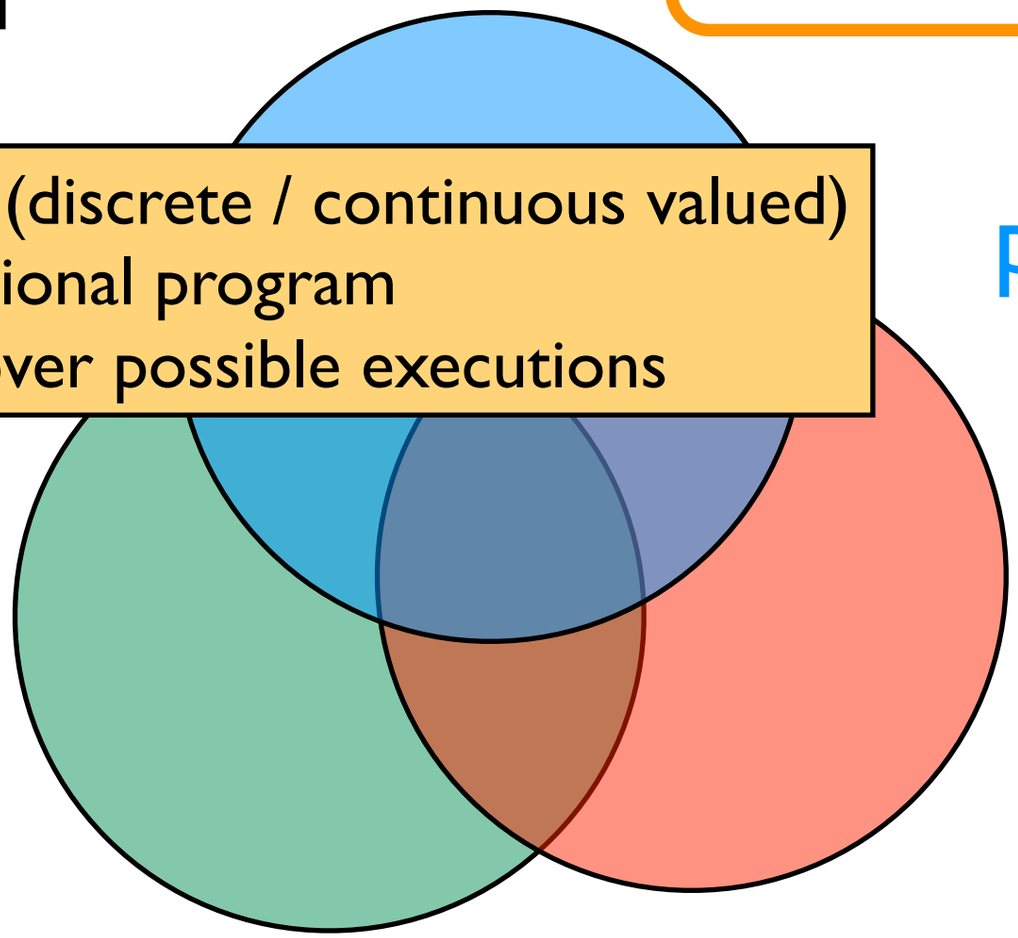
[Goodman et al, UAI 08]

several possible executions

```
(define randplus5
 (lambda (x) (if (flip 0.6)
                 (+ x 5)
                 x)))

(map randplus5 '(1 2 3))
```

probabilistic primitives (discrete / continuous valued)
+ functional program
→ distribution over possible executions



random primitives

functional programming

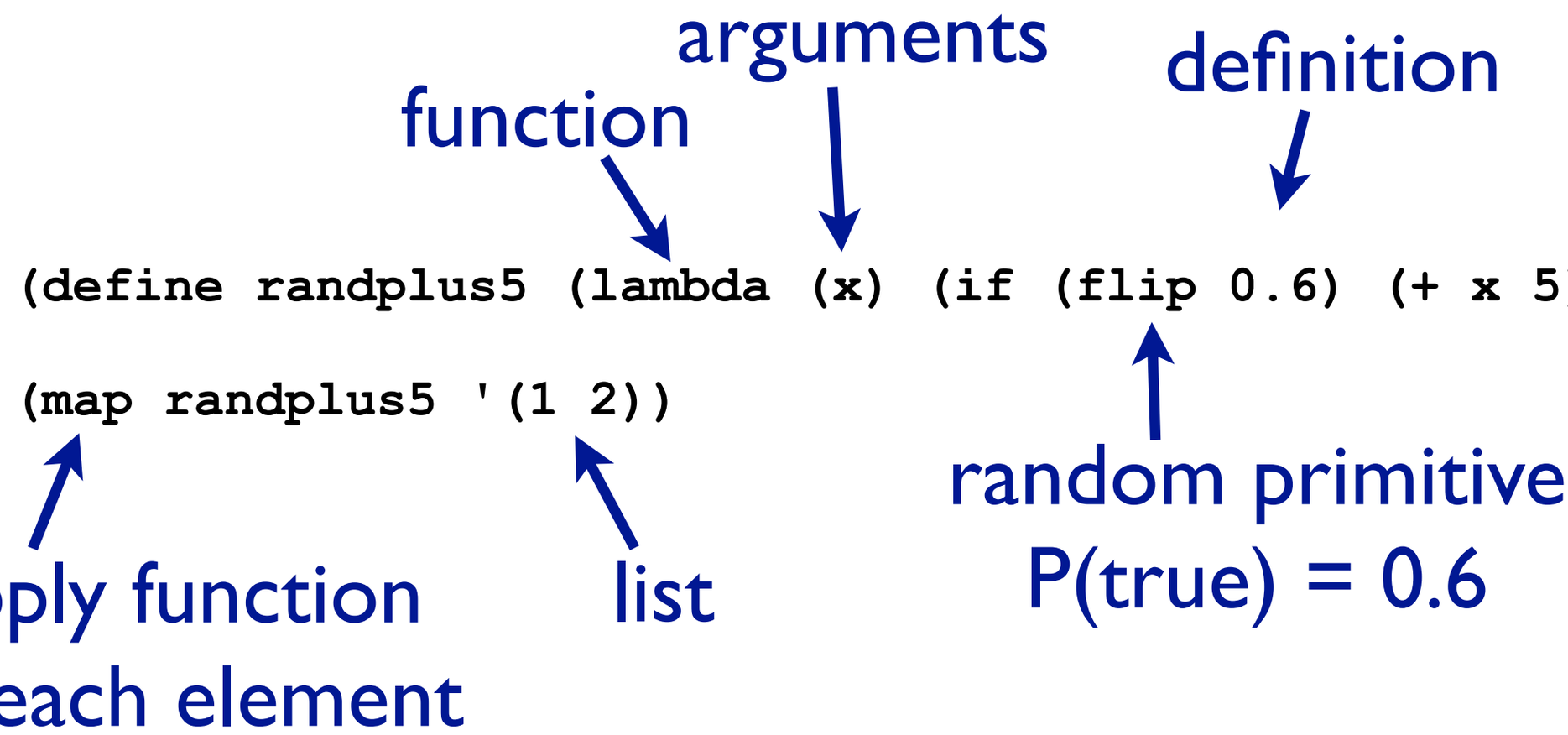
Learning

one execution

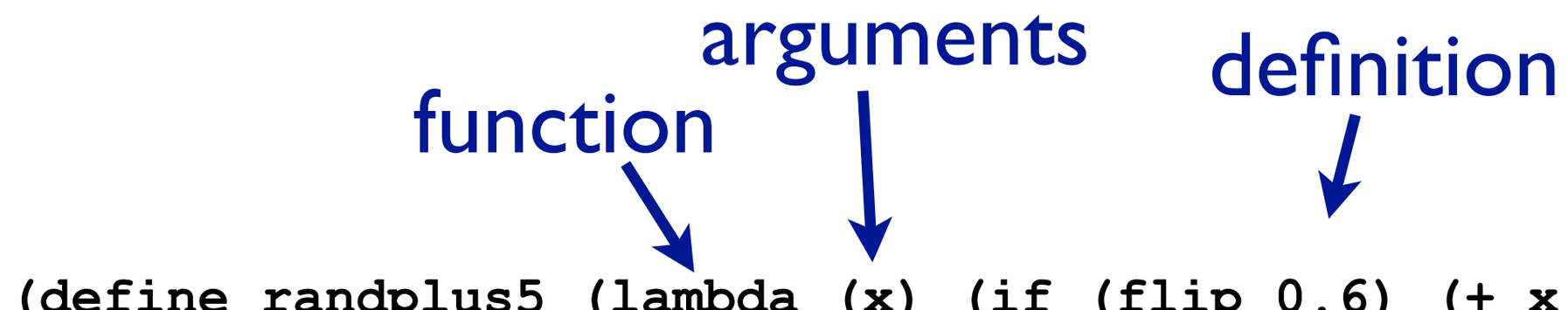
```
(define plus5 (lambda (x) (+ x 5)))

(map plus5 '(1 2 3))
```

Church Example



Church Example




```
(map randplus5 '(1 2))
```

apply function to each element

list

result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

in ProbLog?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

in ProbLog?

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(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

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```

```
0.4::p5(N,N);0.6::p5(N,M) <- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M),
    lp5(L,K).

query(lp5([1,2],_)).
```

in ProbLog?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

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query(lp5([1,2],_)).
```

result: (1 2) with 0.4×0.4
(1 7) with 0.4×0.6
(6 2) with 0.6×0.4
(6 7) with 0.6×0.6

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

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query(lp5([1,1],_)).
```

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

Church result: (1 1) with 0.4×0.4
(1 6) with 0.4×0.6
(6 1) with 0.6×0.4
(6 6) with 0.6×0.6

```
0.4::p5(N,N);0.6::p5(N,M) <- M is N+5.

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results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

Church result: (1 1) with 0.4×0.4
(1 6) with 0.4×0.6
(6 1) with 0.6×0.4
(6 6) with 0.6×0.6

```
0.4::p5(N,N);0.6::p5(N,M) <- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
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query(lp5([1,1],_)).
```

ProbLog result: (1 1) with 0.4
(1 6) with 0.0
(6 1) with 0.0
(6 6) with 0.6

results for [1,1]?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 1))
```

Church result: (1 1) with 0.4×0.4
(1 6) with 0.4×0.6
(6 1) with 0.6×0.4
(6 6) with 0.6×0.6

```
0.4::p5(N,N);0.6::p5(N,M) <- M is N+5.

lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M),
    lp5(L,K).

query(lp5([1,1],_)).
```

ProbLog result: (1 1) with 0.4
(1 6) with 0.0
(6 1) with 0.0
(6 6) with 0.6

stochastic memoization

in ProbLog?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

in ProbLog?

```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

```
0.4::p5(N,N,ID);0.6::p5(N,M,ID) <- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M,L),
    lp5(L,K).

query(lp5([1,1],_)).
```

in ProbLog?

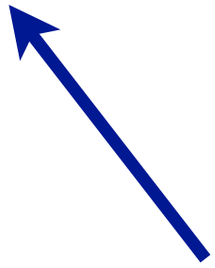
```
(define randplus5 (lambda (x) (if (flip 0.6) (+ x 5) x)))

(map randplus5 '(1 2))
```

```
0.4::p5(N,N,ID);0.6::p5(N,M,ID) <- M is N+5.
lp5([],[]).
lp5([N|L],[M|K]) :-
    p5(N,M,L),
    lp5(L,K).

query(lp5([1,1],_)).
```

identifier distinguishes calls

Stochastic Memoization

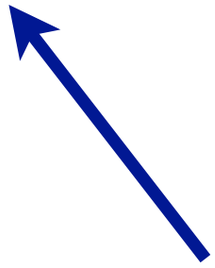
```
(define randplus5 (mem (lambda (x) (if (flip 0.6) (+ x 5) x))))

(map randplus5 '(1 1))
```



remember first value &
reuse for all later calls

Stochastic Memoization

```
(define randplus5 (mem (lambda (x) (if (flip 0.6) (+ x 5) x))))

(map randplus5 '(1 1))
```



remember first value &
reuse for all later calls

ProbLog always memoizes
PRISM never memoizes
Church allows fine-grained choice

ProbLog	**PRISM**	**Church**
probabilistic facts & choices	probabilistic choices	random primitives
all RVs memoized	no RVs memoized	user-defined per RV
Prolog	Prolog with mutually exclusive derivations	λ-calculus functions
distribution over worlds	distribution over derivations / answers	distribution over computations / answers

Roadmap

- Modeling (with detours to related work)
- Reasoning (and a bit of learning)
- Language extensions

Reasoning

- Exact inference with knowledge compilation
  - using proofs
  - using models
  - in PRISM
- Approximate inference

Answering Questions

Given:

program

queries

evidence

?

Find:

marginal probabilities

conditional probabilities

MPE state

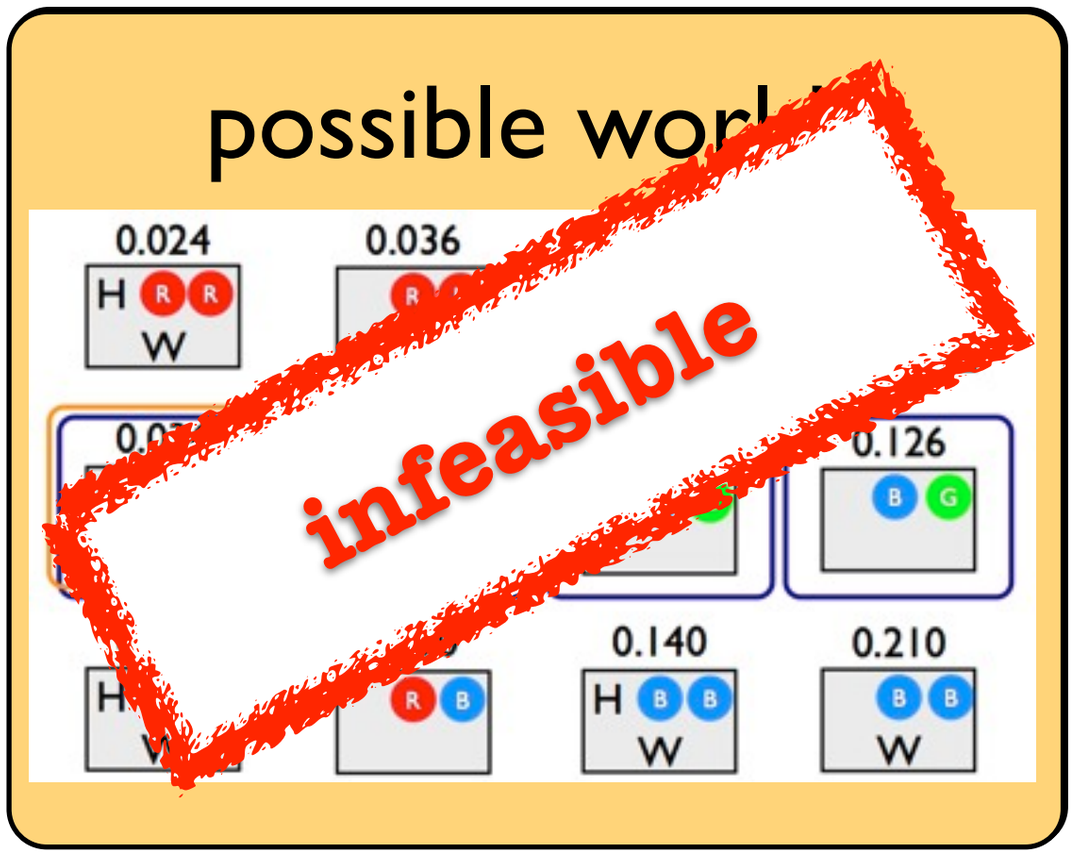
Answering Questions

Given:

program

queries

evidence



Find:

marginal probabilities

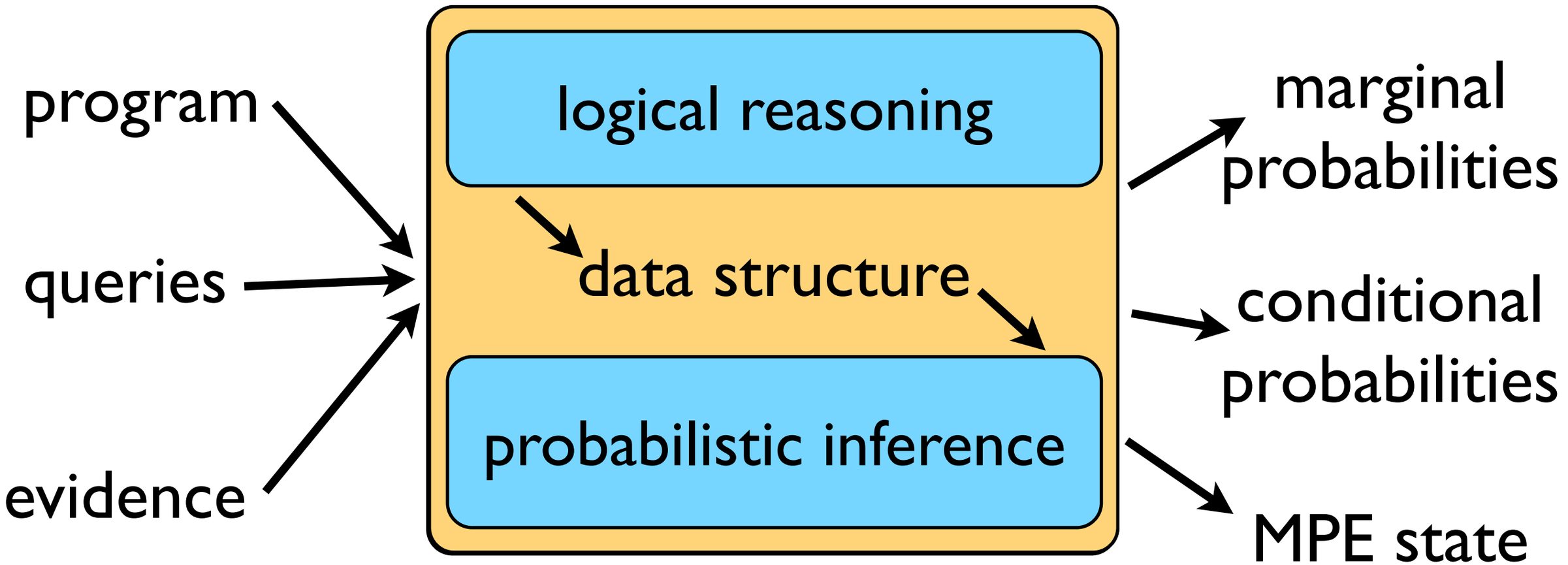
conditional probabilities

MPE state

Answering Questions

Given:

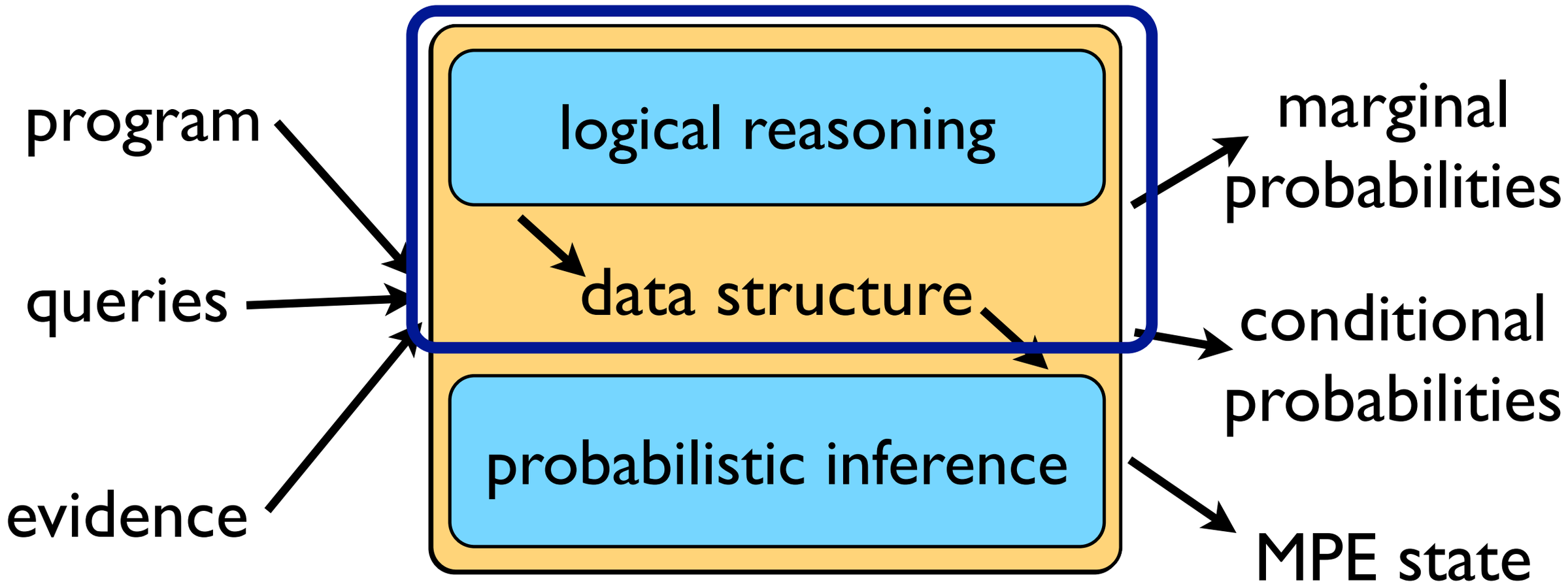
Find:



Answering Questions

Given:

Find:

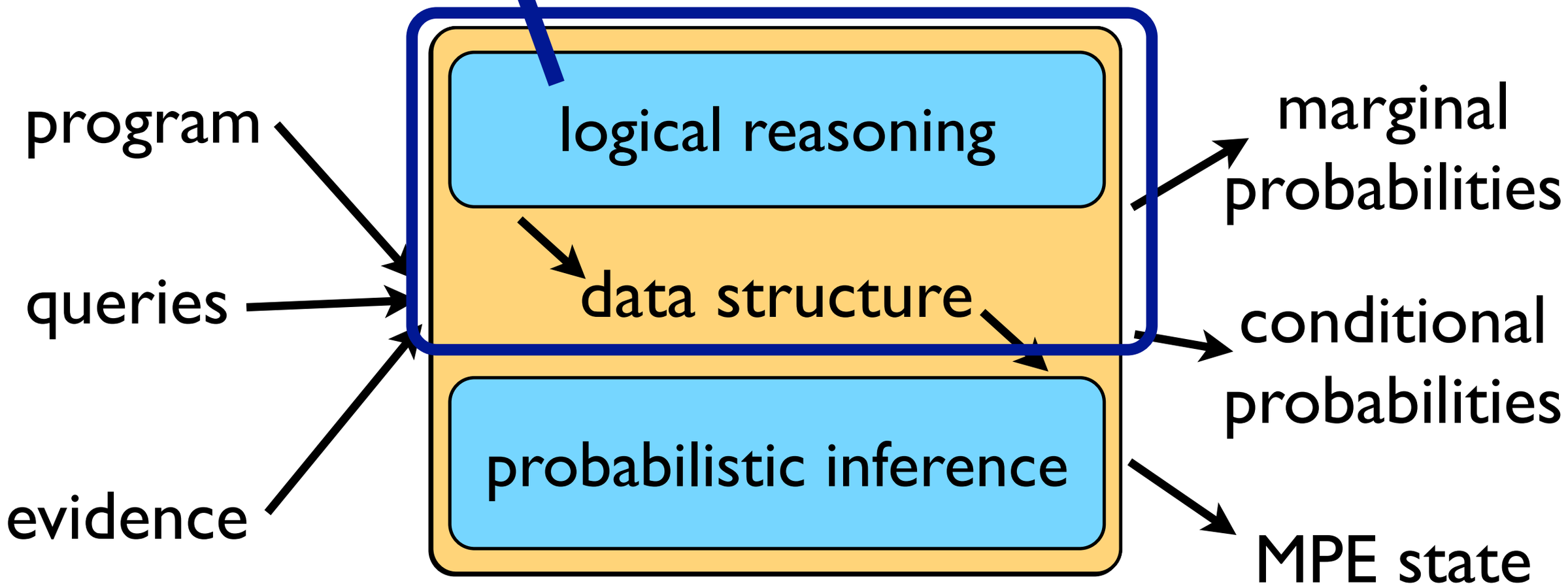


Answering Questions

1. using proofs
2. using models

knowledge compilation

Given:



Find:

Logical Reasoning: Proofs in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
      smokes(Y).
```

Logical Reasoning: Proofs in Prolog

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```

```
?- smokes(carl).
```

Logical Reasoning: Proofs in Prolog

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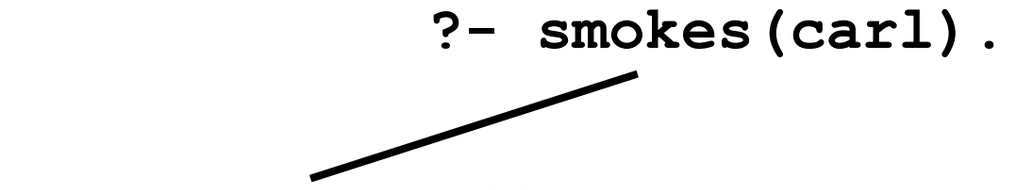

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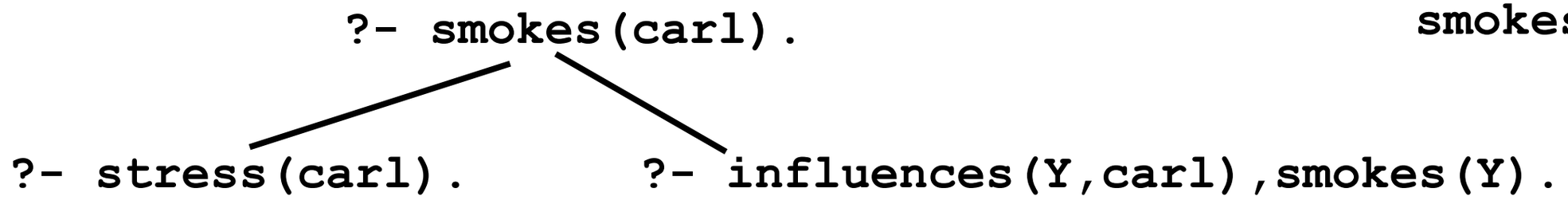

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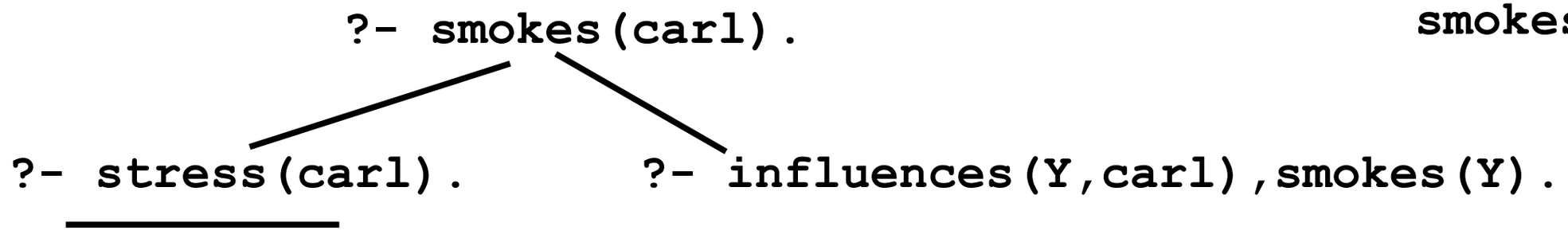

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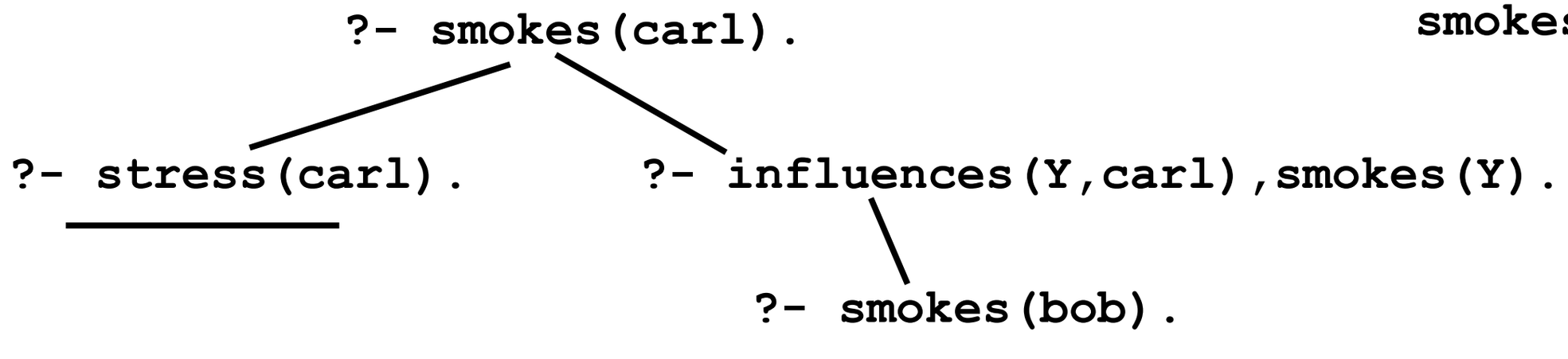

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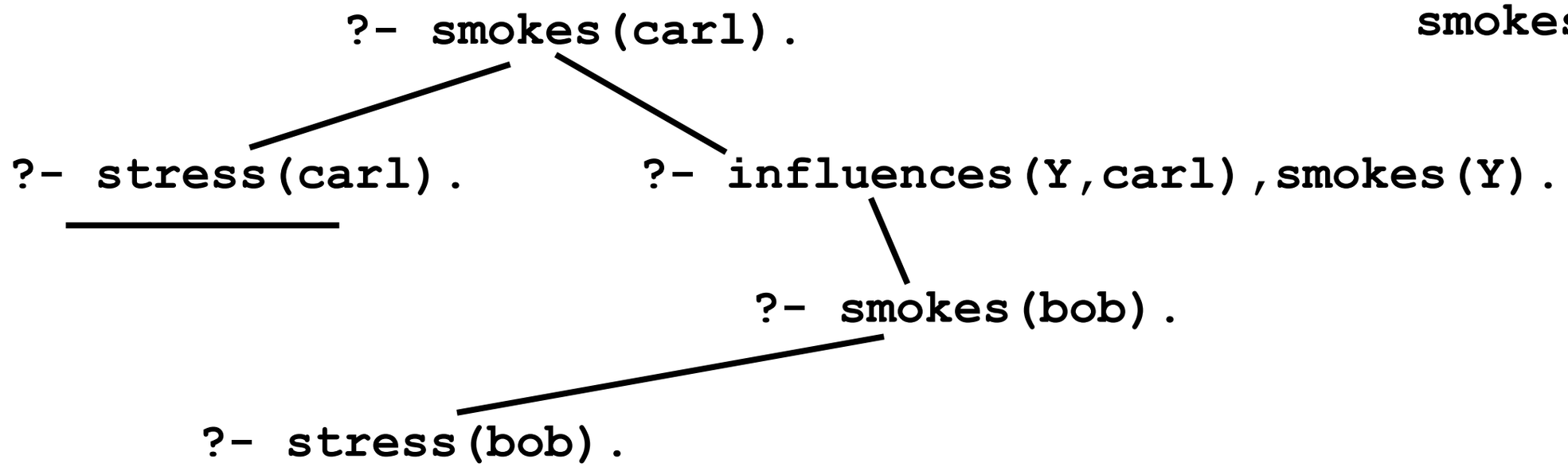

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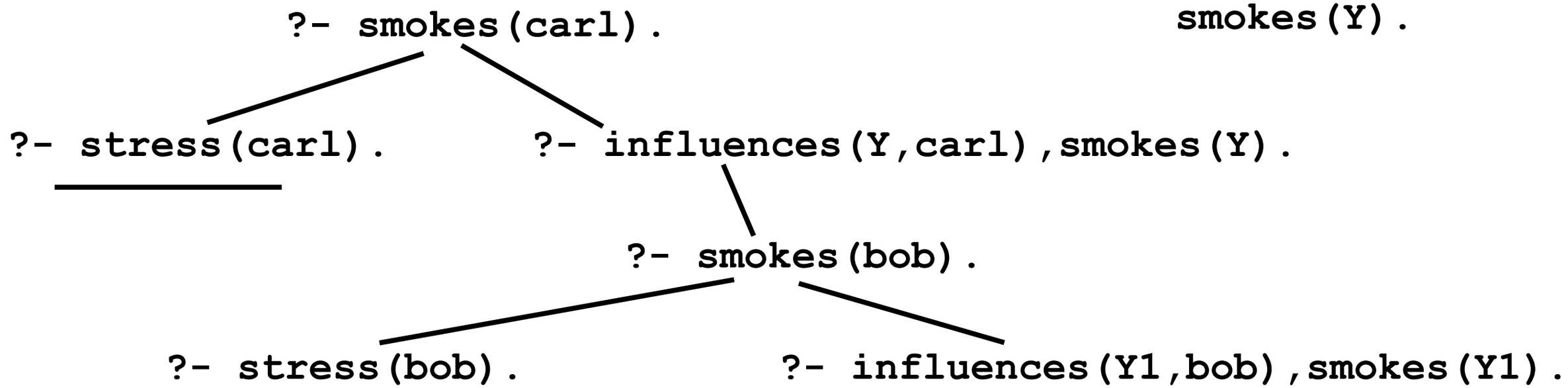

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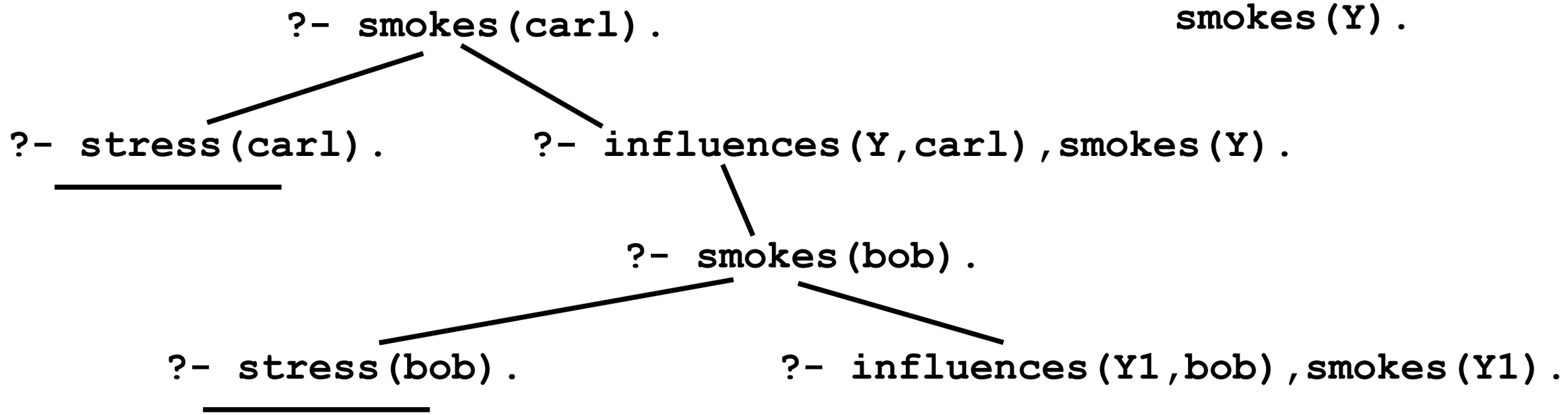

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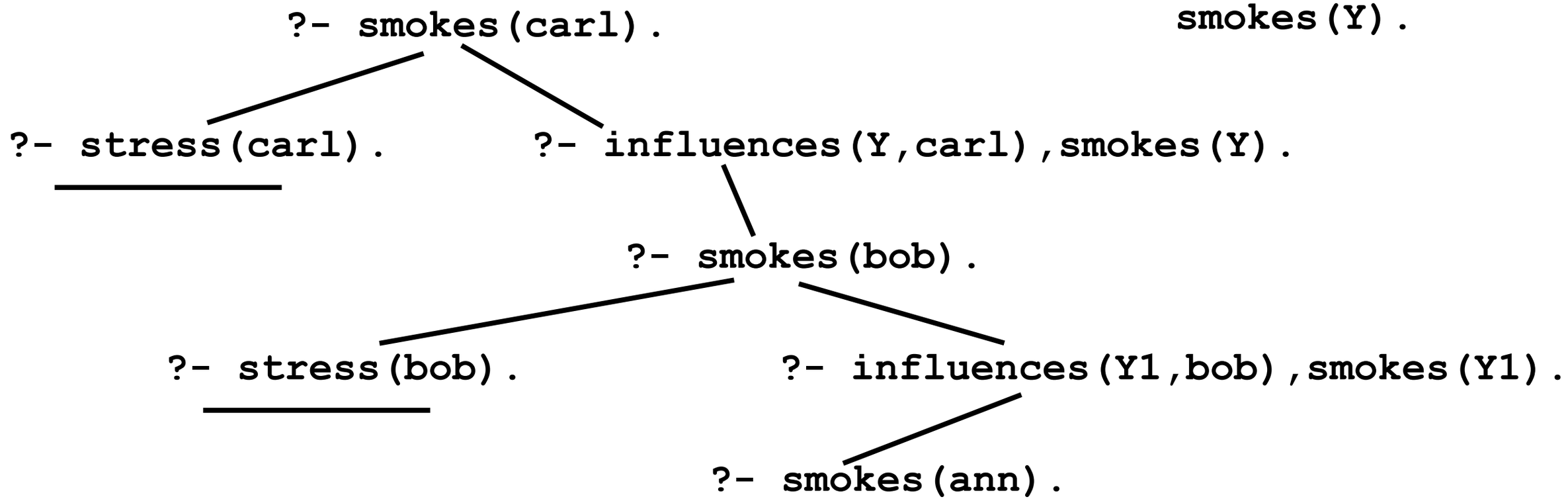

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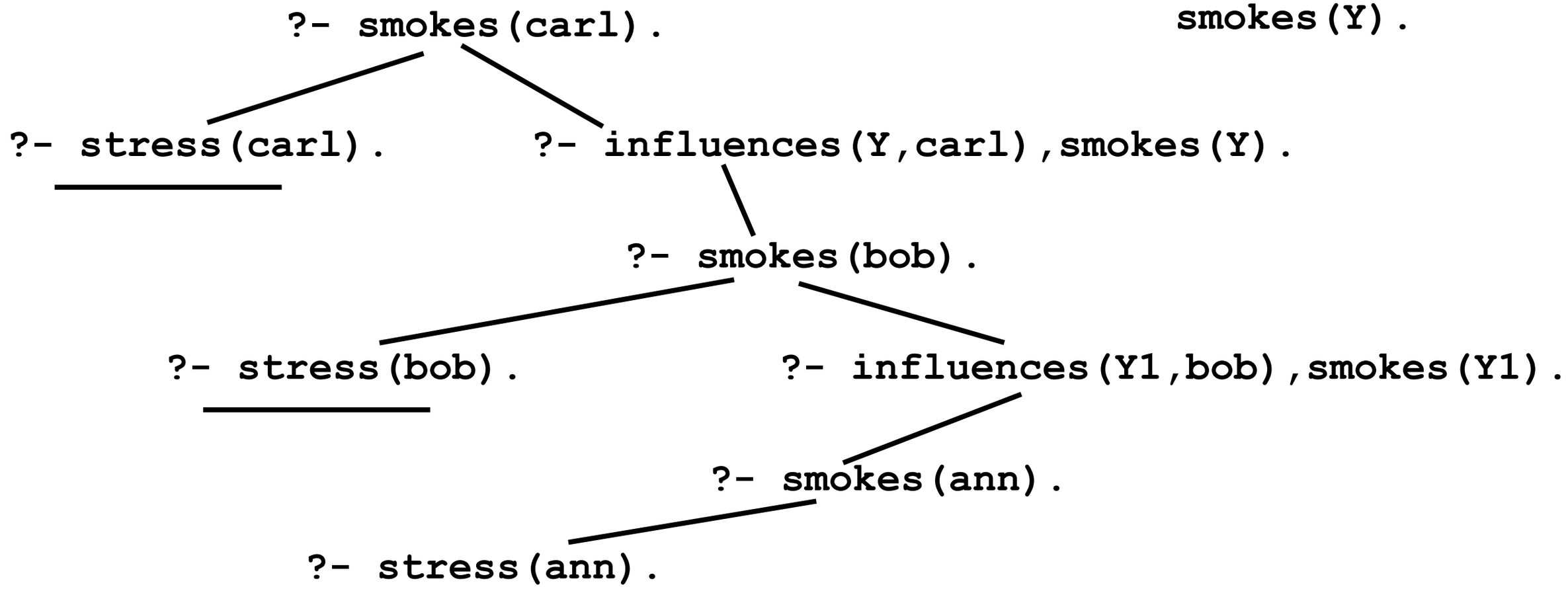

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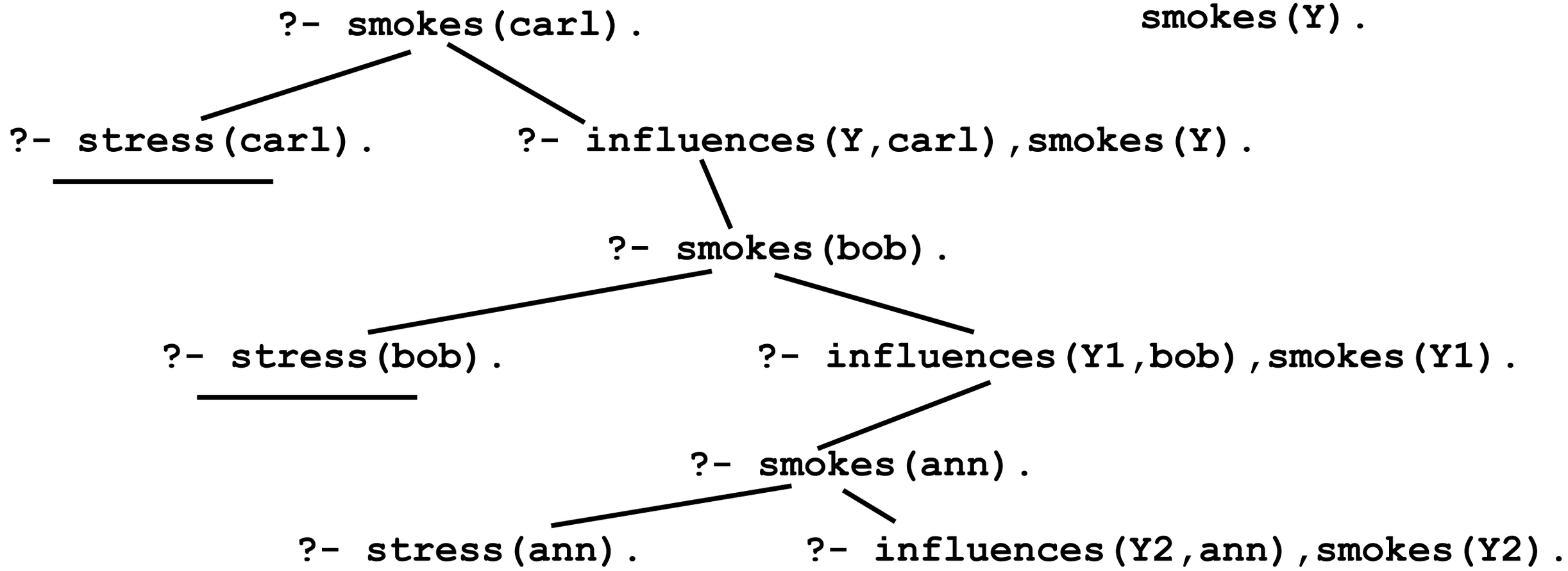

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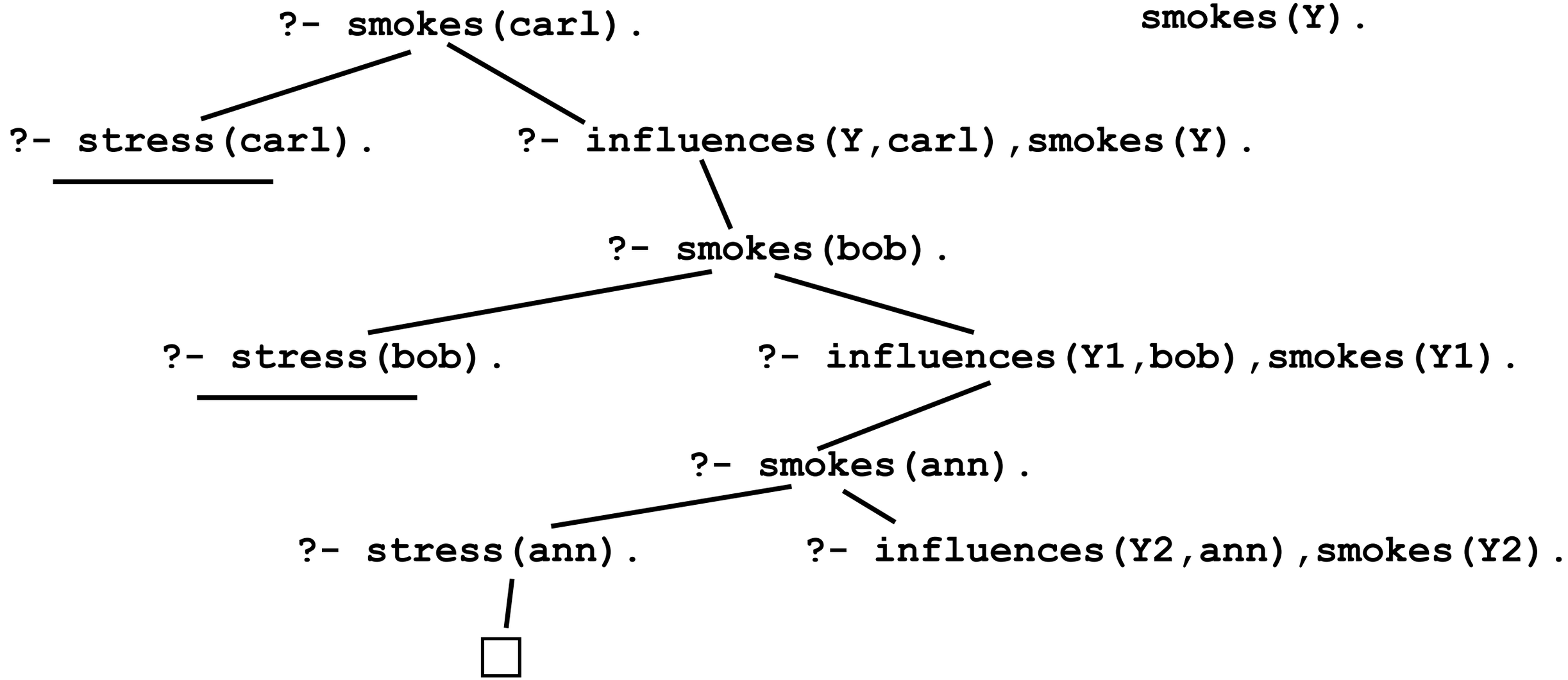

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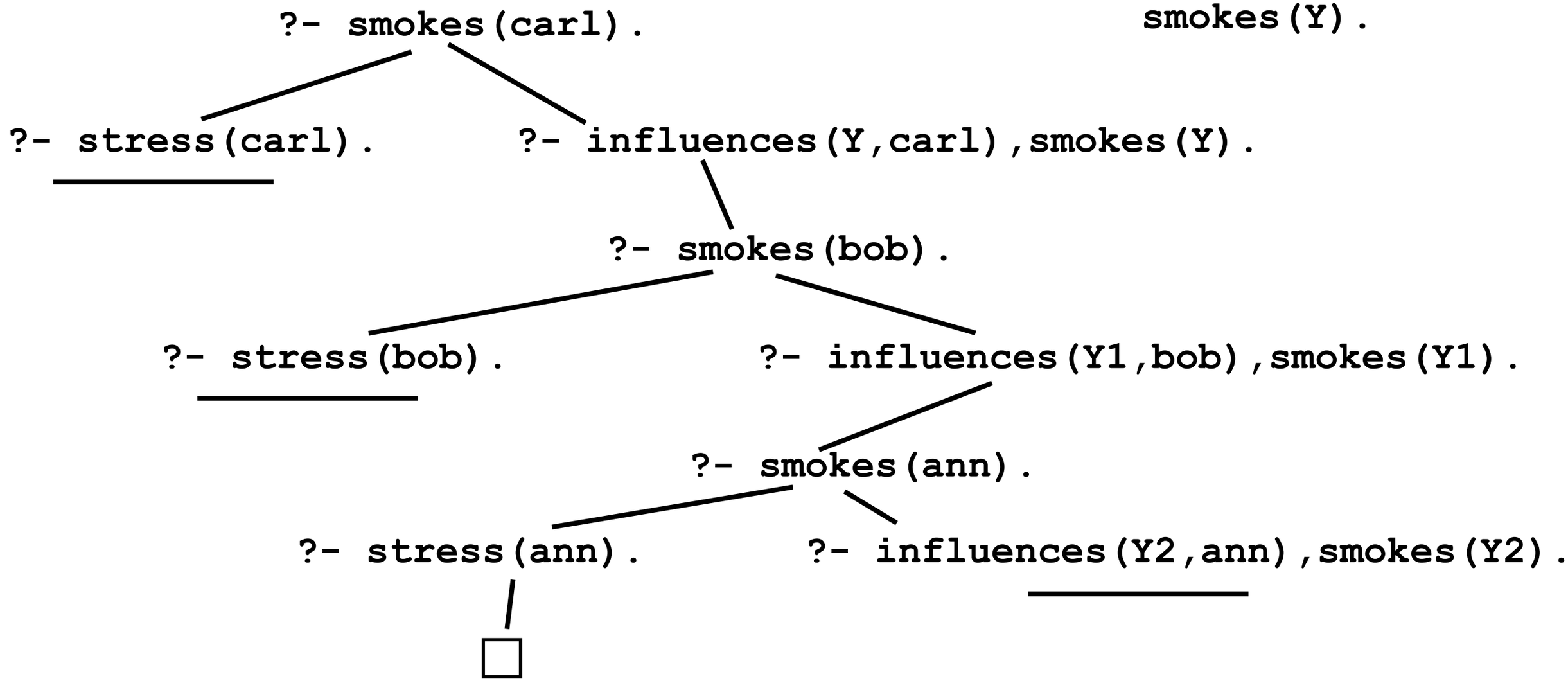

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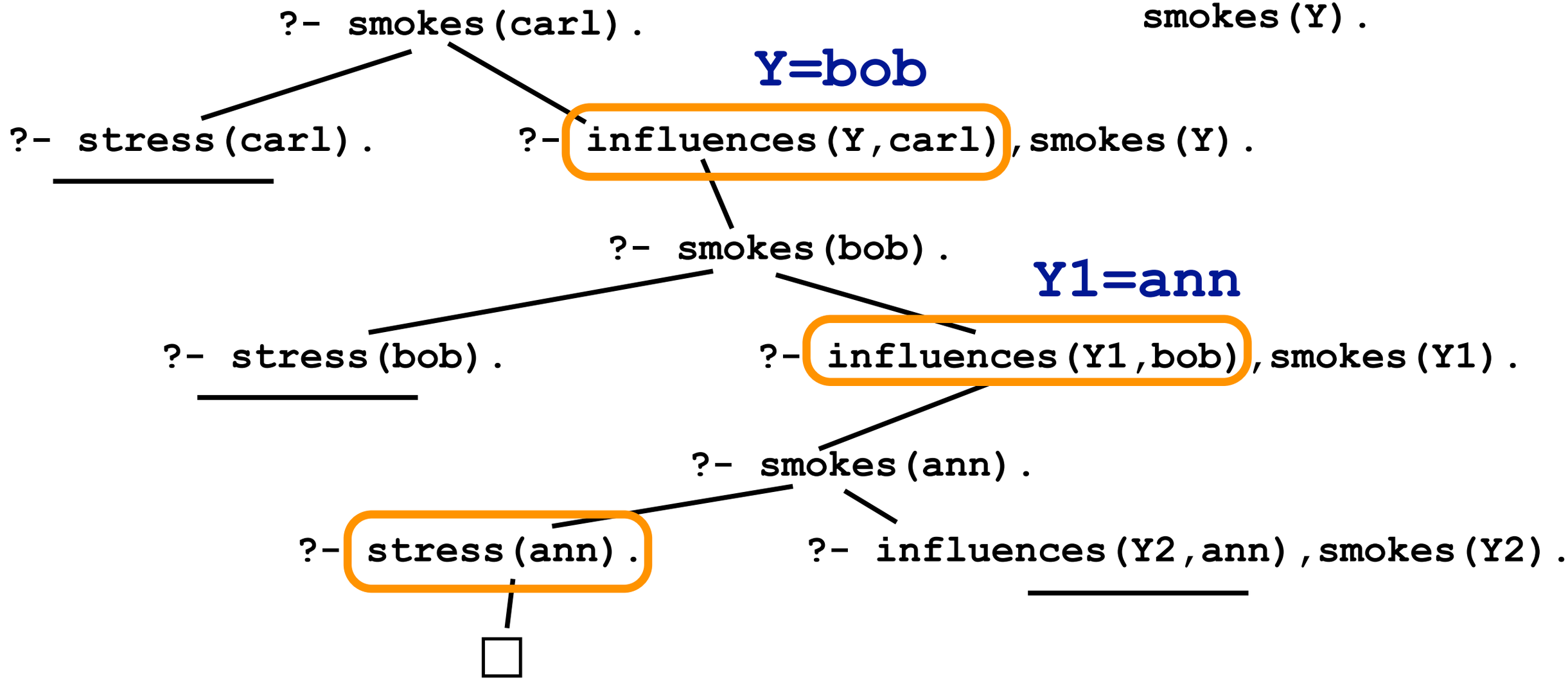


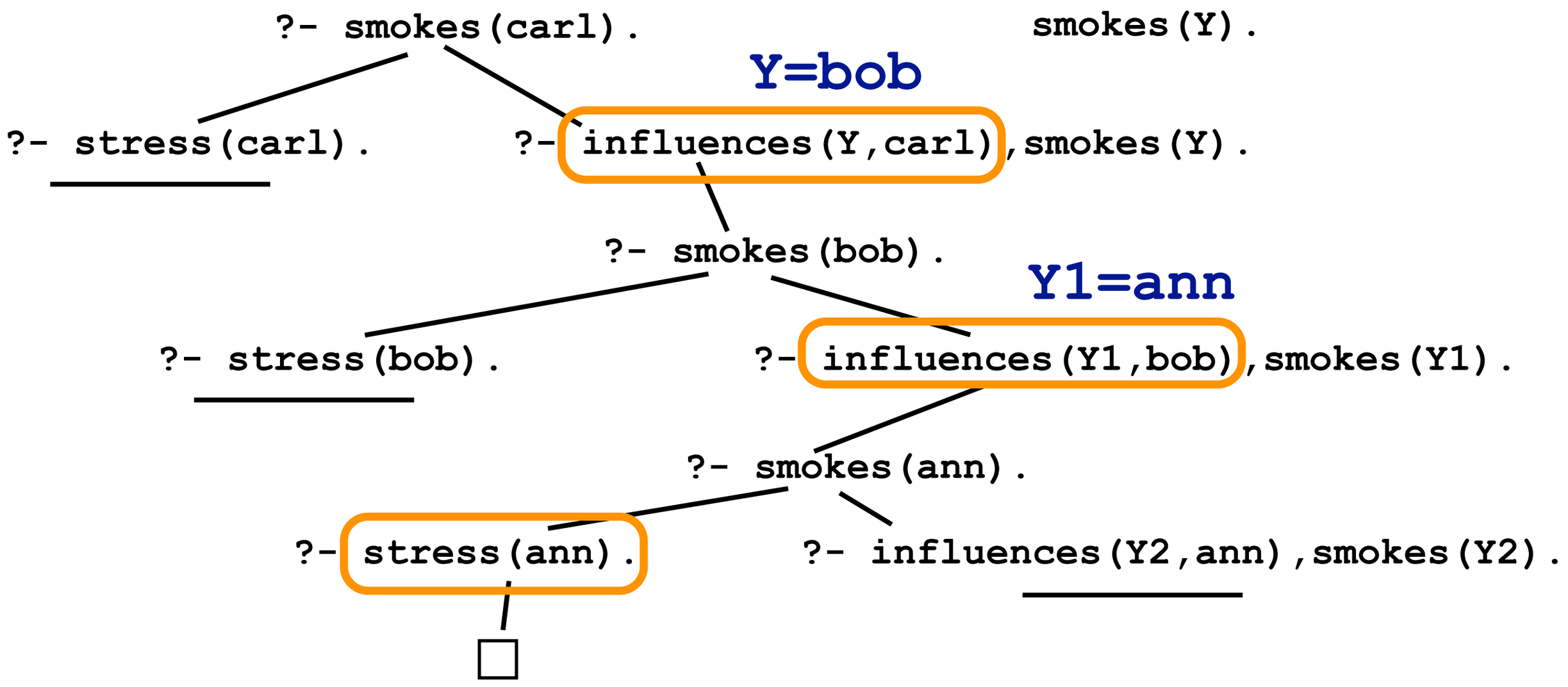
proof = facts used in successful derivation:

`influences(bob,carl) &influences(ann,bob) &stress(ann)`

Proofs in ProbLog

```
0.8::stress(ann).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
     influences(Y,X),
     smokes(Y).
```



`influences(bob,carl)&influences(ann,bob)&stress(ann)`

probability of proof = 0.2 × 0.6 × 0.8 = 0.096

Proofs in ProbLog

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
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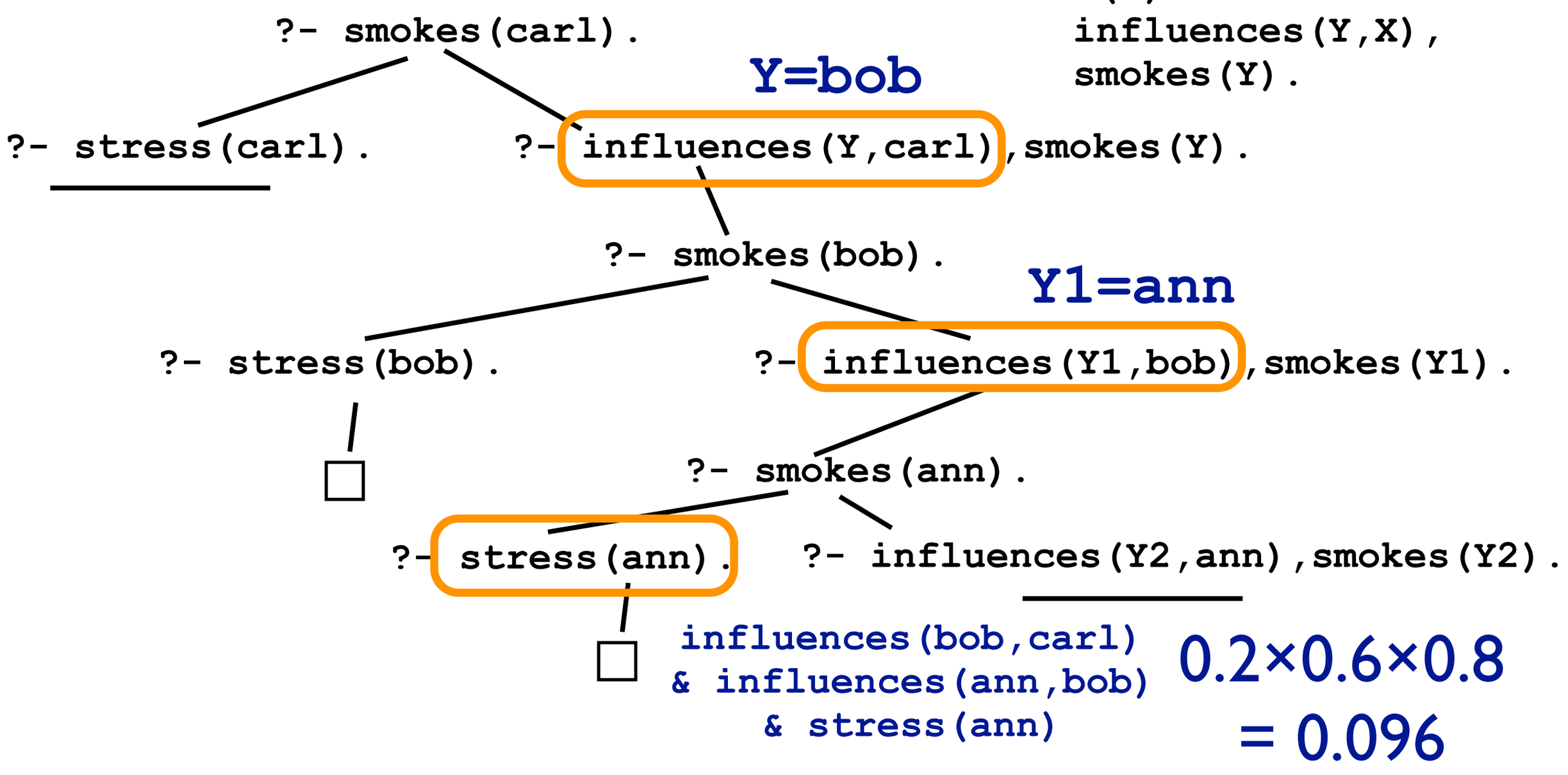

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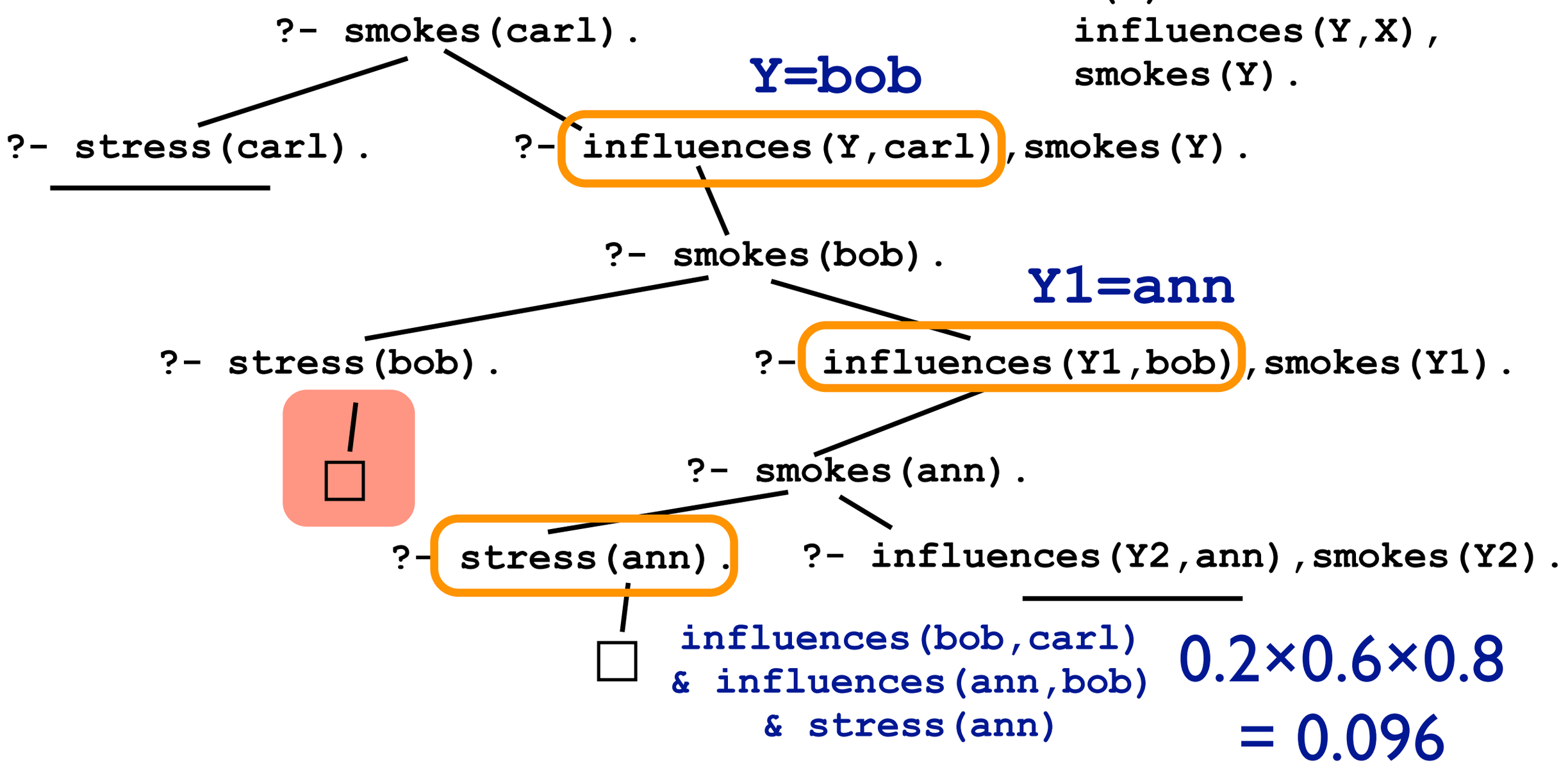

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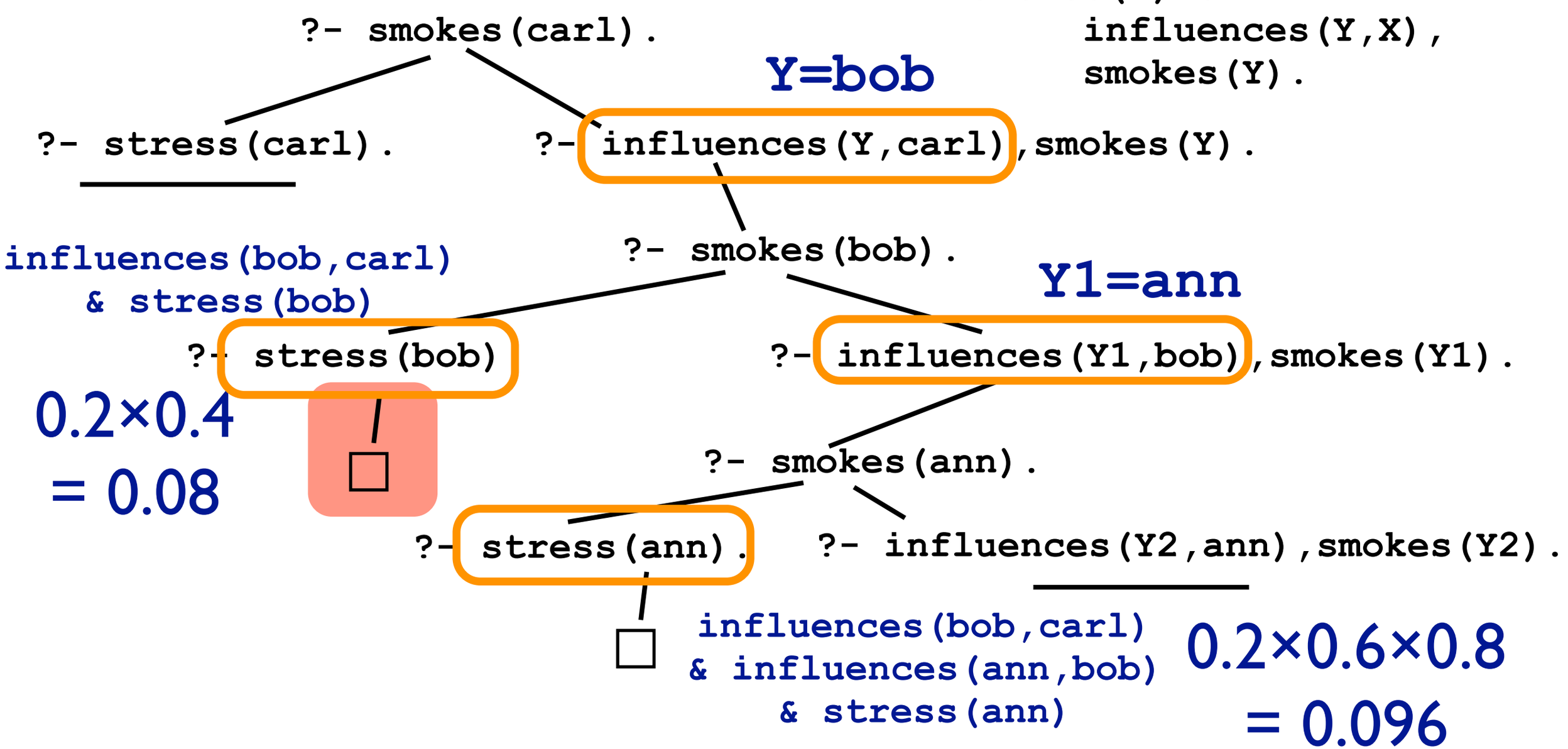

Proofs in ProbLog

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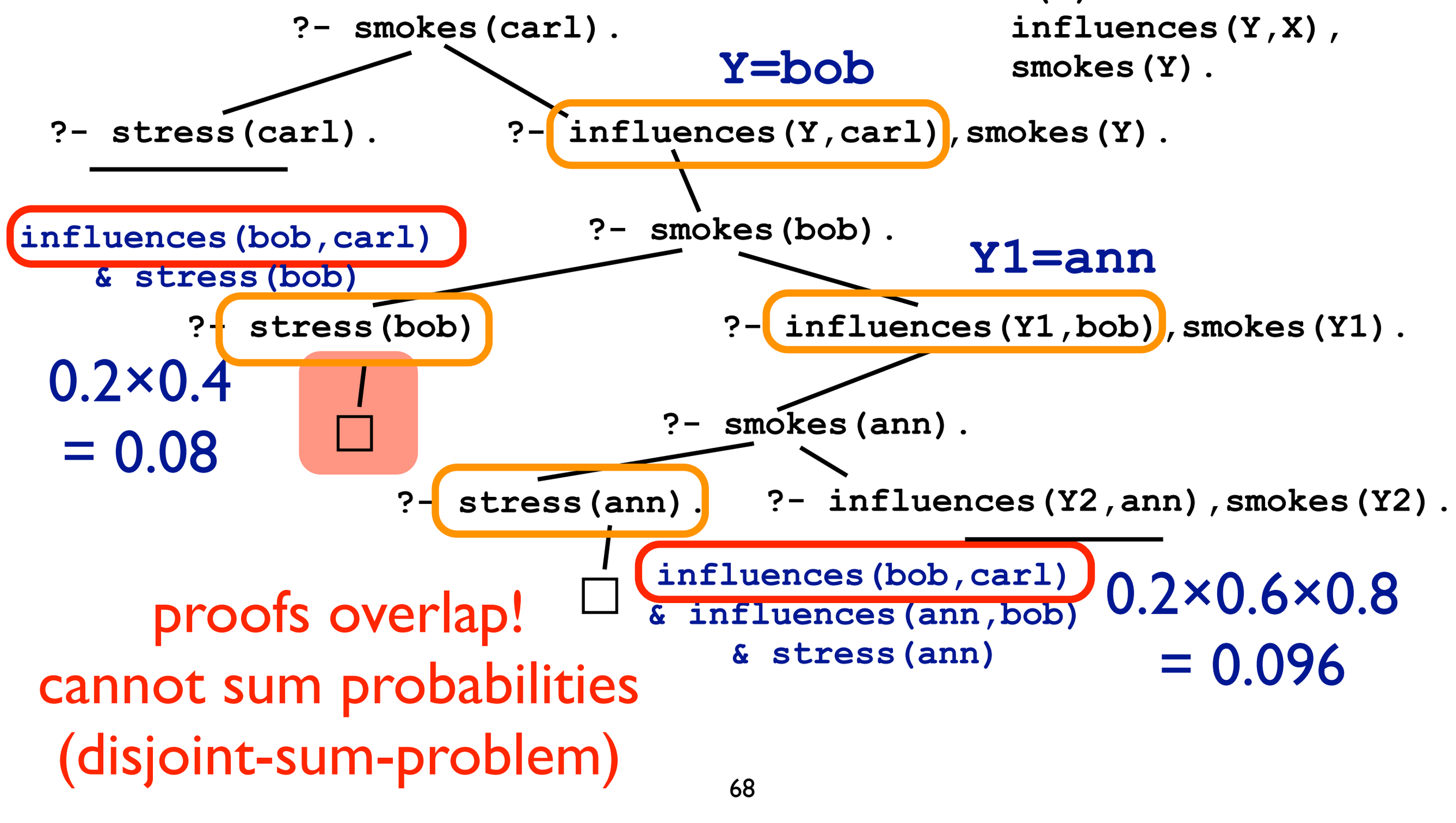


proofs overlap!
cannot sum probabilities
(disjoint-sum-problem)

Disjoint-Sum-Problem

possible worlds

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)

infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)

infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)

infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)

...
```

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
```

... **influences(bob,carl) & stress(bob)**

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

```
infl(bob,carl) &   infl(ann,bob) &   st(ann) & \+st(bob)
infl(bob,carl) &   infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) &   st(ann) &   st(bob)
infl(bob,carl) &   infl(ann,bob) & \+st(ann) &   st(bob)
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) &   st(bob)
```

... **influences(bob,carl) & stress(bob)**

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

influences(bob,carl) &
influences(ann,bob) & stress(ann)

infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)	0.0576
infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)	0.0384
infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)	0.0256
infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)	0.0096
infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)	0.0064
... **influences(bob,carl) & stress(bob)**	∑ = 0.1376

sum of proof probabilities: 0.096+0.08 = 0.1760

Disjoint-Sum-Problem

possible worlds

solution: knowledge compilation

`infl(bob,carl) & infl(ann,bob) & st(ann) & \+st(bob)`	0.0576
`infl(bob,carl) & infl(ann,bob) & st(ann) & st(bob)`	0.0384
`infl(bob,carl) & \+infl(ann,bob) & st(ann) & st(bob)`	0.0256
`infl(bob,carl) & infl(ann,bob) & \+st(ann) & st(bob)`	0.0096
`infl(bob,carl) & \+infl(ann,bob) & \+st(ann) & st(bob)`	0.0064
`...` `influences(bob,carl) & stress(bob)`	$\sum = 0.1376$

sum of proof probabilities: 0.096+0.08 = 0.1760

Binary Decision Diagrams [Bryant 86]

- compact graphical representation of Boolean formula
- popular in many branches of CS
- automatically disjoins proofs
  → efficient probability computation

Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$

Binary Decision Diagrams
[Bryant 86]

$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

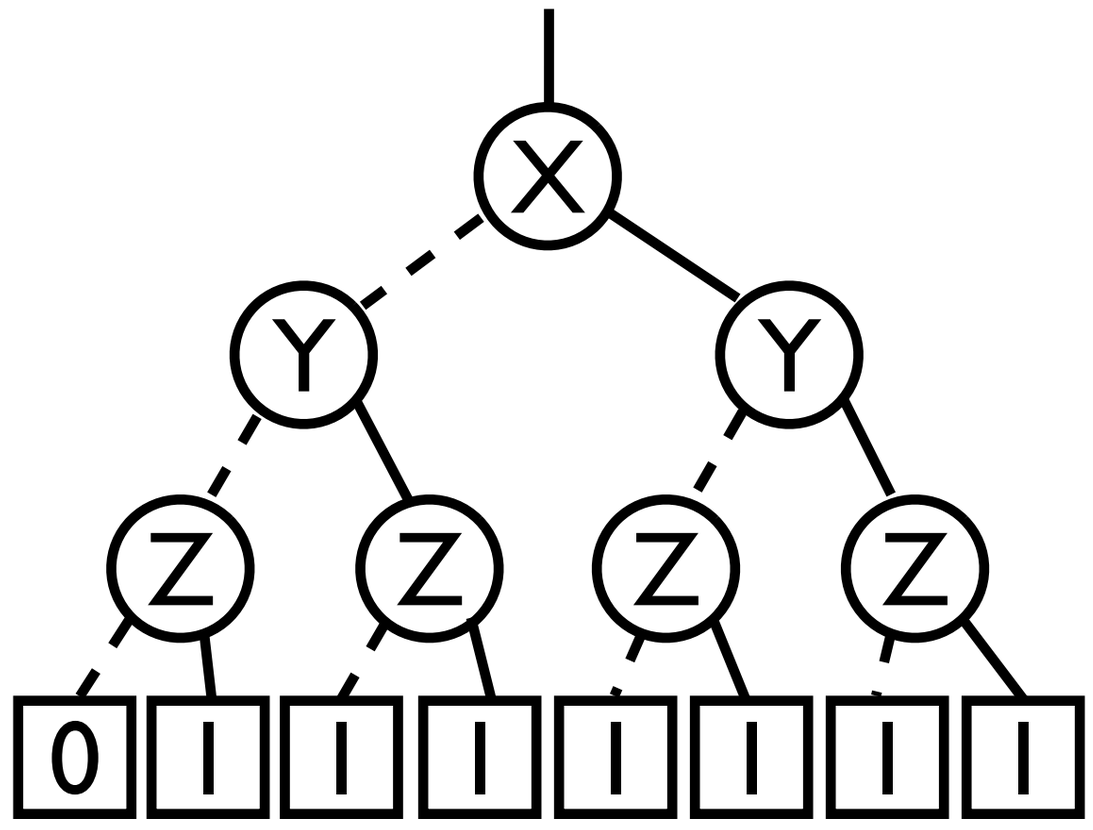
$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

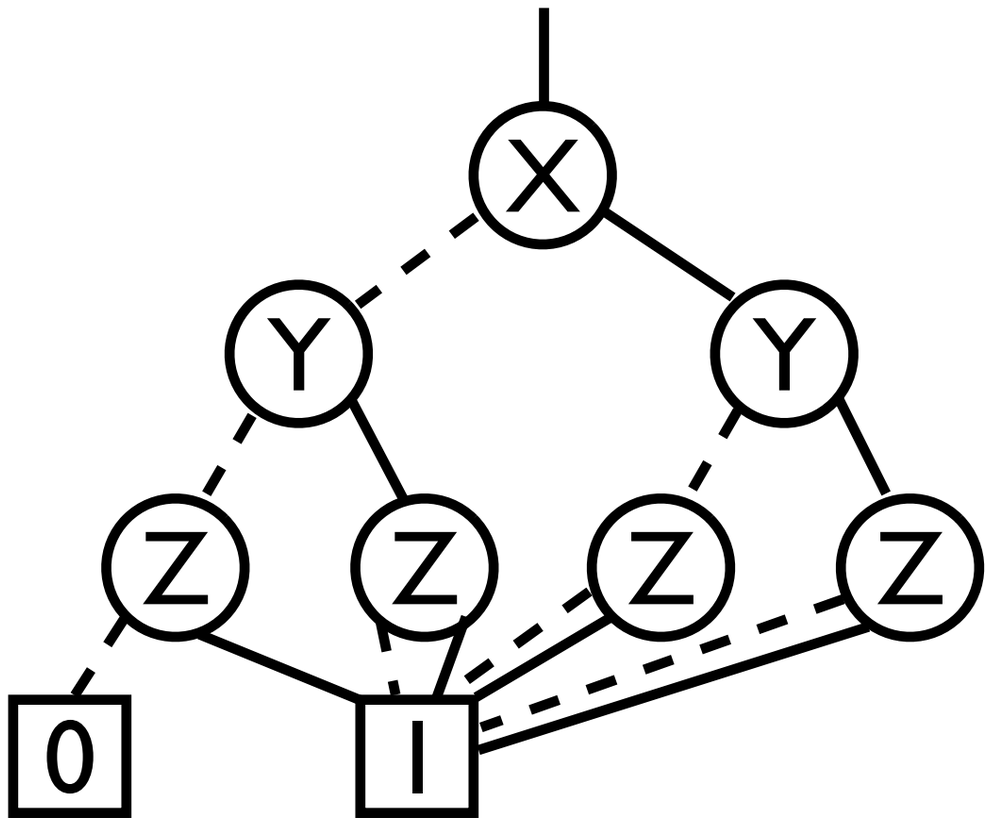
$X \vee Y \vee Z$



X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

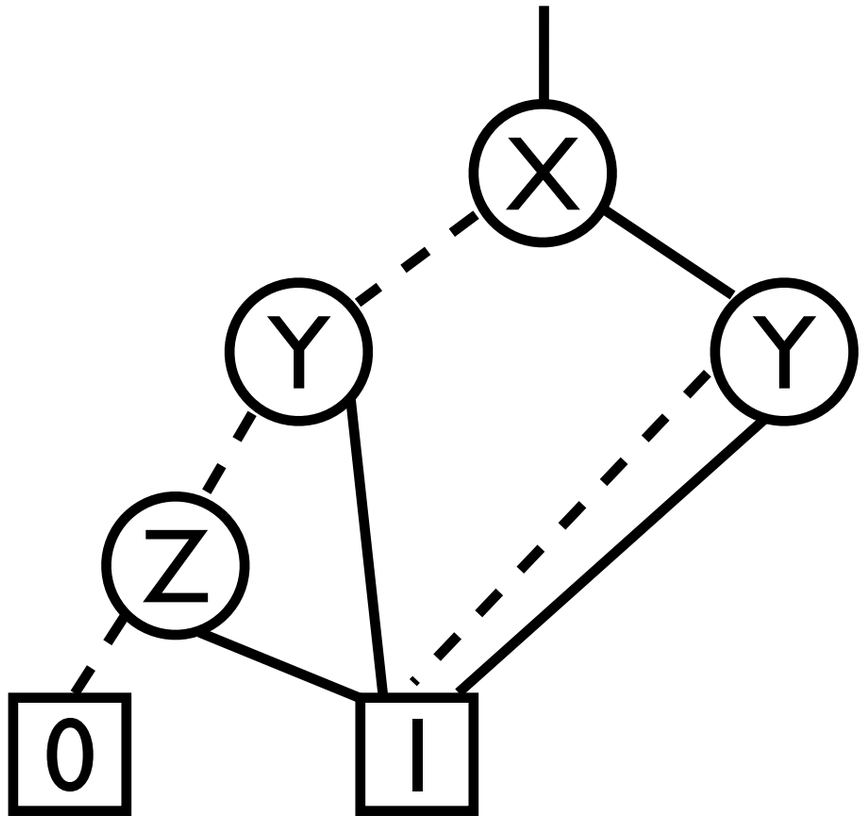


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

[Bryant 86]

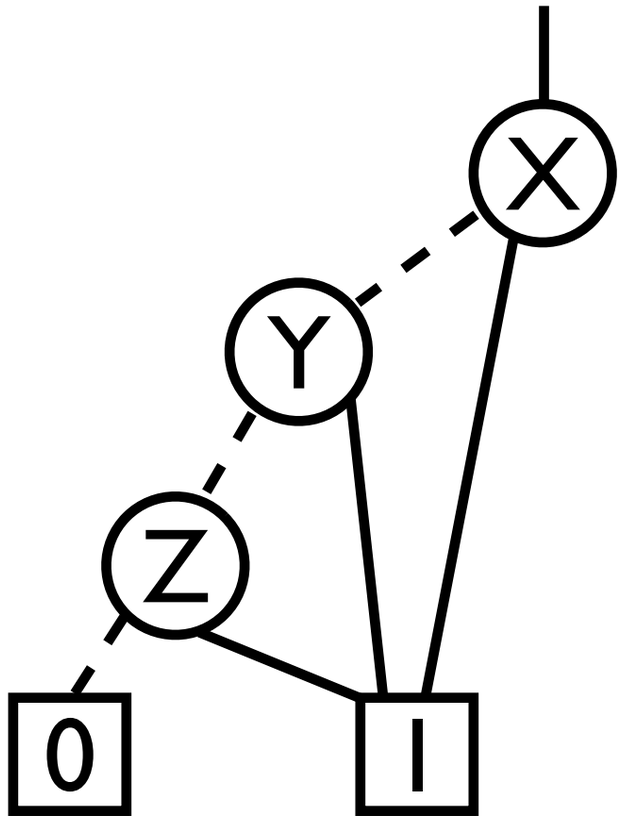


$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams

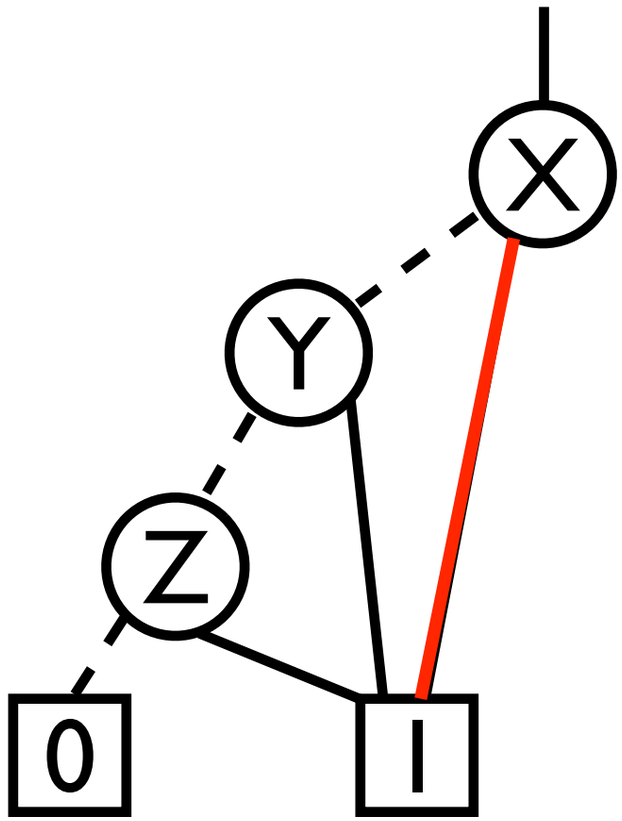
[Bryant 86]



$X \vee Y \vee Z$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

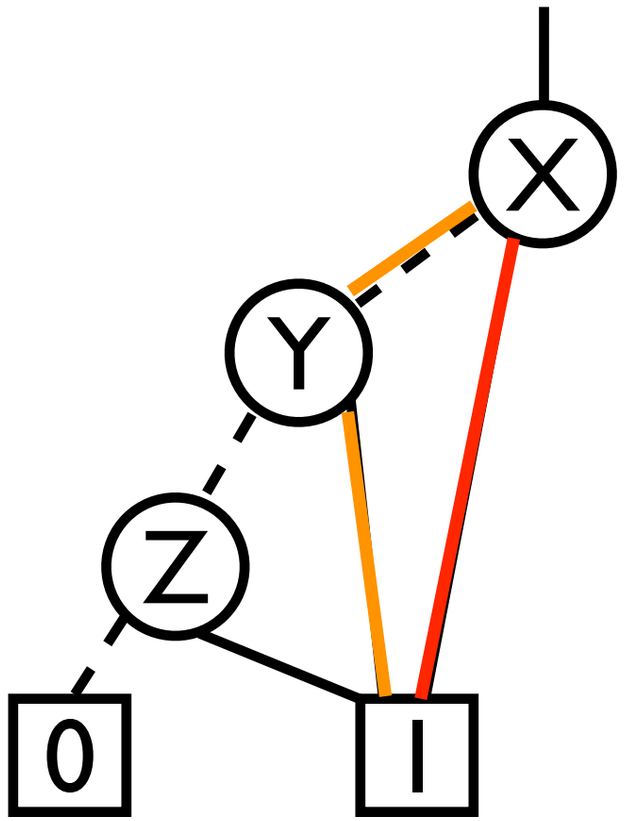


$X \vee Y \vee Z$

$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

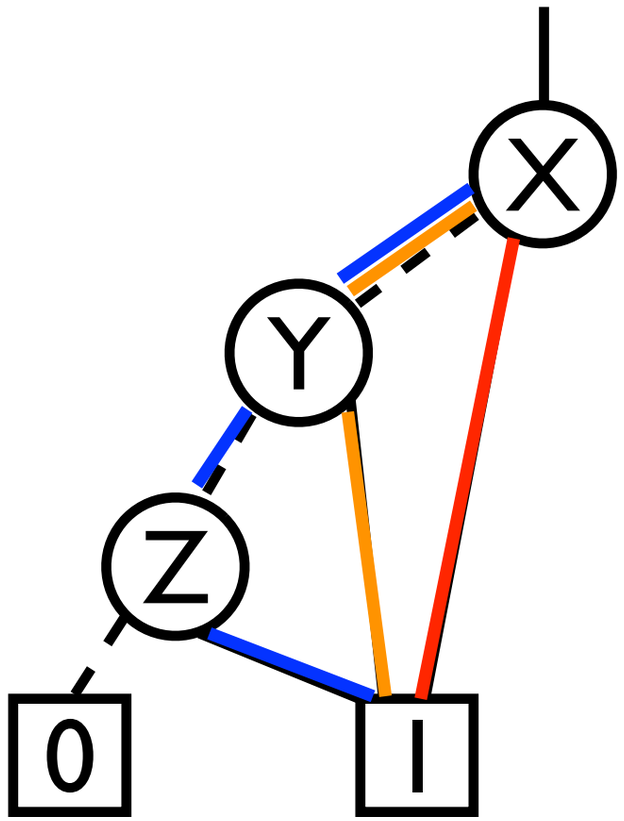


$X \vee Y \vee Z$

$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]

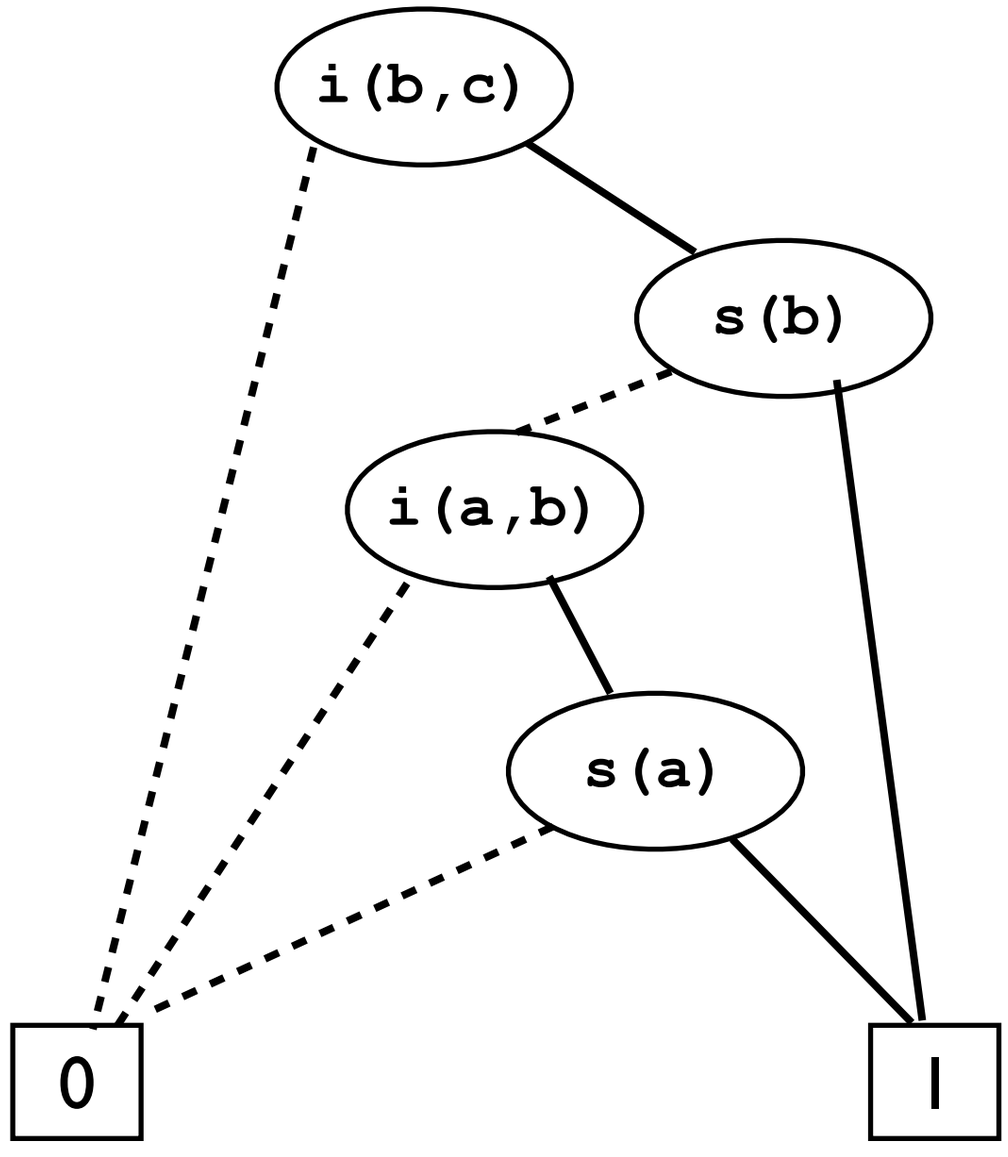


$$X \vee Y \vee Z$$

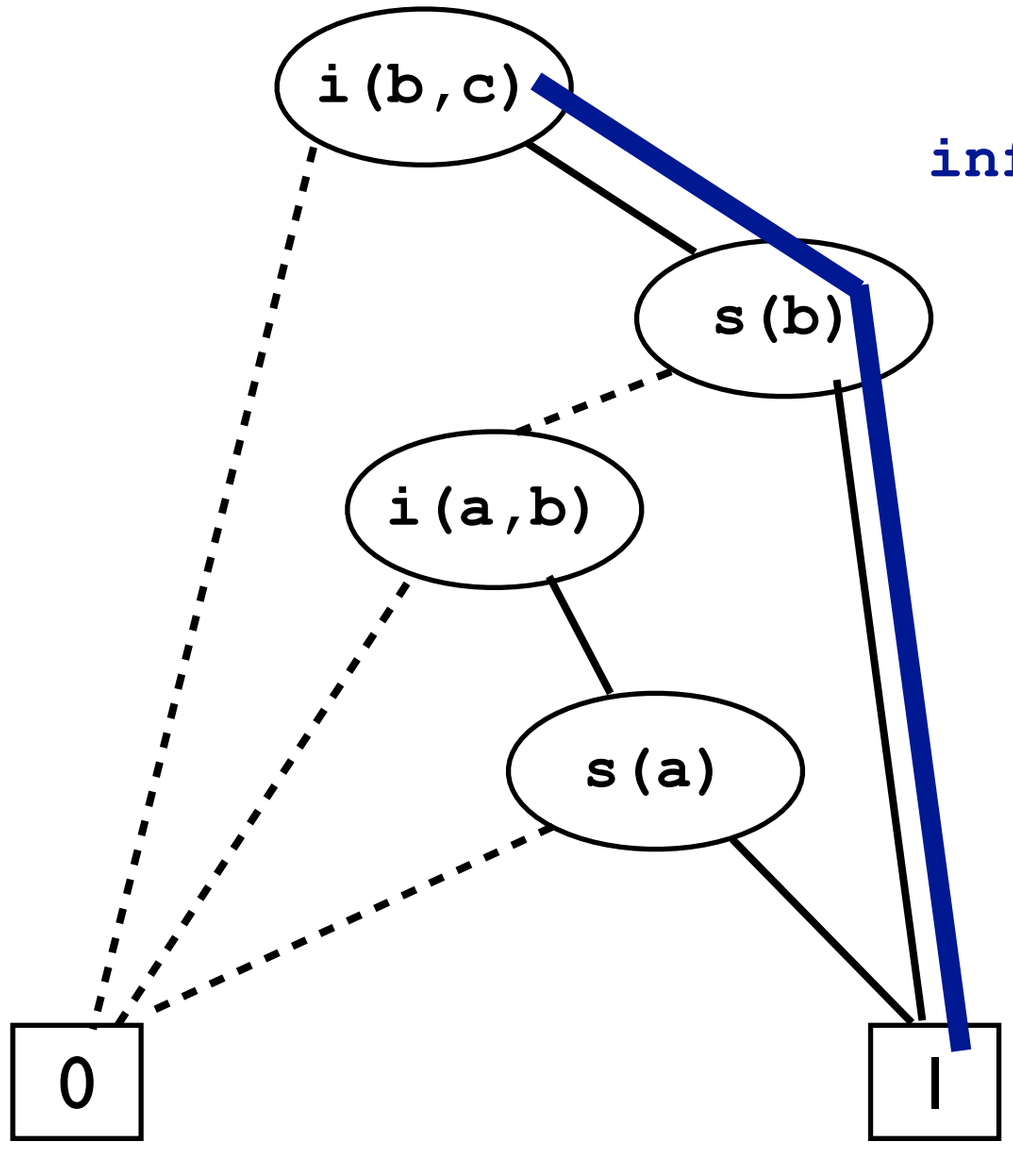
$$X \vee (\neg X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z)$$

X	0	0	0	0	1	1	1	1
Y	0	0	1	1	0	0	1	1
Z	0	1	0	1	0	1	0	1

Binary Decision Diagrams
[Bryant 86]



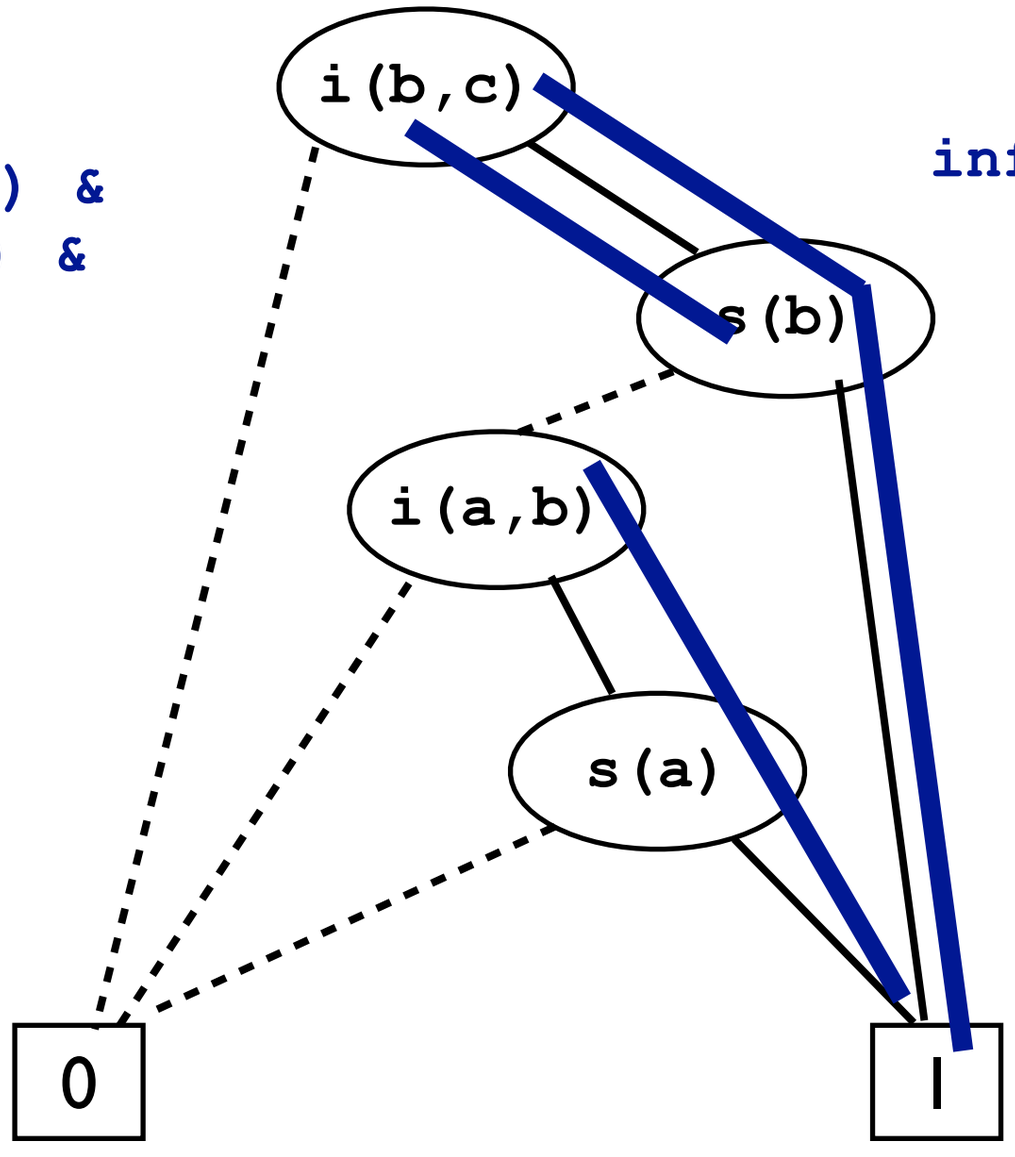
Binary Decision Diagrams
[Bryant 86]



influences(bob,carl) & stress(bob)

Binary Decision Diagrams
[Bryant 86]

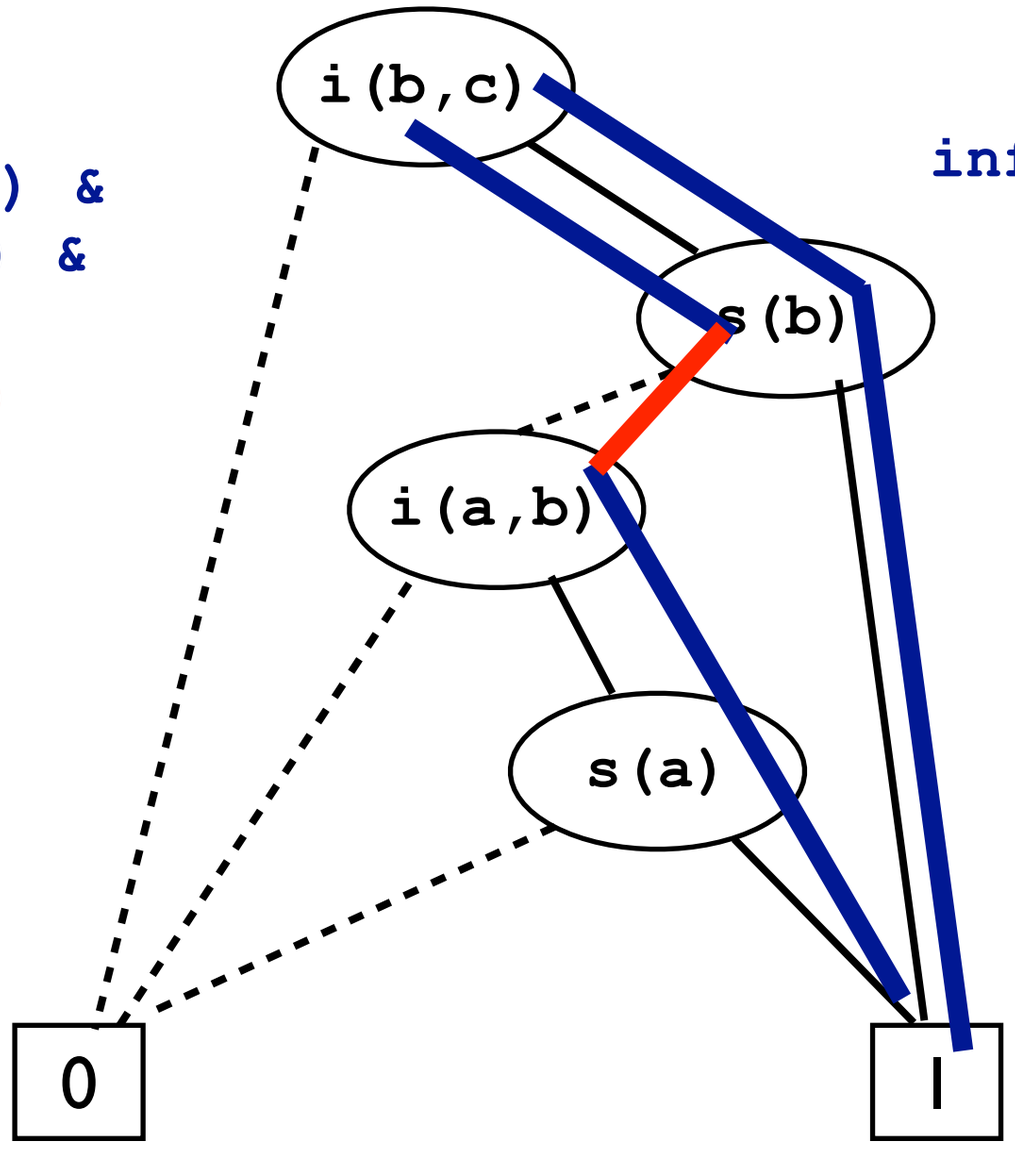
`influences(bob,carl) & influences(ann,bob) & stress(ann)`



`influences(bob,carl) & stress(bob)`

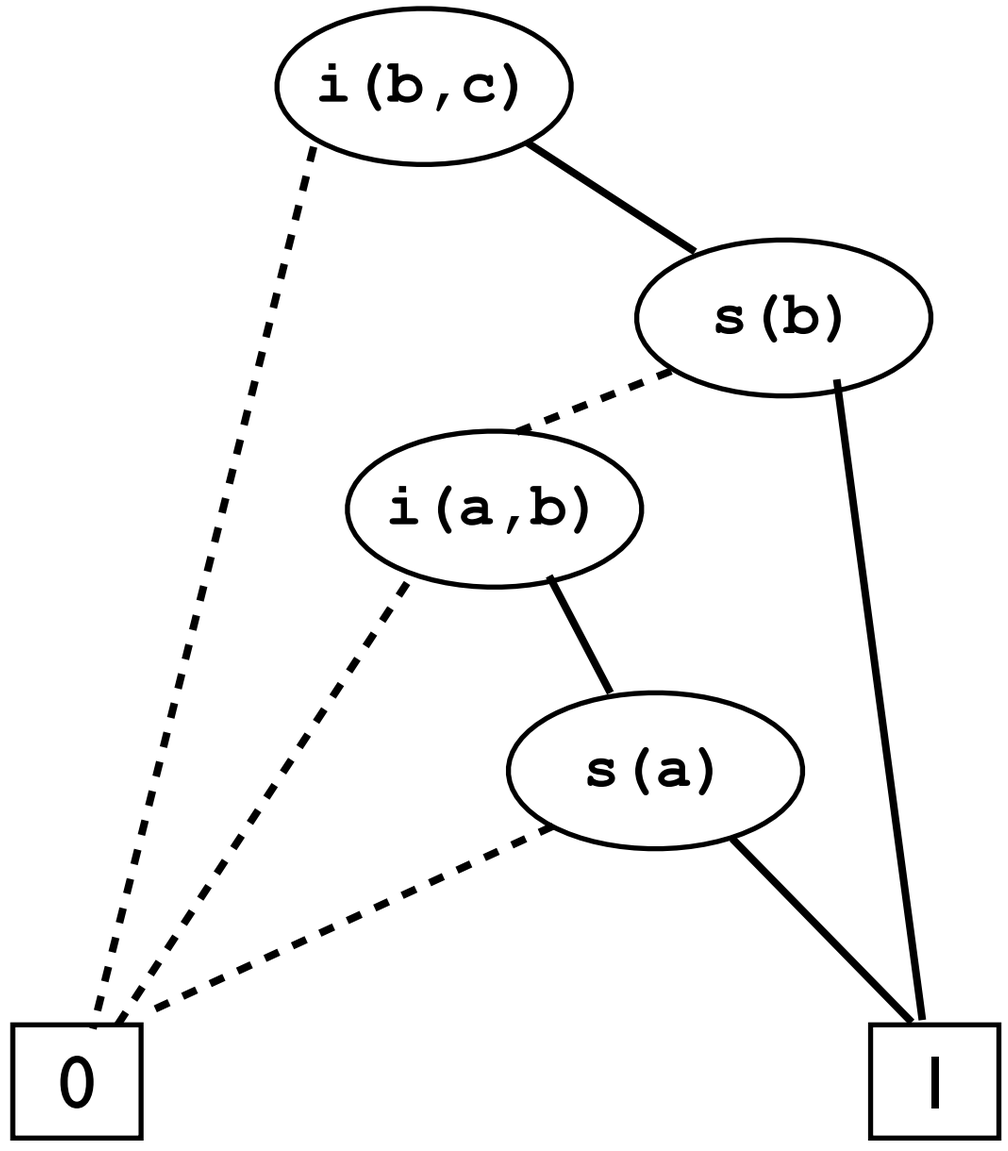
Binary Decision Diagrams
[Bryant 86]

influences(bob,carl) &
influences(ann,bob) &
stress(ann)
& not stress(bob)

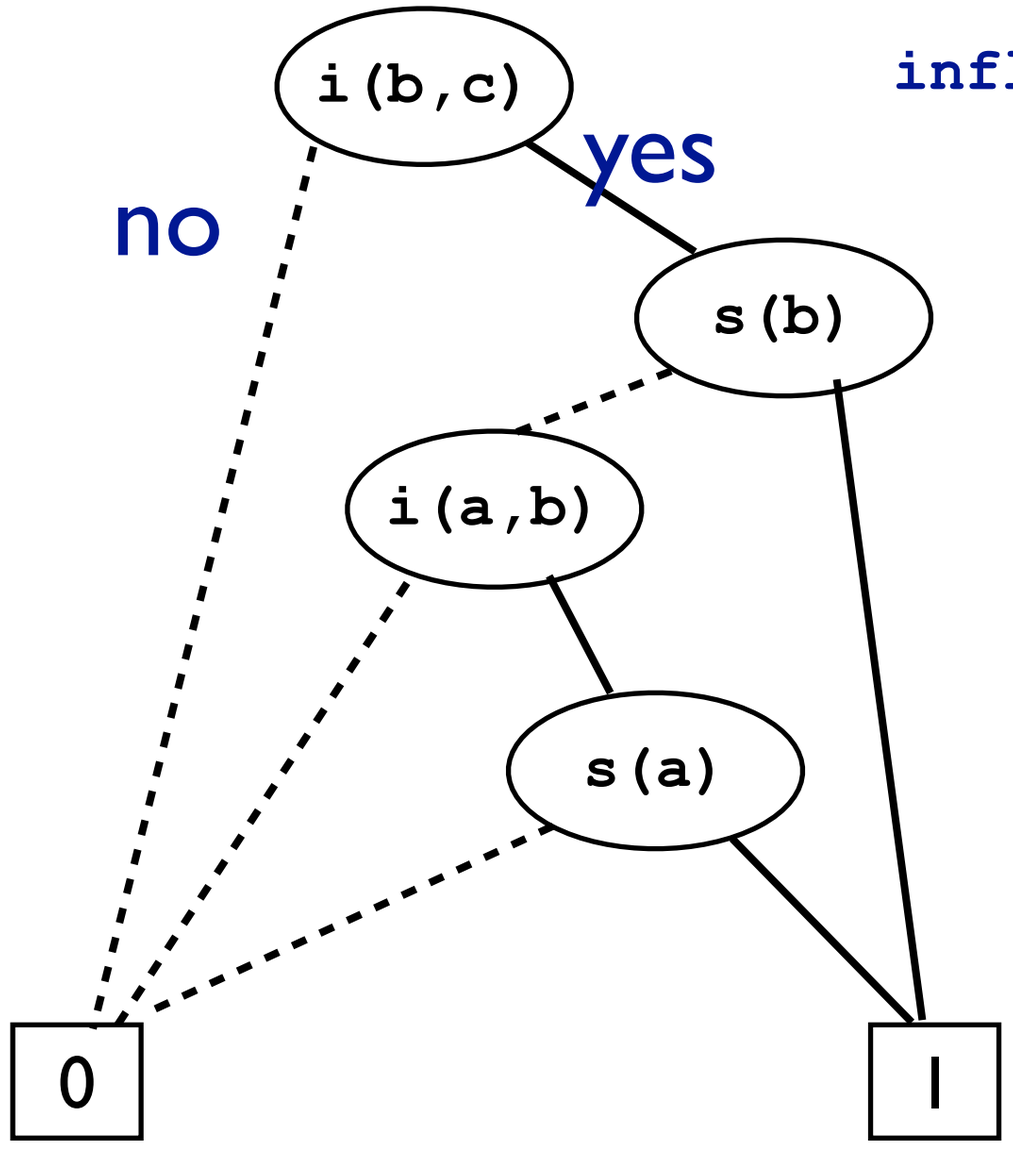


influences(bob,carl) &
stress(bob)

Binary Decision Diagrams

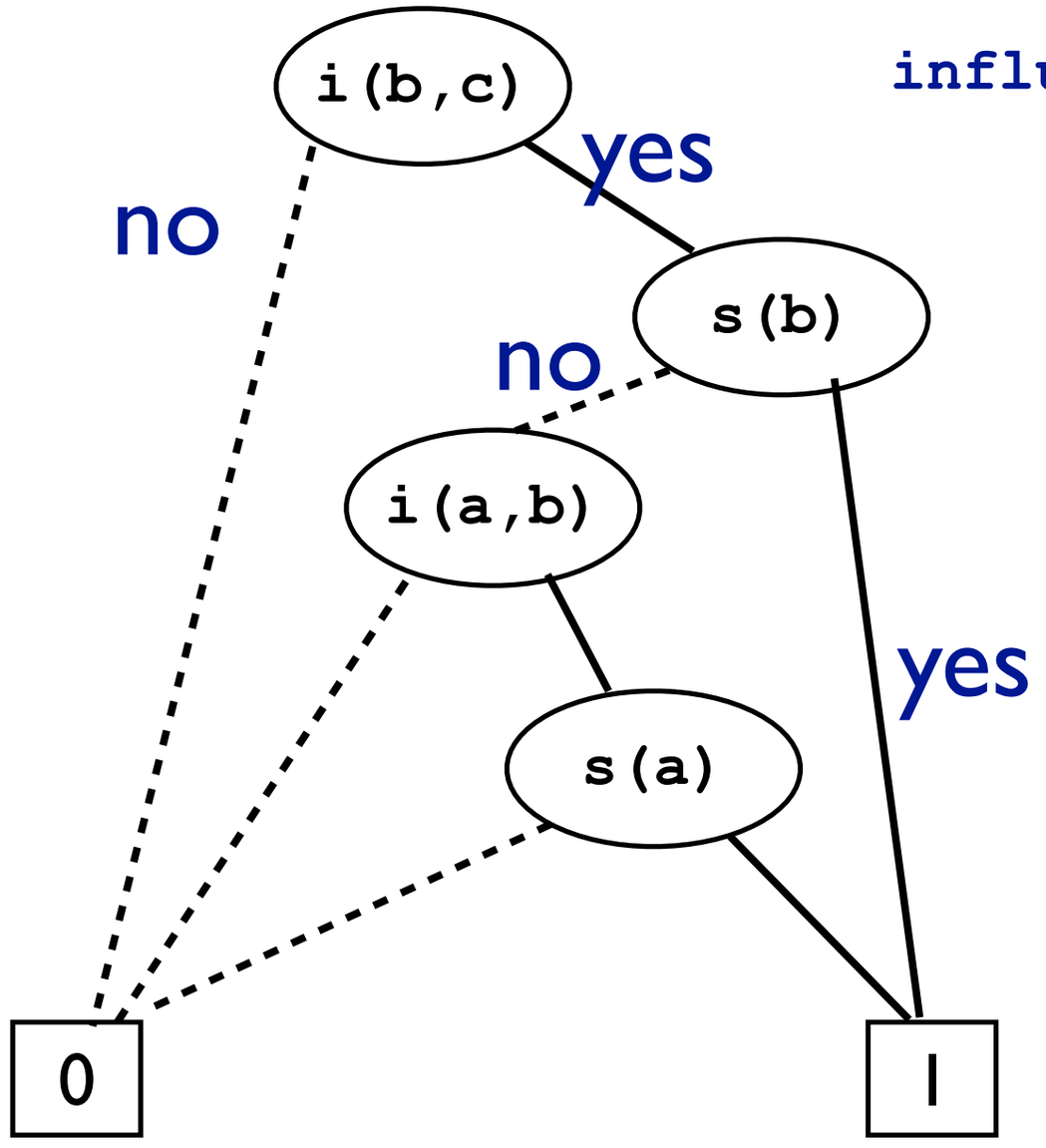


Binary Decision Diagrams



influences(bob,carl)?

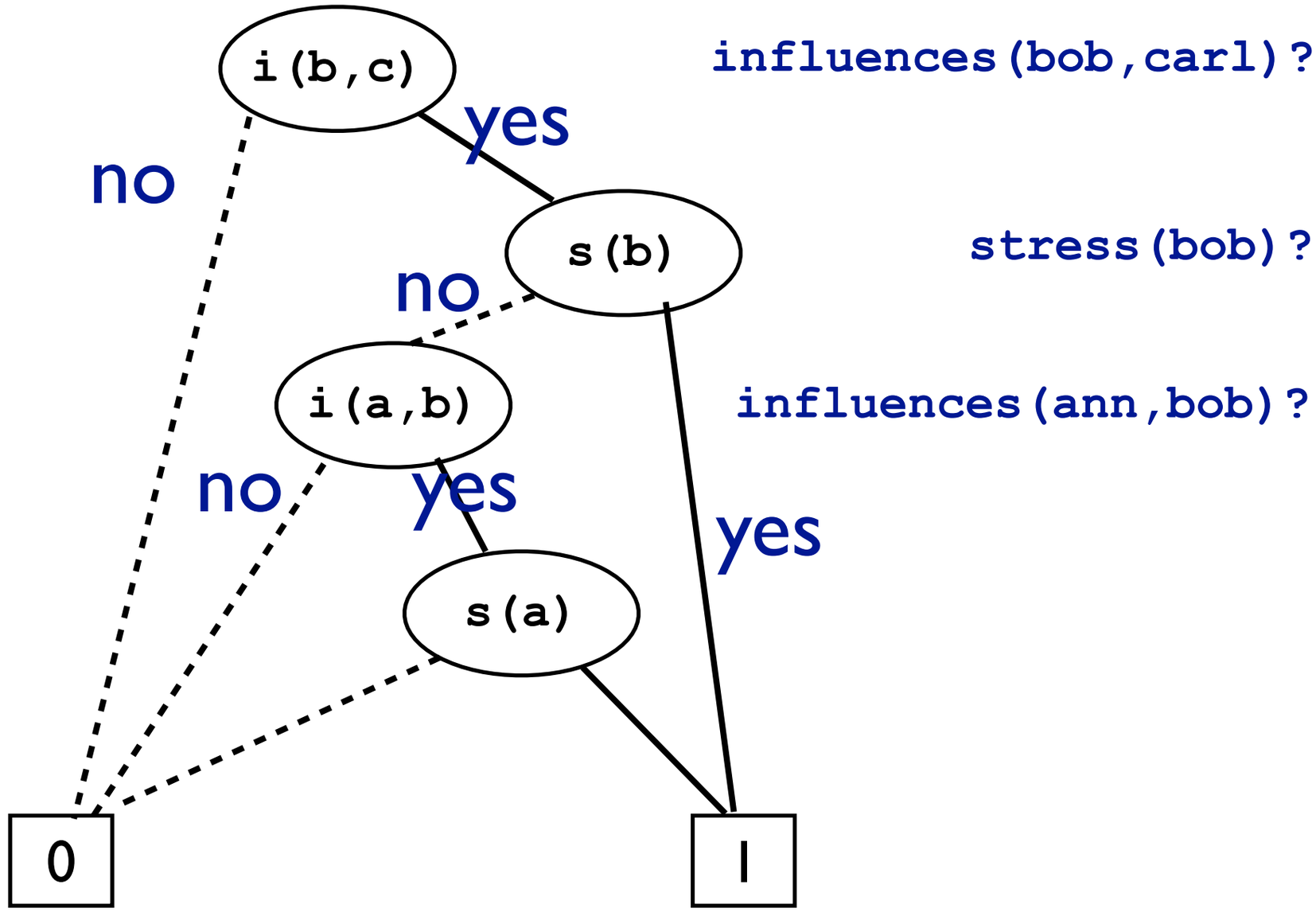
Binary Decision Diagrams



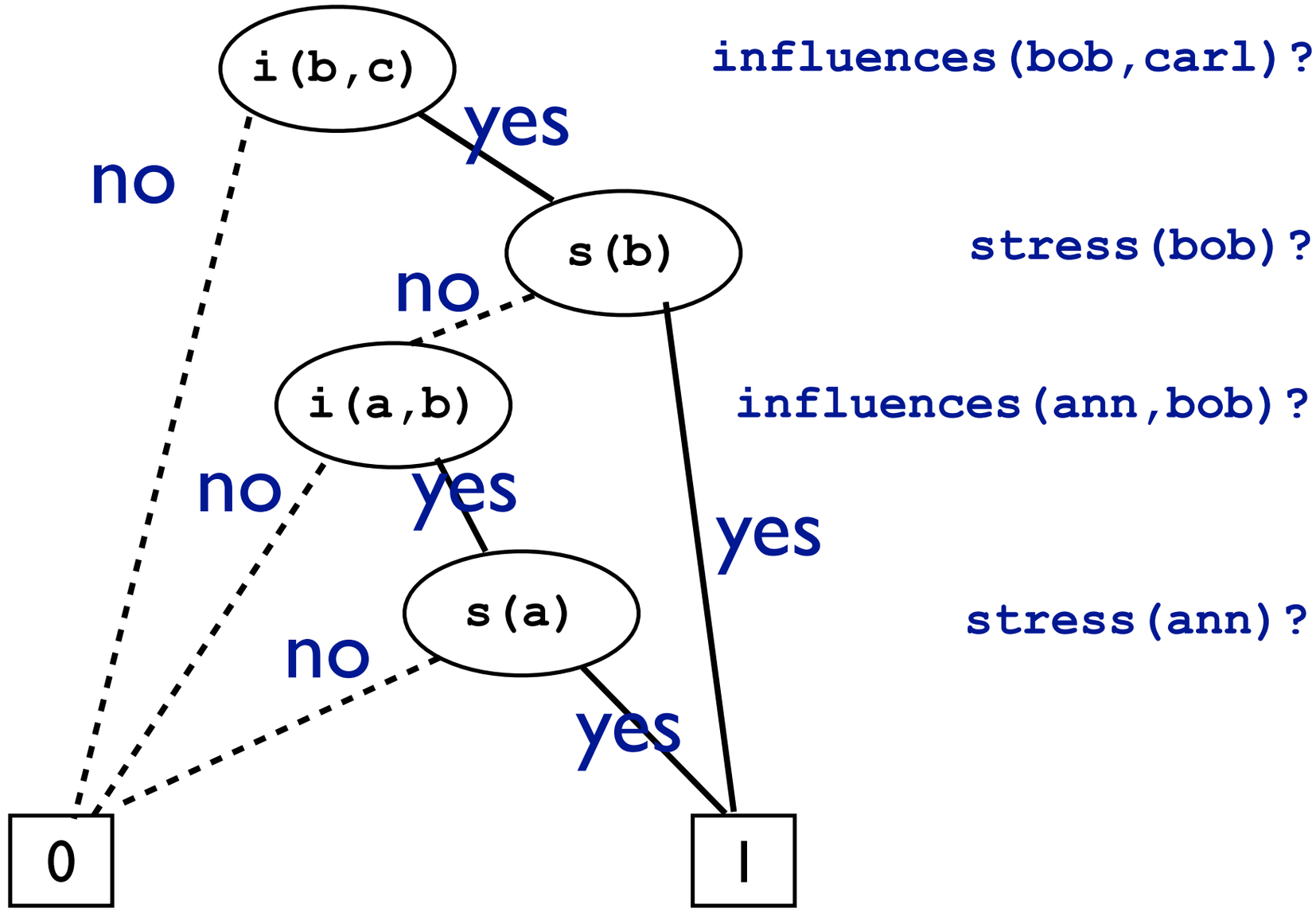
influences(bob,carl)?

stress(bob)?

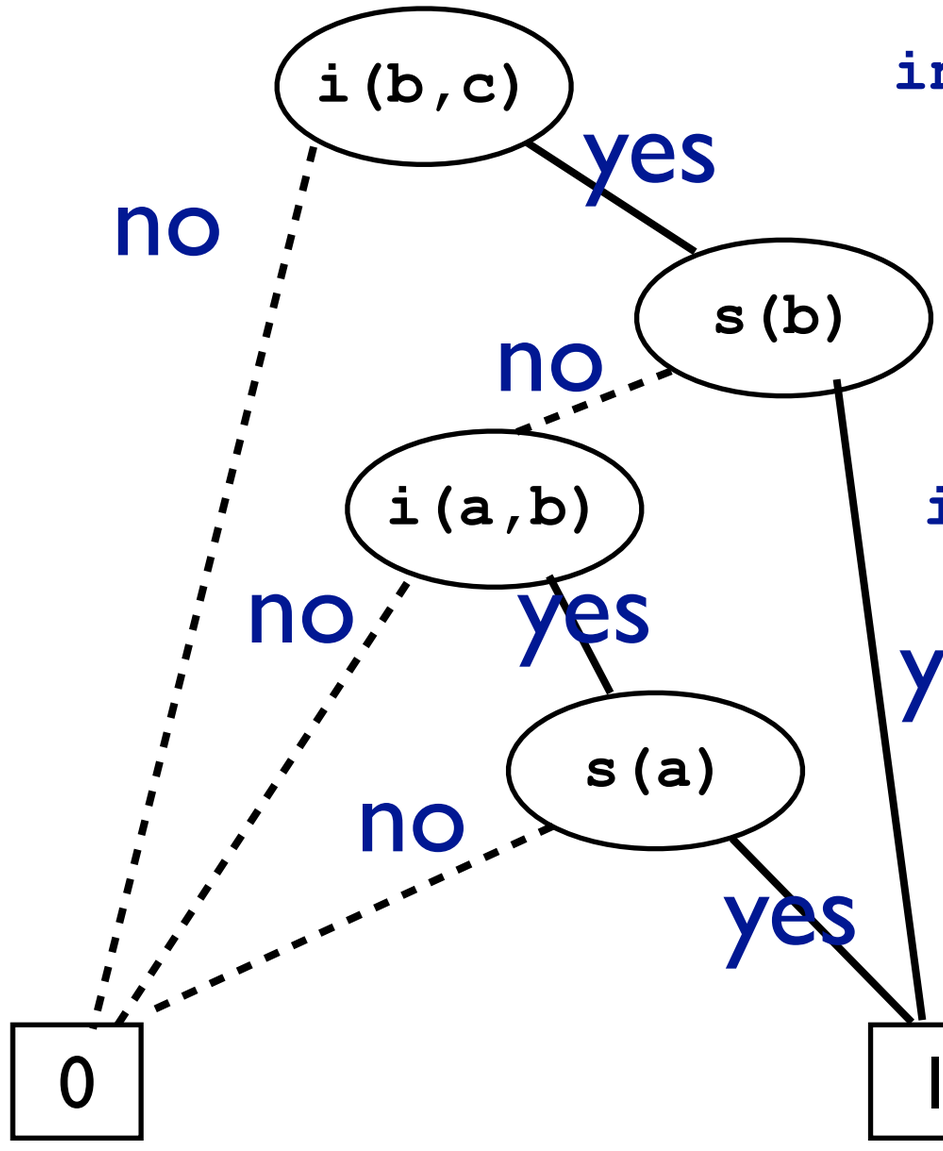
Binary Decision Diagrams



Binary Decision Diagrams



Binary Decision Diagrams



influences(bob,carl)?

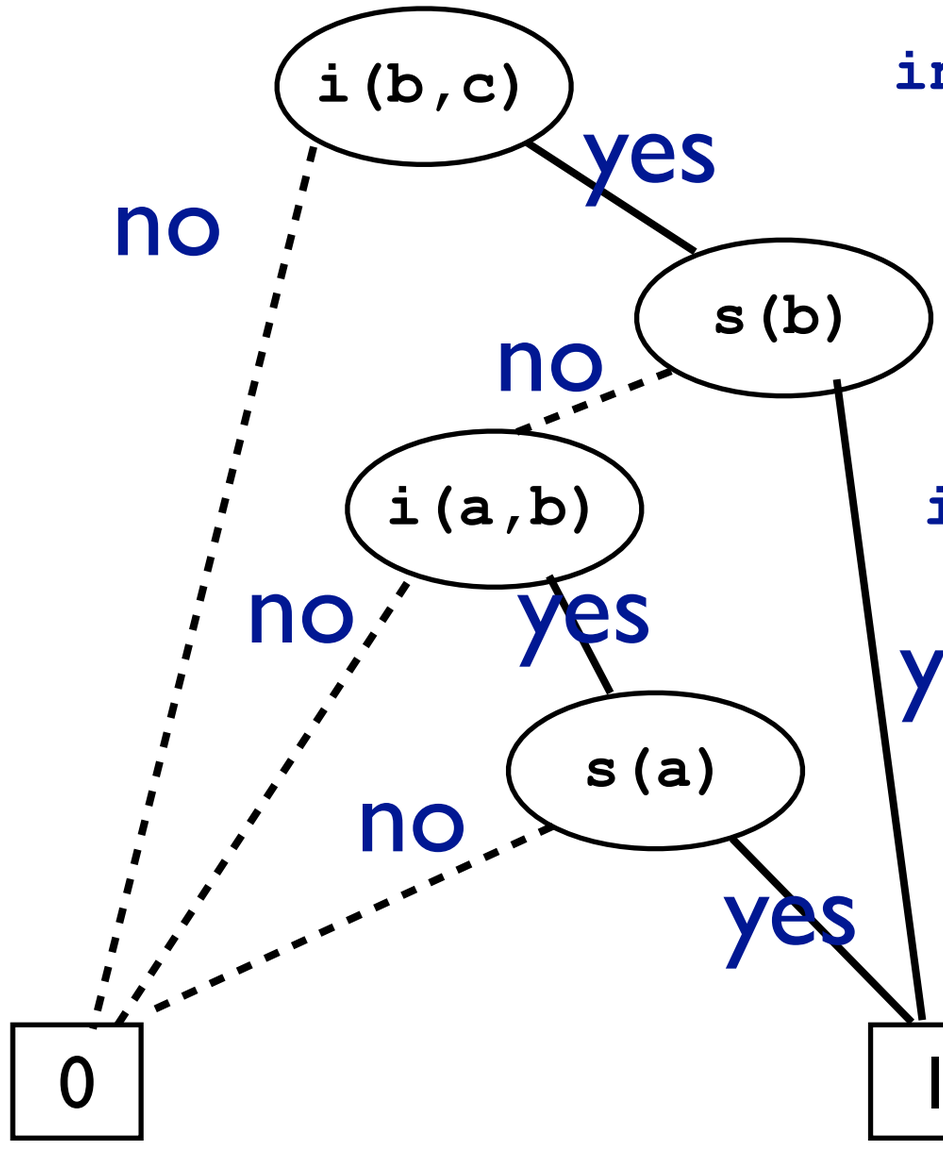
stress(bob)?

influences(ann,bob)?

stress(ann)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

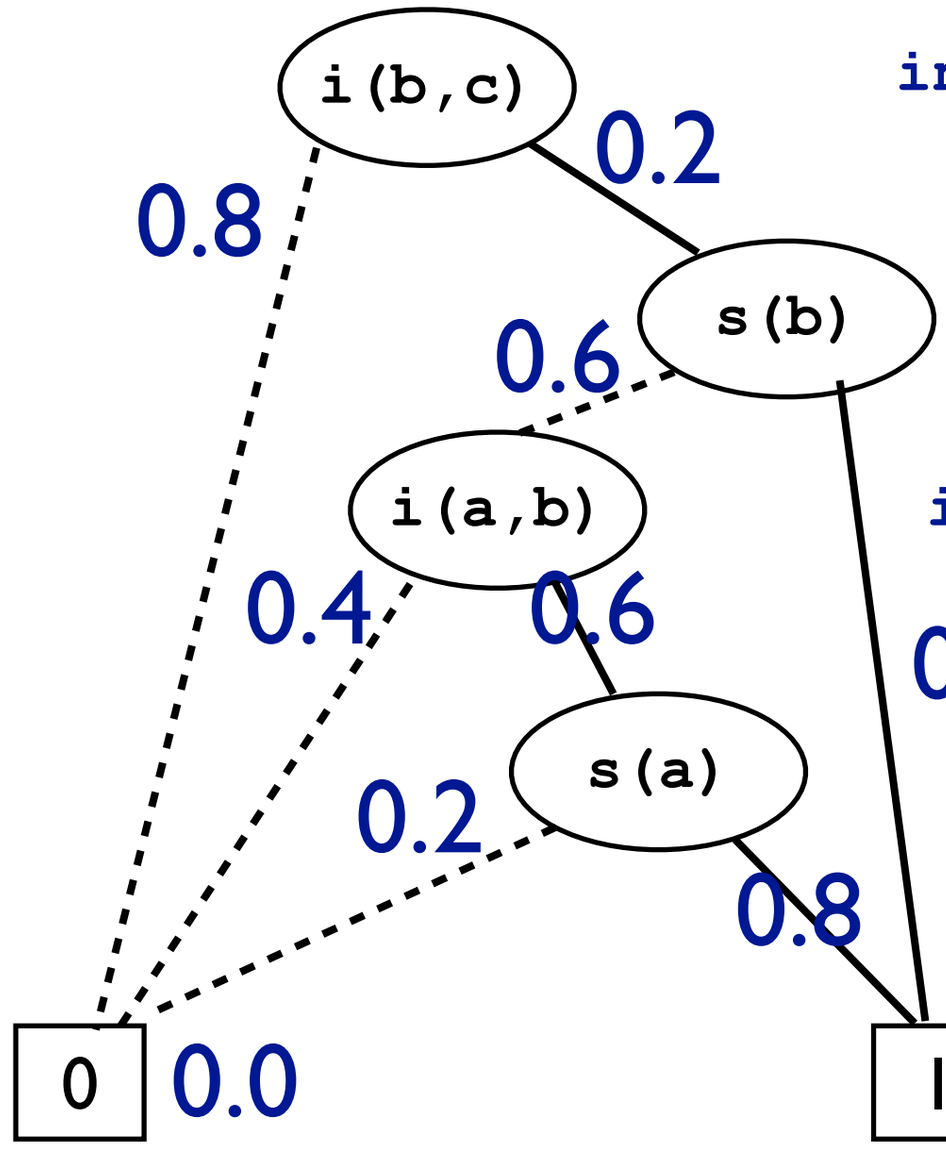
stress(ann)?

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

`influences(ann,bob)?`

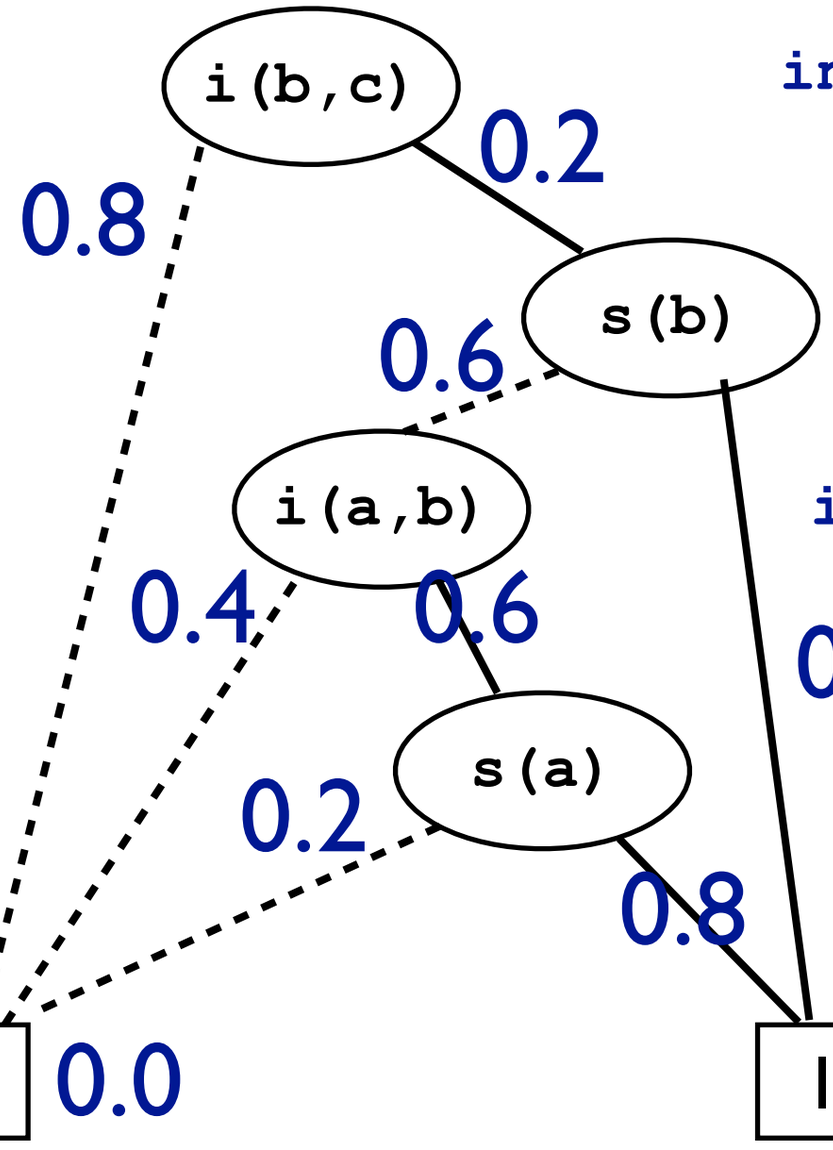
`stress(ann)?`

probability of

`smokes(c)?`

`smokes(c) = i(b,c)&s(b) v i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

influences(ann,bob)?

stress(ann)?

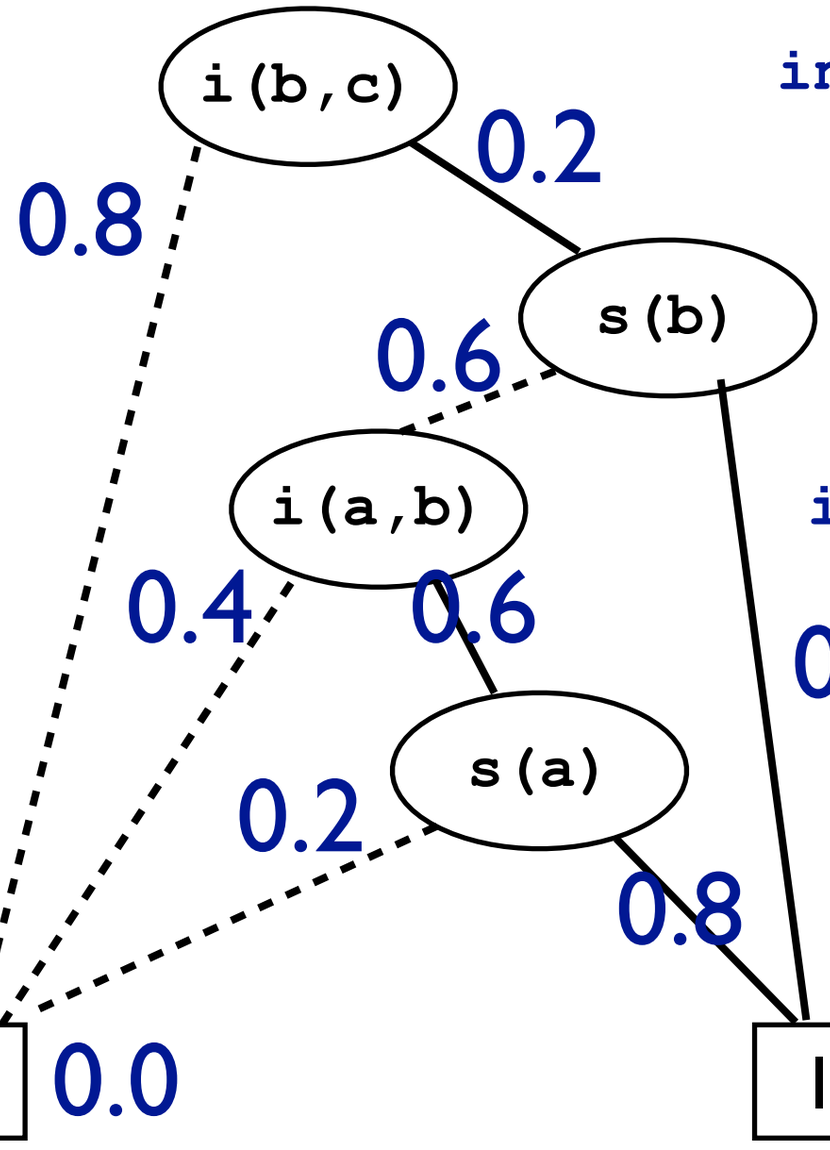
0.2×0.0+0.8×1.0=0.8

probability of

smokes(c)?

smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



`influences(bob,carl)?`

`stress(bob)?`

0.4×0.0+0.6×0.8=0.48

`influences(ann,bob)?`

0.2×0.0+0.8×1.0=0.8

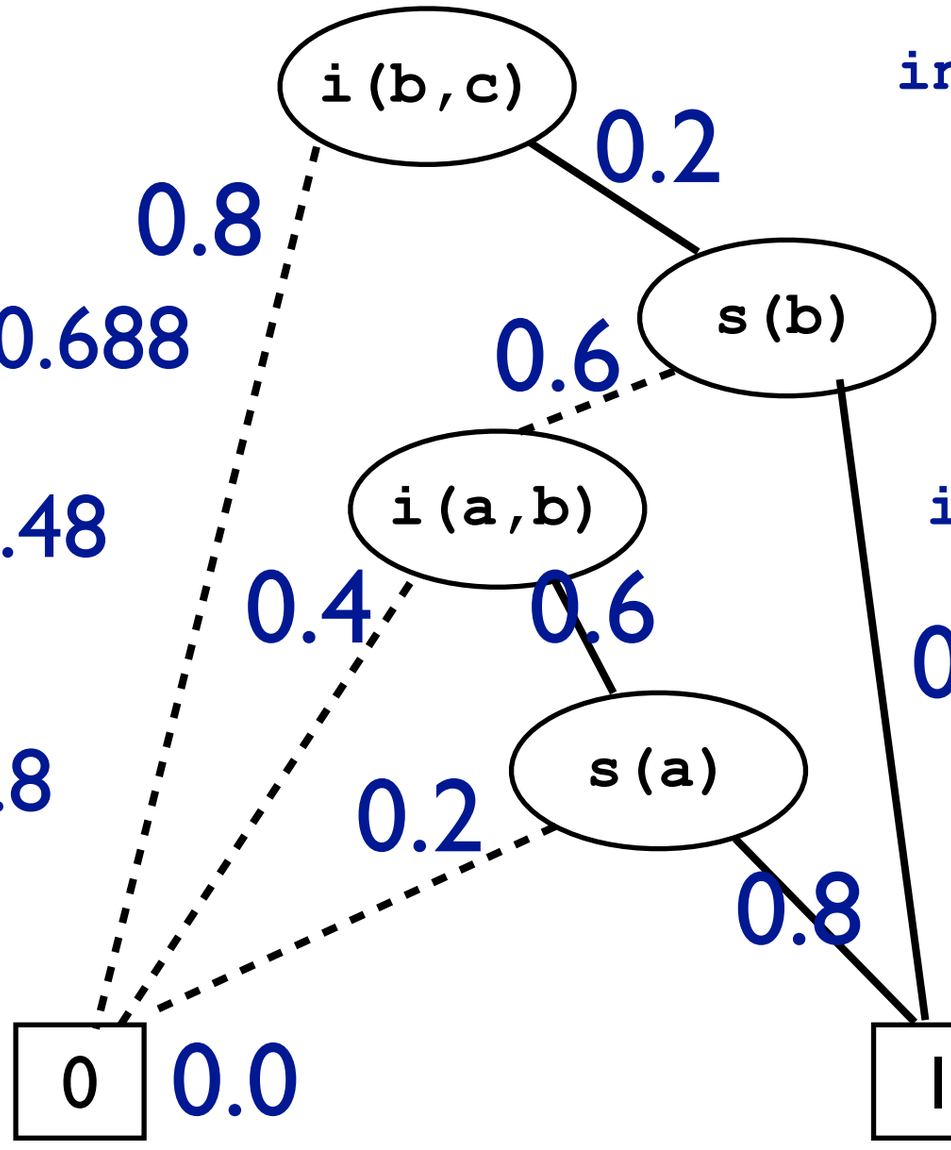
`stress(ann)?`

probability of

`smokes(c)?`

`smokes(c) = i(b,c)&s(b) v`
`i(b,c)&i(a,b)&s(a)`

Binary Decision Diagrams



influences(bob,carl)?

stress(bob)?

0.6×0.48+0.4×1.0=0.688

0.4×0.0+0.6×0.8=0.48

influences(ann,bob)?

0.2×0.0+0.8×1.0=0.8

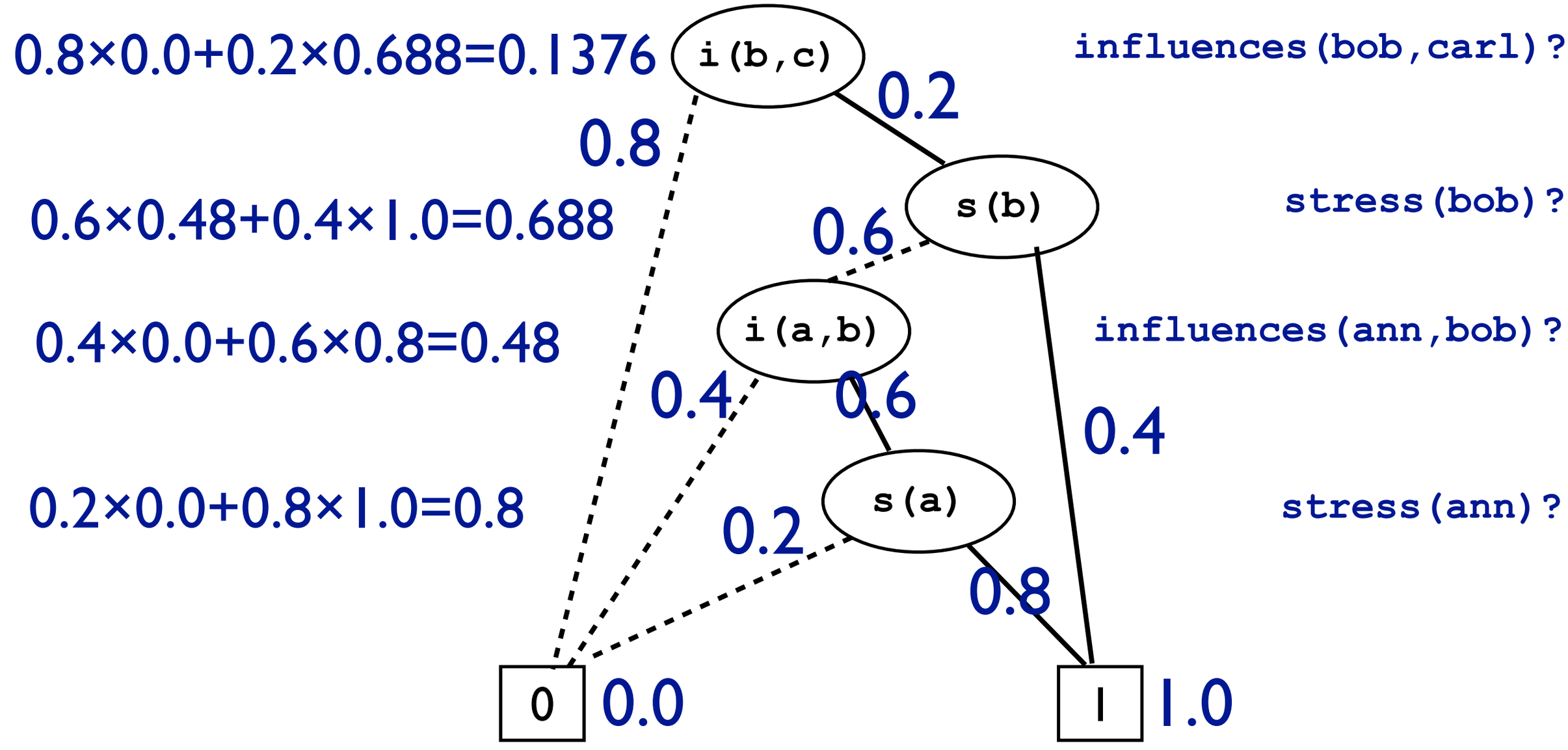
stress(ann)?

probability of

smokes(c)?

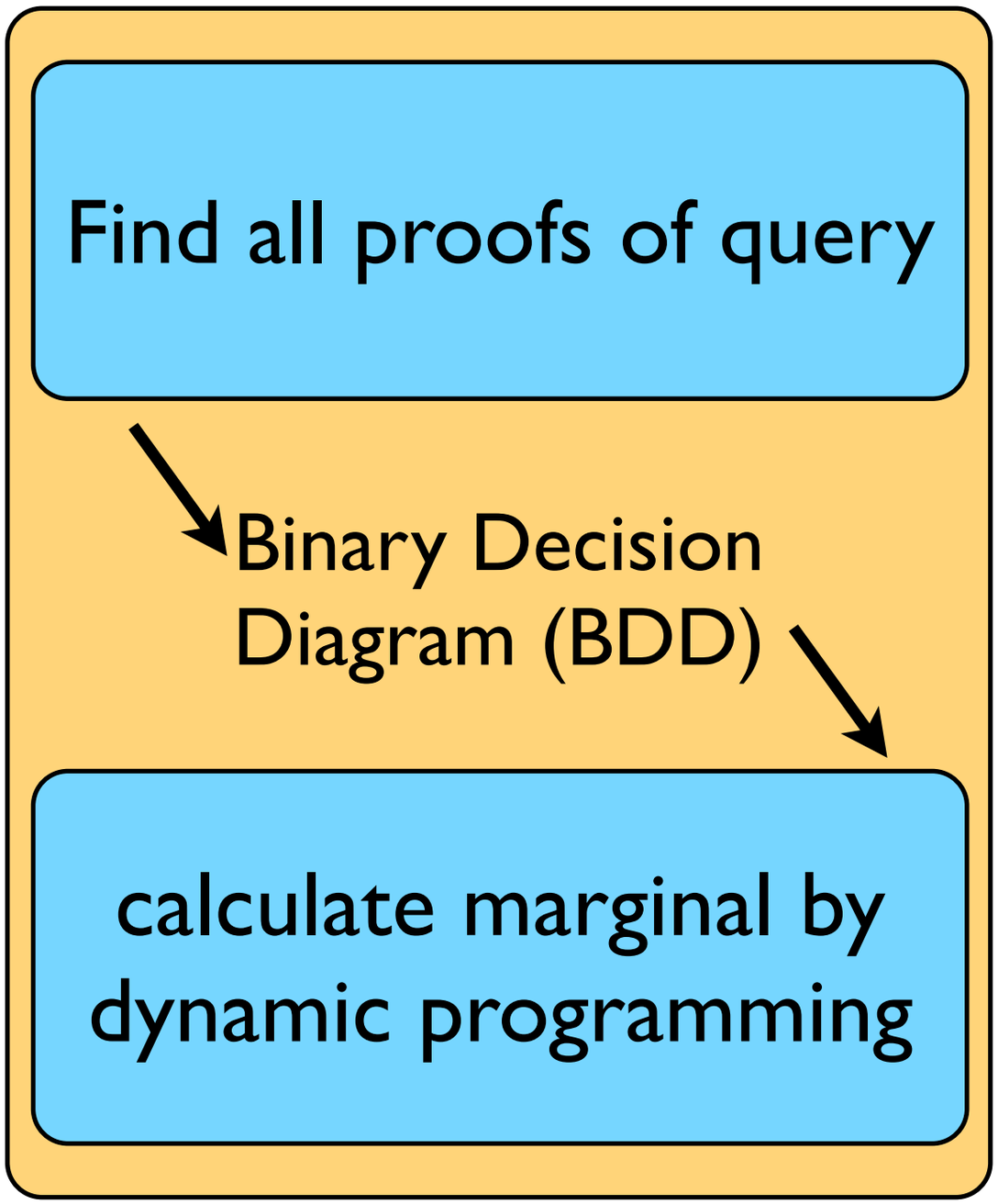
smokes(c) = i(b,c)&s(b) v
i(b,c)&i(a,b)&s(a)

Binary Decision Diagrams



Initial Approach

(ProbLog1 & others)

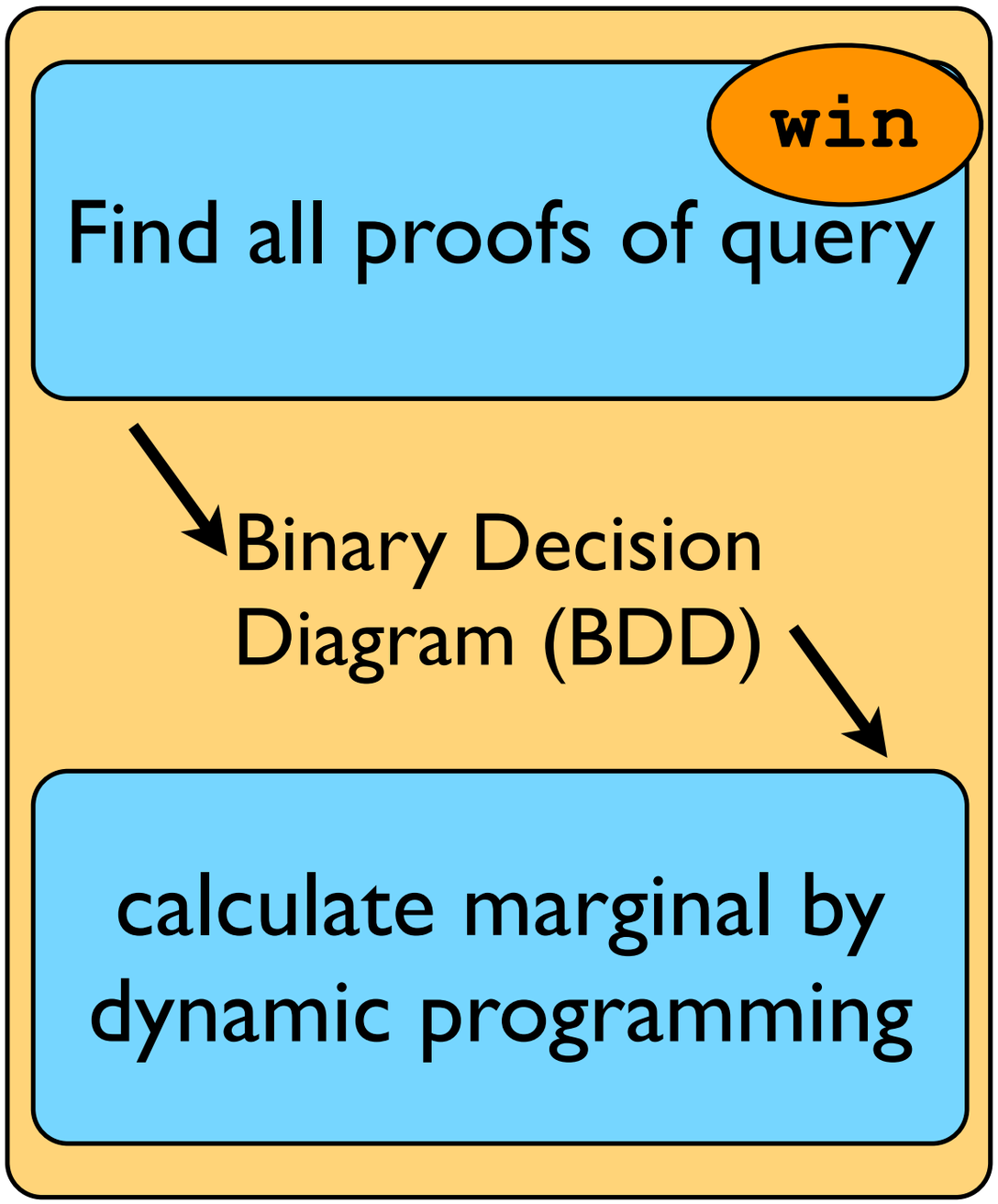


[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

```
0.4::heads(1).
0.7::heads(2).
0.5::heads(3).
win :- heads(1).
win :- heads(2),heads(3).
```



[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

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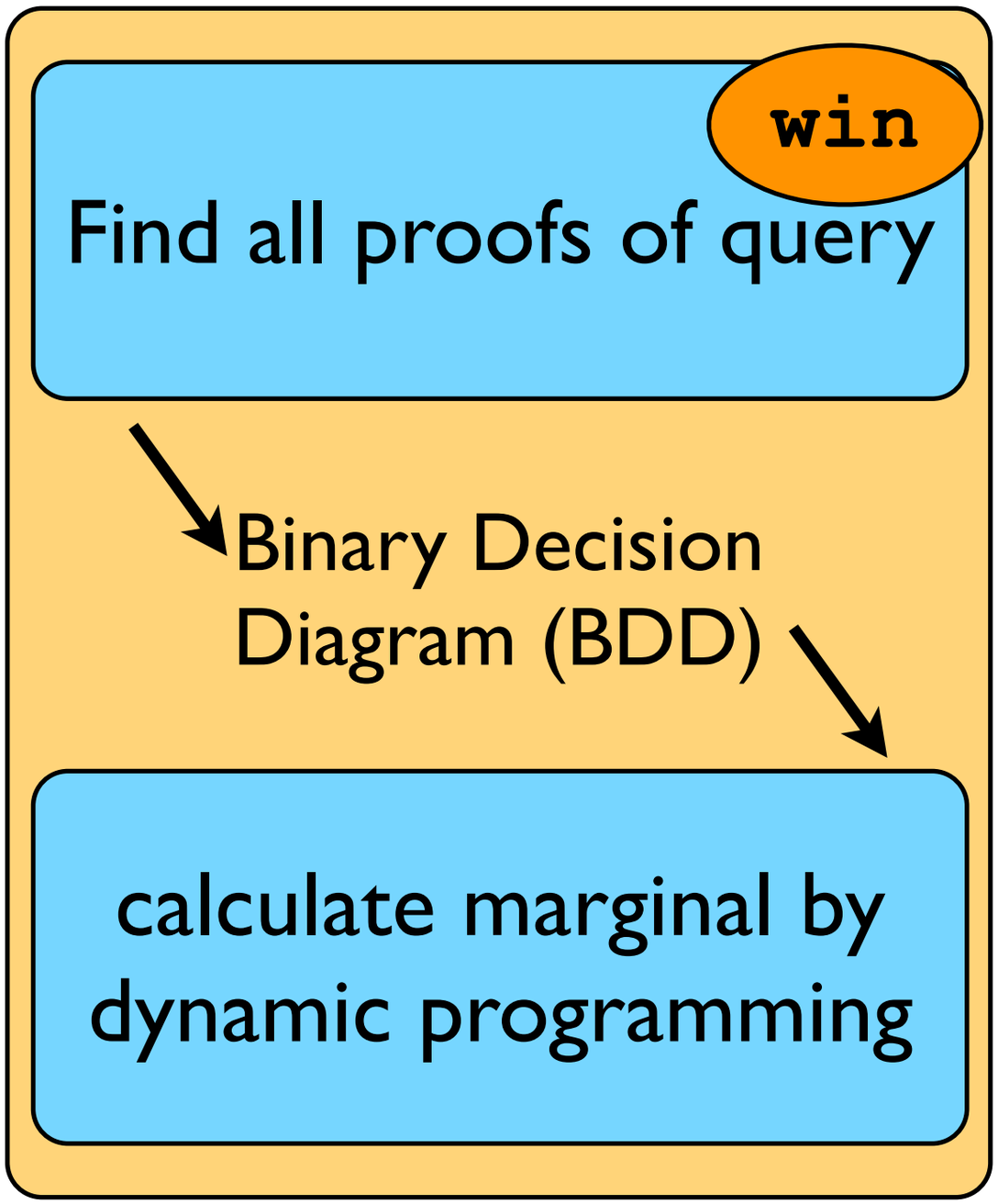


```
heads(1)
heads(2)  & heads(3)
```


[De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

```
0.4::heads(1).
0.7::heads(2).
0.5::heads(3).
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```
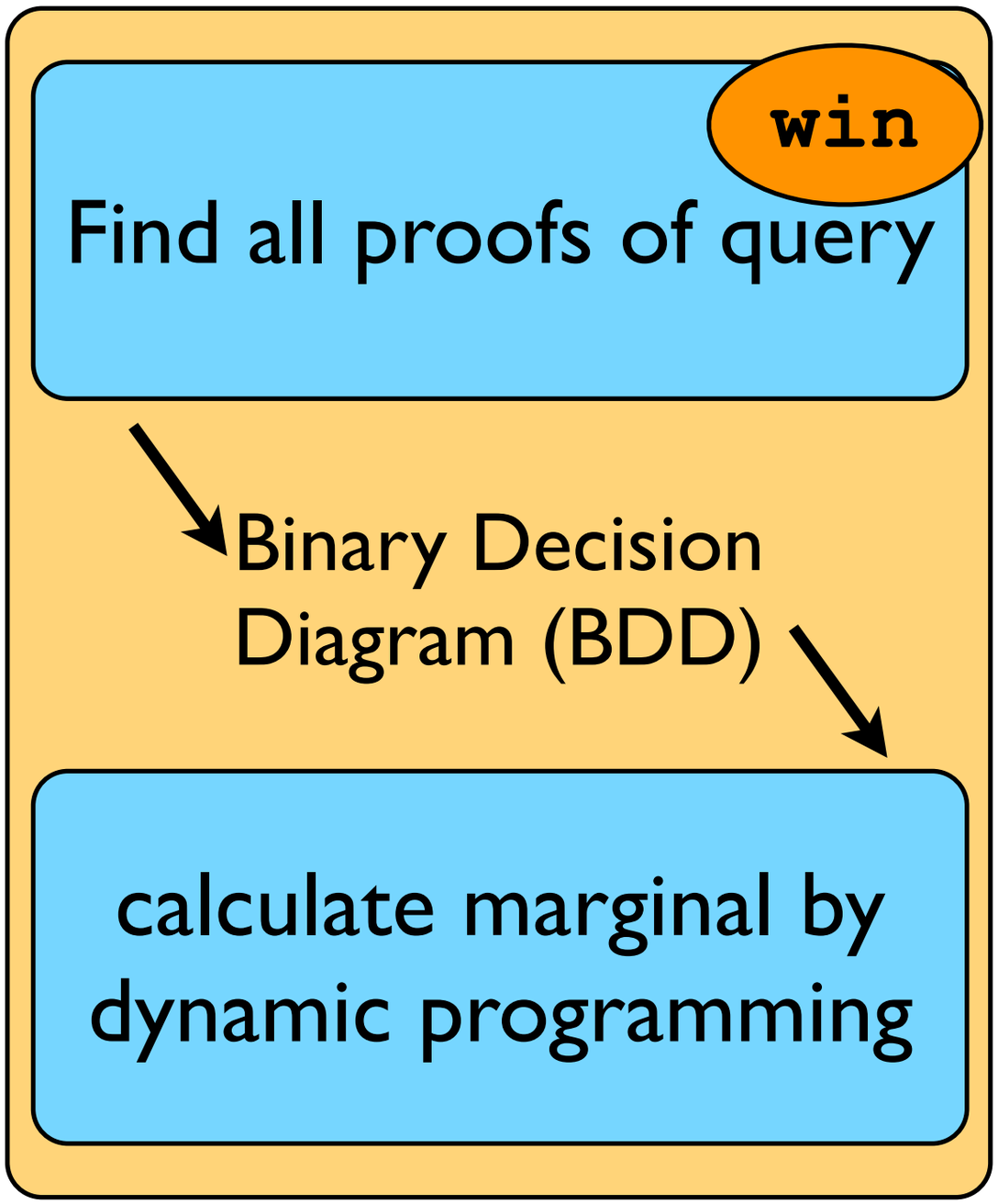
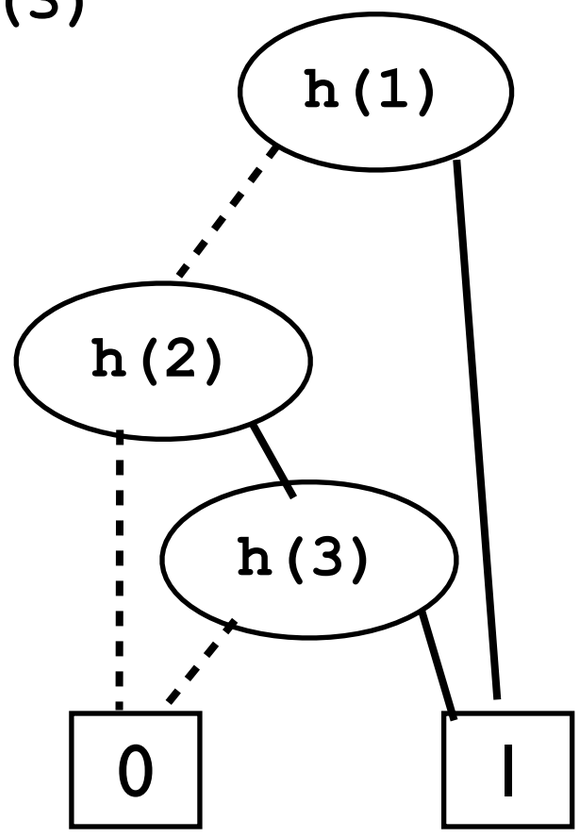




```
heads(1)
heads(2)  & heads(3)
```



 [De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

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0.4::heads(1).
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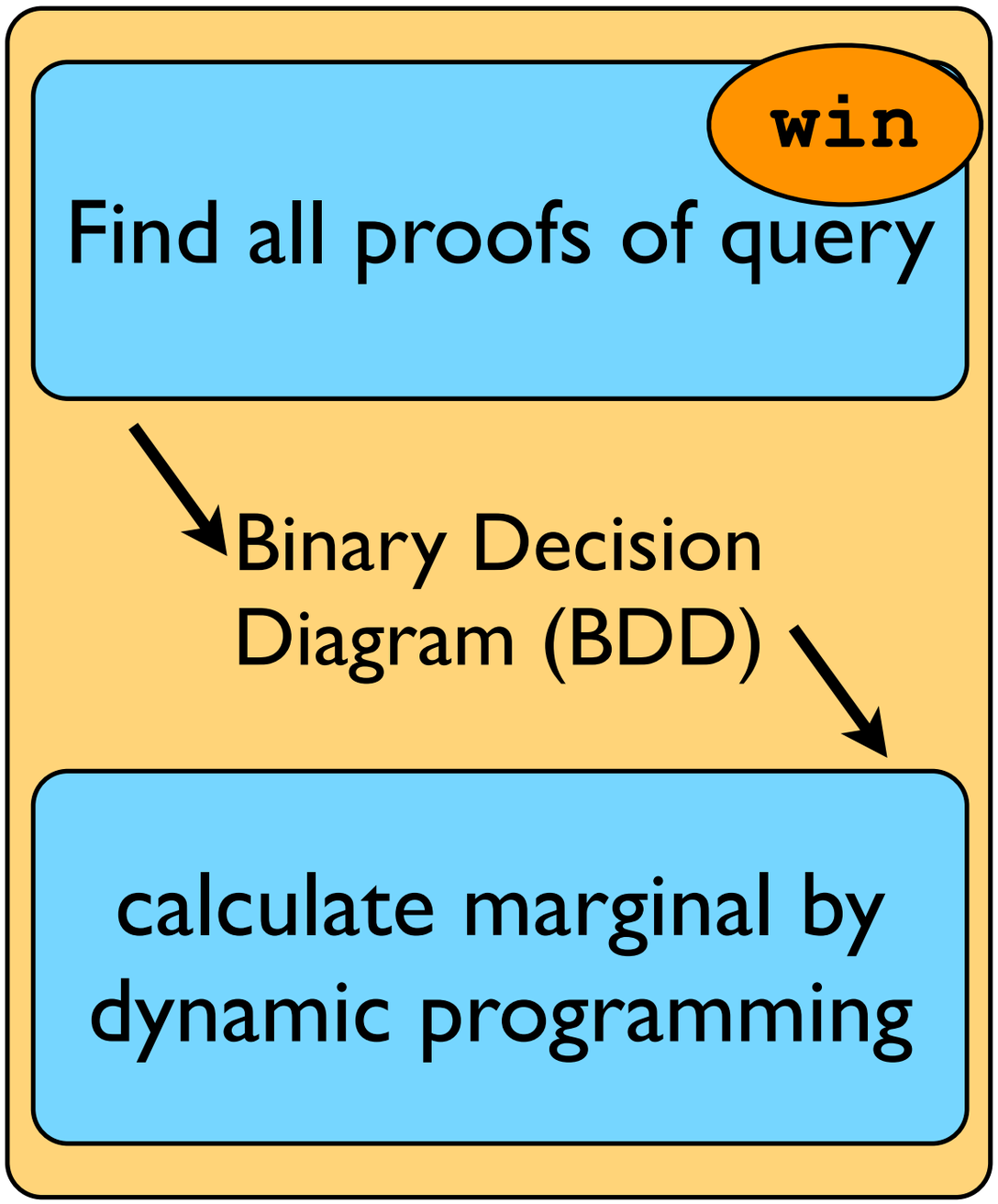
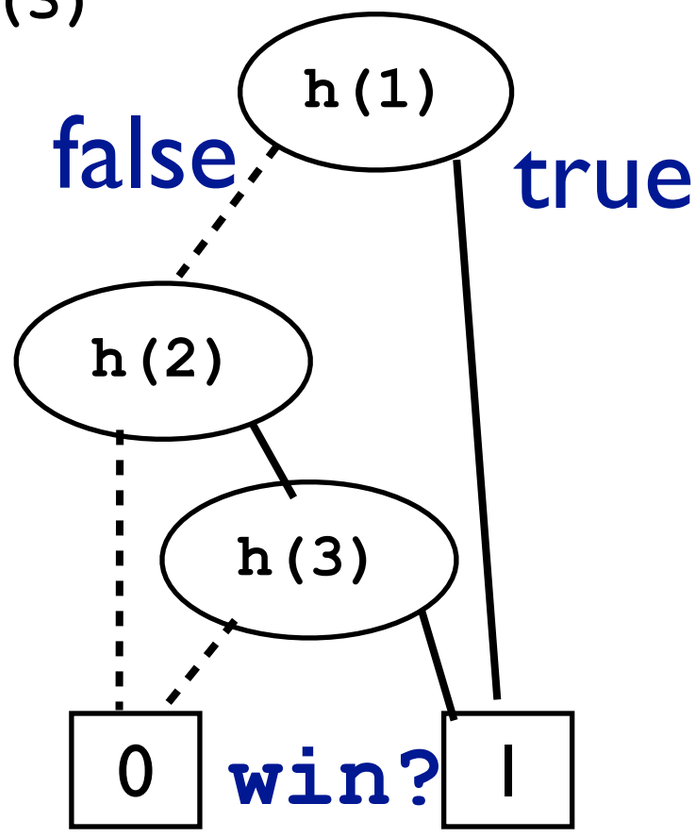
```




```
heads(1)
heads(2)  & heads(3)
```



 [De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Initial Approach

(ProbLog1 & others)

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0.7::heads(2).
0.5::heads(3).
win :- heads(1).
win :- heads(2),heads(3).
```
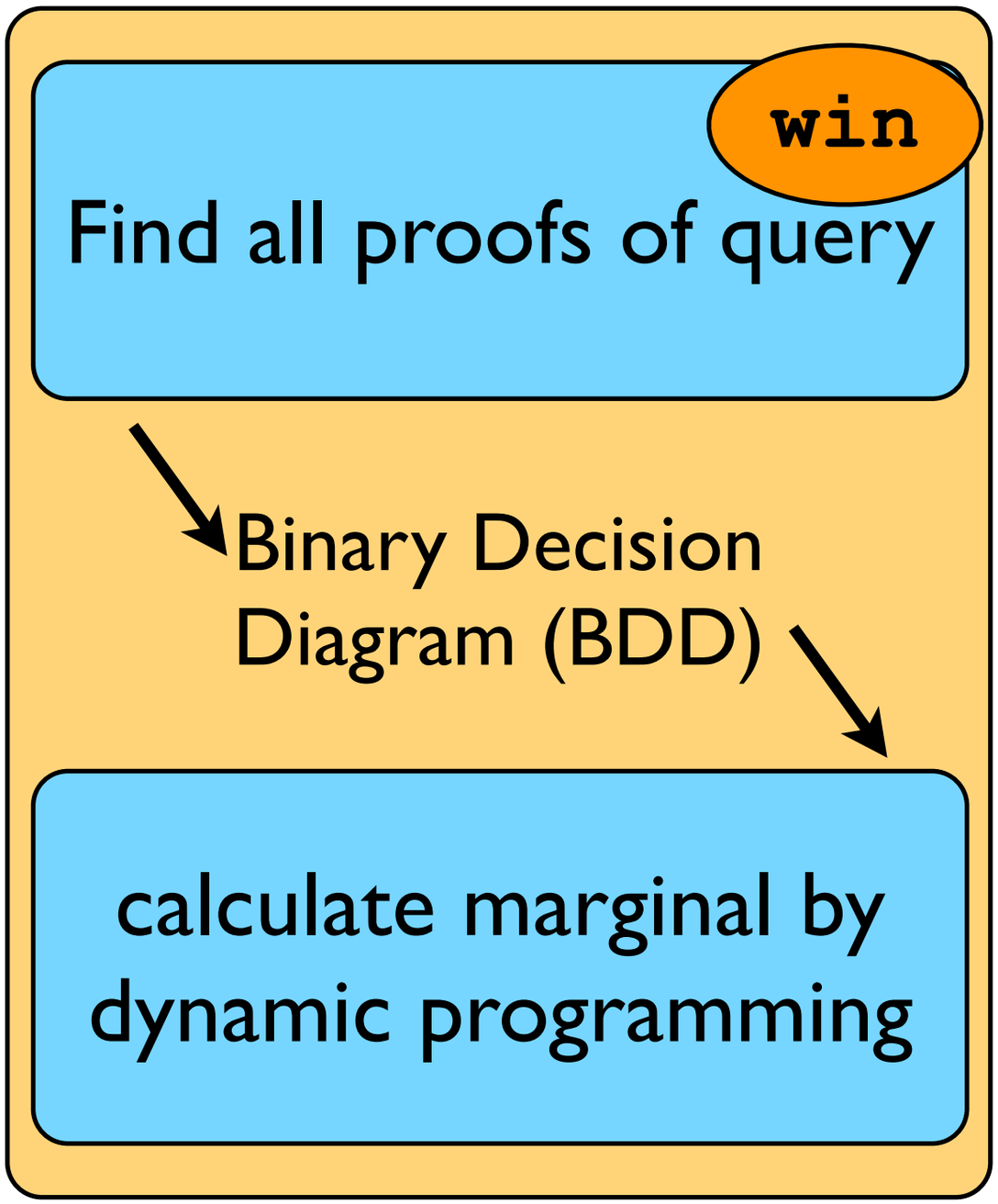
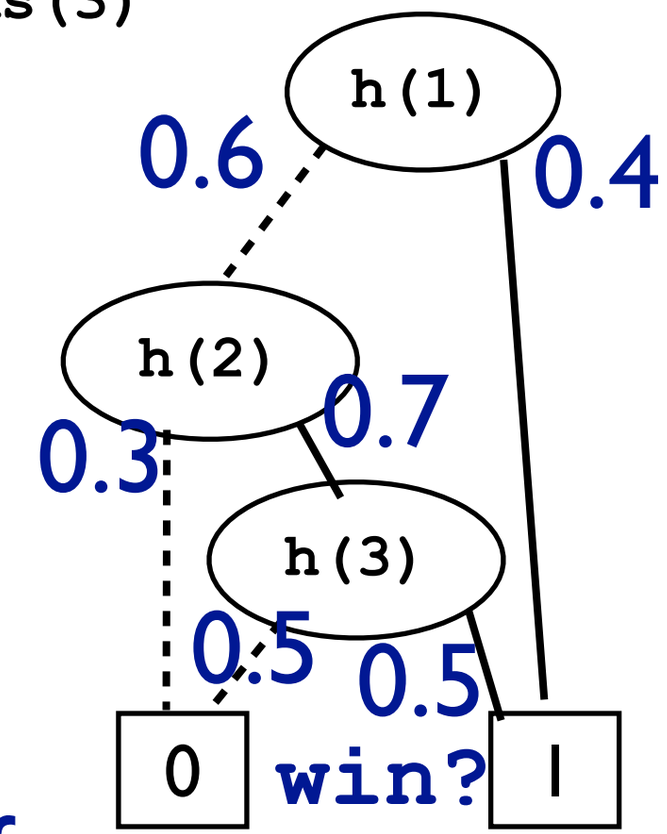




```
heads(1)
heads(2)  & heads(3)
```



P(**win**) = probability of reaching 1-leaf

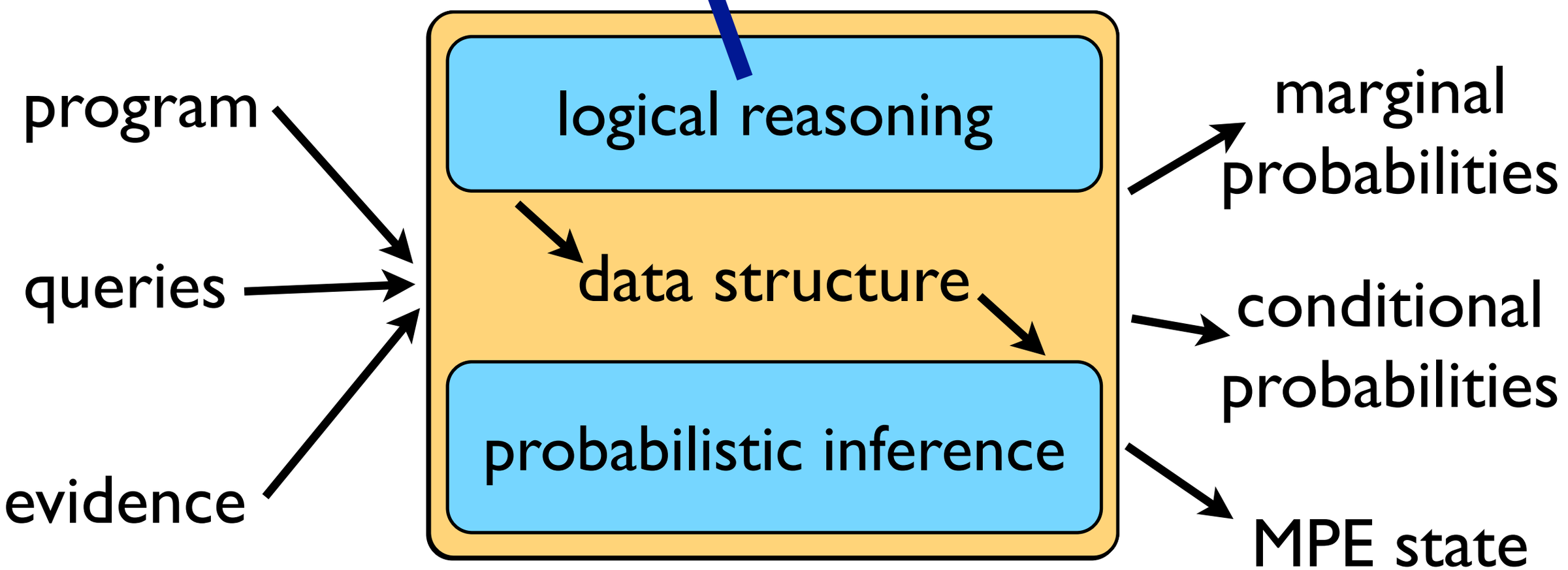
 [De Raedt et al, IJCAI 07; Kimmig et al, TPLP 11]

Answering Questions

1. using proofs
2. using models

Given:

Find:



Logical Reasoning: Models in Prolog

```
stress(ann).
influences(ann,bob).
influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
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      smokes(Y).
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```
?- smokes(carl).
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- Forward reasoning to construct unique model:

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- Forward reasoning to construct unique model:
  - Start with database facts
  - Use rules to add more facts

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influences(bob,carl).

smokes(ann).
```

Logical Reasoning: Models in Prolog

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- **ProbLog**: each possible world is a model, probability of query is sum over models where query is true

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→ weighted model counting

Weighted Model Counting

$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I} w(l)$$

Weighted Model Counting

propositional formula in conjunctive normal form (CNF)

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propositional formula in conjunctive normal form (CNF)

given by ProbLog program & query

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possible worlds

weight of literal

for p::f,
w(f) = p
w(not f) = 1−p

Weighte

$$P(Q) = \sum_{F \cup R \models Q} \prod_{f \in F} p(f) \prod_{f \notin F} 1 - p(f)$$

propositional formula in conjunctive normal form (CNF)

given by ProbLog program & query

$$WMC(\phi) = \sum_{I_V \models \phi} \prod_{l \in I} w(l)$$

interpretations (truth value assignments) of propositional variables

possible worlds

weight of literal

for p::f,
w(f) = p
w(not f) = 1−p

ProbLog → CNF

```
?- smokes(carl).
```

```
0.8::stress(ann).
0.4::stress(bob).
0.6::influences(ann,bob).
0.2::influences(bob,carl).

smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
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```

ProbLog → CNF

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0.8::stress(ann).
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smokes(X) :- stress(X).
smokes(X) :-
      influences(Y,X),
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```
?- smokes(carl).
```

- Find relevant ground rules by backward reasoning

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     smokes(Y).
```

```
?- smokes(carl).
```

- Find relevant ground rules by backward reasoning

```
smokes(carl) :- influences(bob,carl),smokes(bob).
smokes(bob) :- stress(bob).
smokes(bob) :- influences(ann,bob),smokes(ann).
smokes(ann) :- stress(ann).
```

ProbLog →CNF

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0.8::stress(ann).
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- Find relevant ground rules by backward reasoning

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smokes(carl) :- influences(bob,carl),smokes(bob).
smokes(bob) :- stress(bob).
smokes(bob) :- influences(ann,bob),smokes(ann).
smokes(ann) :- stress(ann).
```

- Convert to propositional logic formula

ProbLog → CNF

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smokes(bob) :- influences(ann,bob),smokes(ann).
smokes(ann) :- stress(ann).
```

- Convert to propositional logic formula

$$sm(c) \leftrightarrow (i(b,c) \wedge sm(b))$$
$$\wedge\ sm(b) \leftrightarrow (st(b) \vee (i(a,b) \wedge sm(a)))$$
$$\wedge\ sm(a) \leftrightarrow st(a)$$

ProbLog → CNF

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0.8::stress(ann).
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smokes(X) :-
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```
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```

- Find relevant ground rules by backward reasoning

```
smokes(carl) :- influences(bob,carl),smokes(bob).
smokes(bob) :- stress(bob).
smokes(bob) :- influences(ann,bob),smokes(ann).
smokes(ann) :- stress(ann).
```

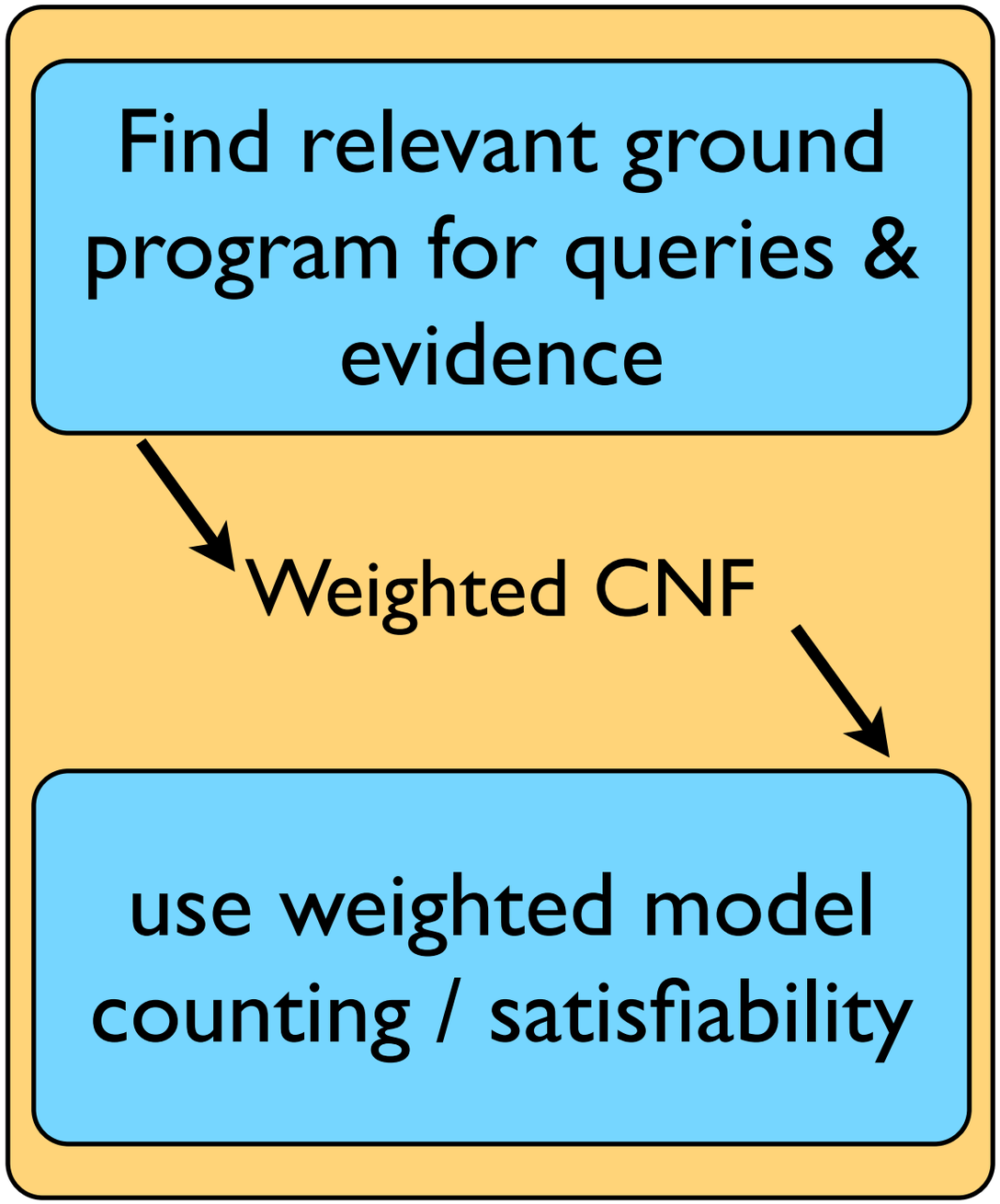
- Convert to propositional logic formula

$$sm(c) \leftrightarrow (i(b,c) \wedge sm(b))$$
$$\wedge\ sm(b) \leftrightarrow (st(b) \vee (i(a,b) \wedge sm(a)))$$
$$\wedge\ sm(a) \leftrightarrow st(a)$$

- Rewrite in CNF (as usual)

Current Approach

(ProbLog2)

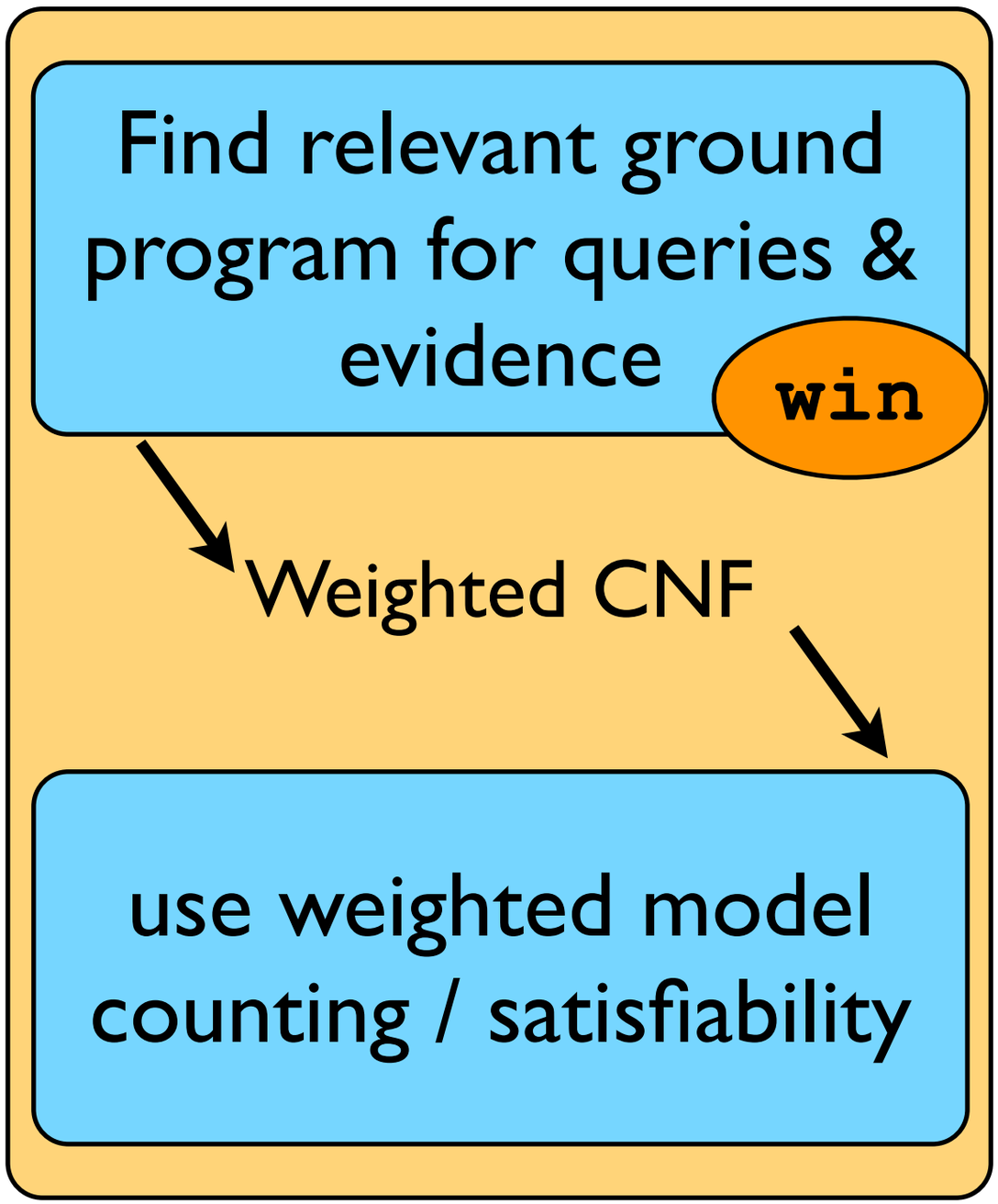


[Fierens et al, TPLP 14]

Current Approach

(ProbLog2)

```
0.4::heads(1).
0.7::heads(2).
0.5::heads(3).
win :- heads(1).
win :- heads(2),
       heads(3).
```



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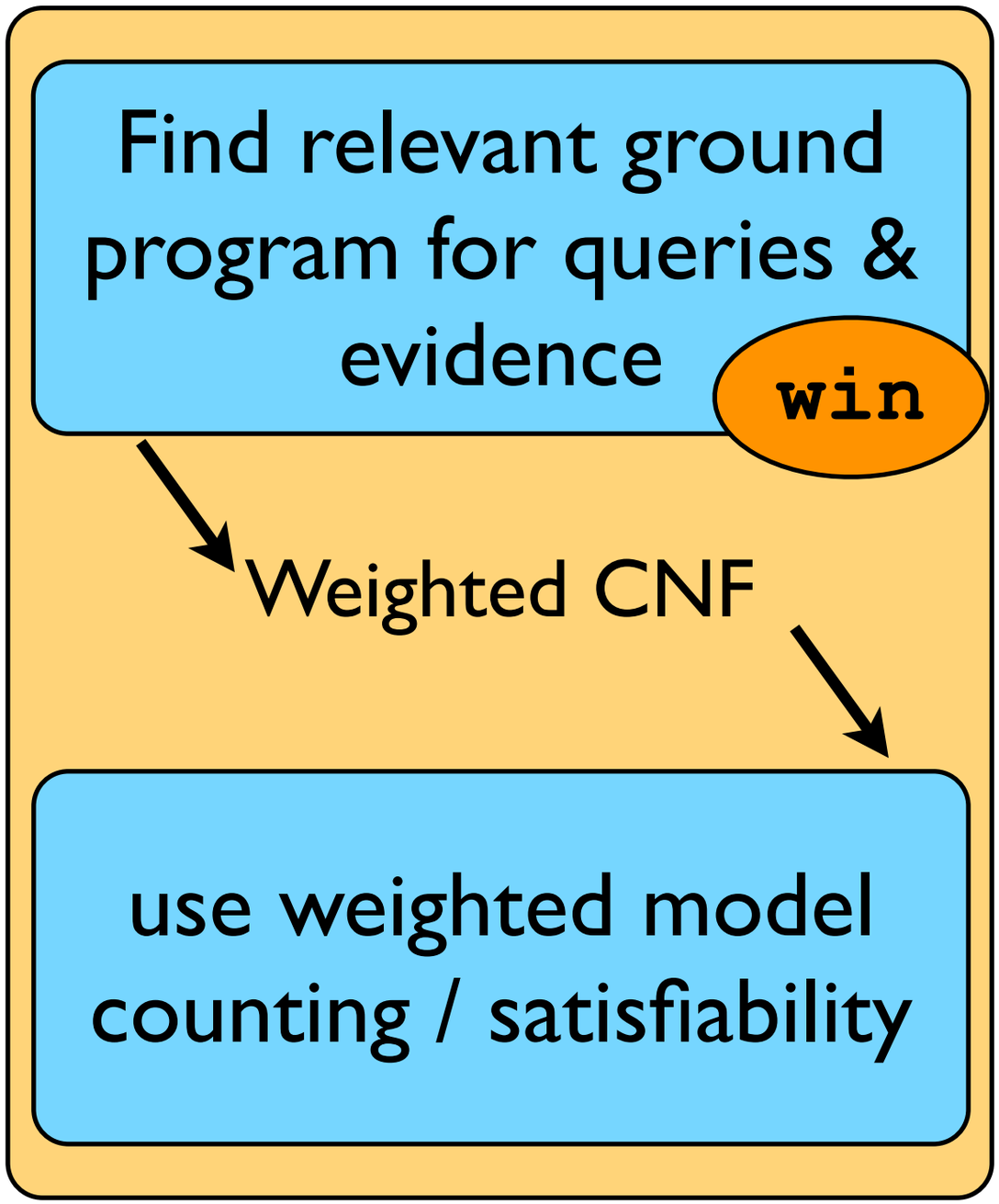


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[Fierens et al, TPLP 14]

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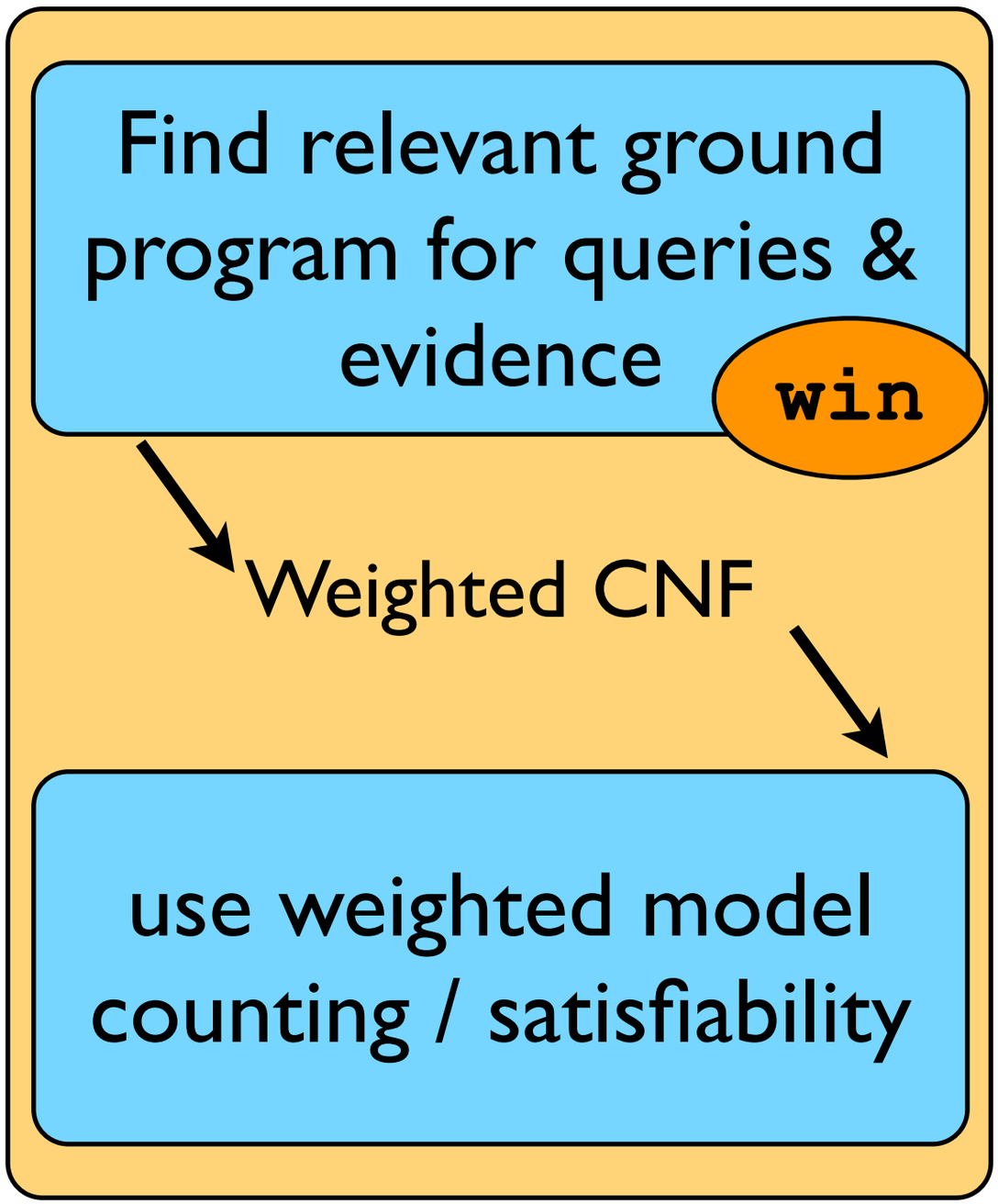


```
win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

[Fierens et al, TPLP 14]

Current Approach

(ProbLog2)

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win :- heads(1).
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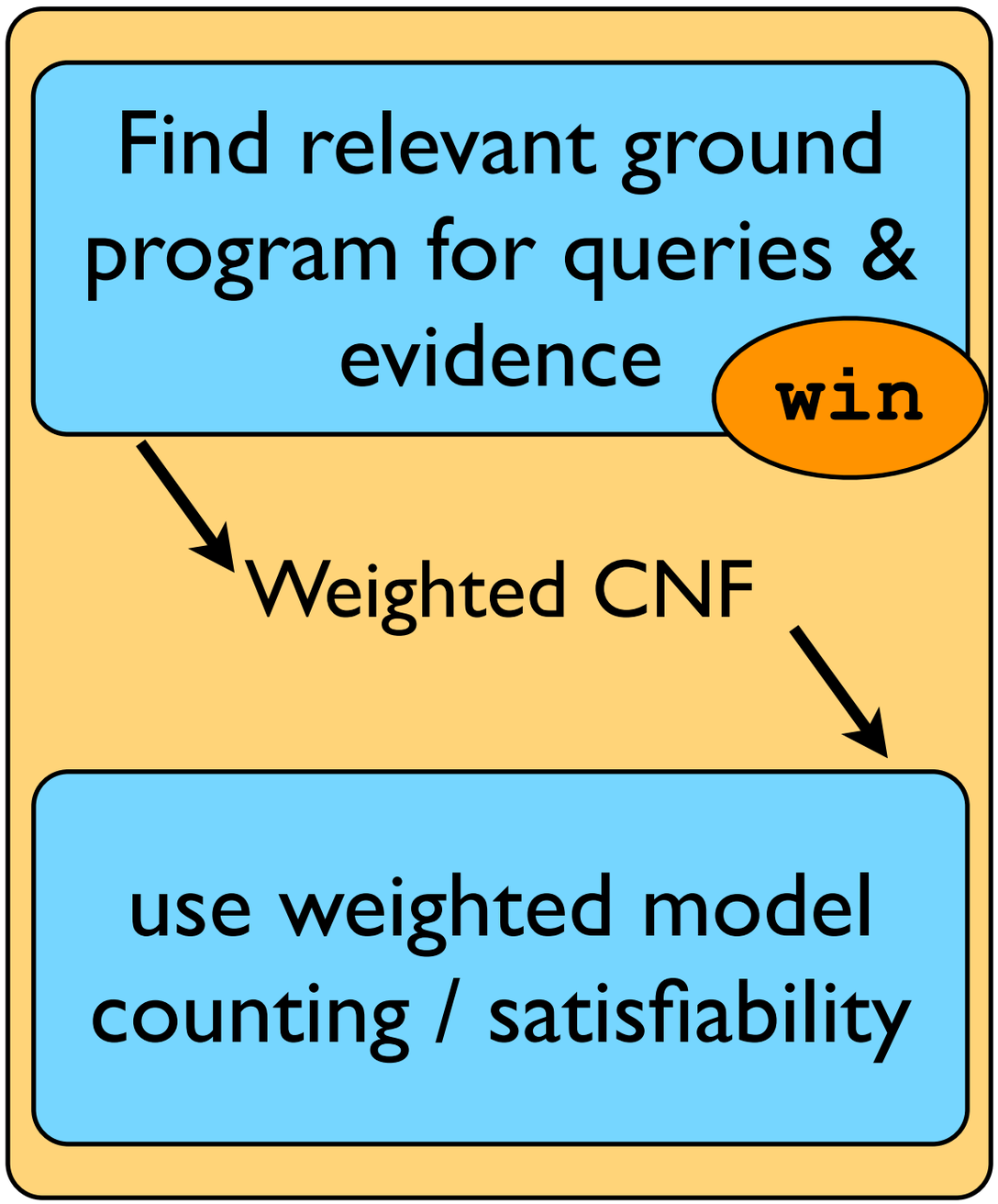


```
win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

$$(\neg\text{win} \vee h(1) \vee h(2)) \wedge (\neg\text{win} \vee h(1) \vee h(3)) \wedge (\text{win} \vee \neg h(1)) \wedge (\text{win} \vee \neg h(2) \vee \neg h(3))$$

[Fierens et al, TPLP 14]

Current Approach
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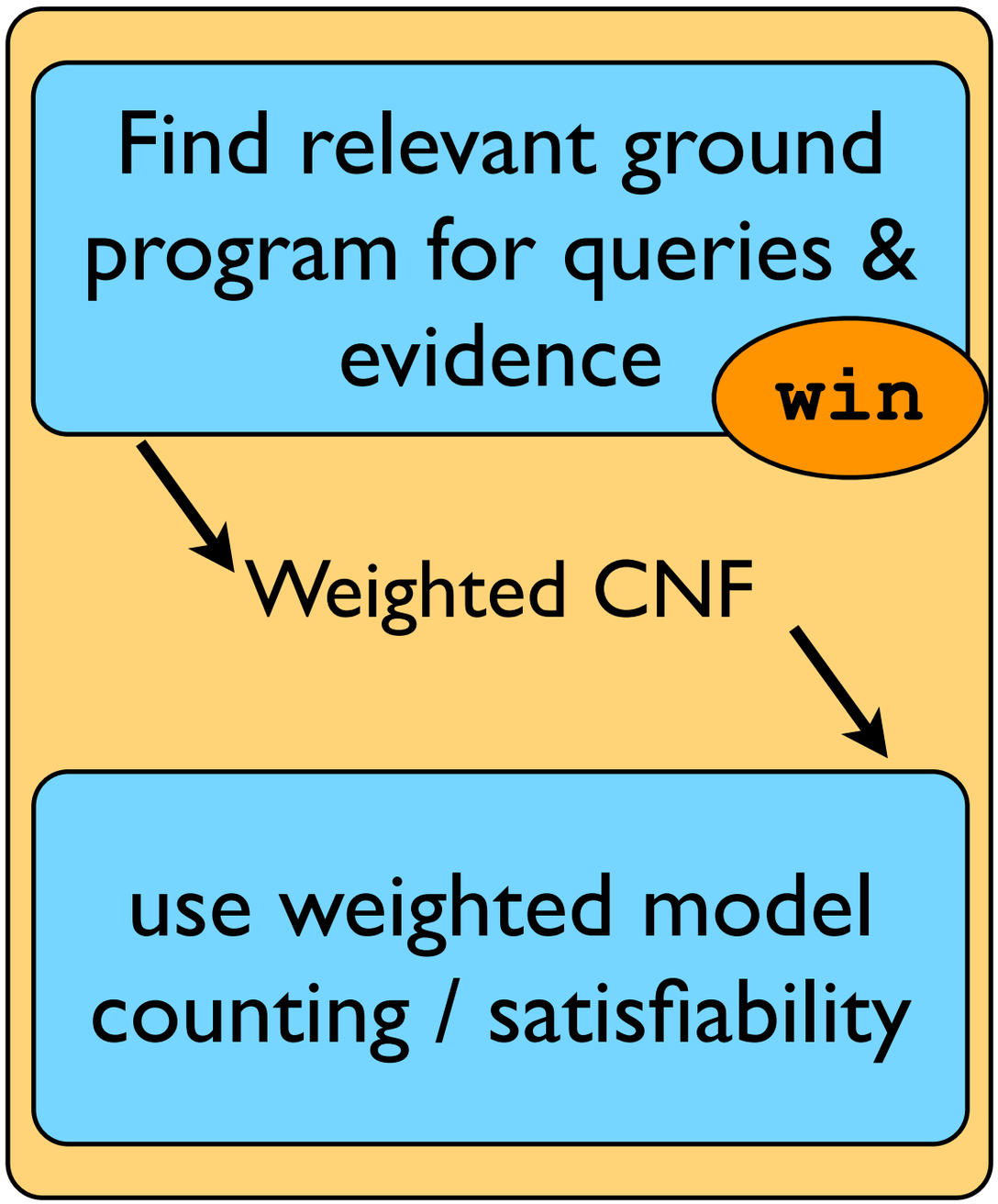


```
win :- heads(1).
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$$(\neg\text{win} \vee h(1) \vee h(2)) \wedge (\neg\text{win} \vee h(1) \vee h(3)) \wedge (\text{win} \vee \neg h(1)) \wedge (\text{win} \vee \neg h(2) \vee \neg h(3))$$

$h(1) \rightarrow 0.4$	$h(2) \rightarrow 0.7$	$h(3) \rightarrow 0.5$
$\neg h(1) \rightarrow 0.6$	$\neg h(2) \rightarrow 0.3$	$\neg h(3) \rightarrow 0.5$

[Fierens et al, TPLP 14]

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0.4::heads(1).
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win :- heads(1).
win :- heads(2),
       heads(3).
```

Find relevant ground program for queries & evidence

win

Weighted CNF

use weighted model counting / satisfiability

```
win :- heads(1).
win :- heads(2), heads(3).
```

$$\text{win} \leftrightarrow h(1) \vee (h(2) \wedge h(3))$$

$$(\neg\text{win} \vee h(1) \vee h(2)) \wedge (\neg\text{win} \vee h(1) \vee h(3)) \wedge (\text{win} \vee \neg h(1)) \wedge (\text{win} \vee \neg h(2) \vee \neg h(3))$$

use standard tool

$h(1) \rightarrow 0.4$ $\quad h(2) \rightarrow 0.7$ $\quad h(3) \rightarrow 0.5$

$\neg h(1) \rightarrow 0.6$ $\quad \neg h(2) \rightarrow 0.3$ $\quad \neg h(3) \rightarrow 0.5$

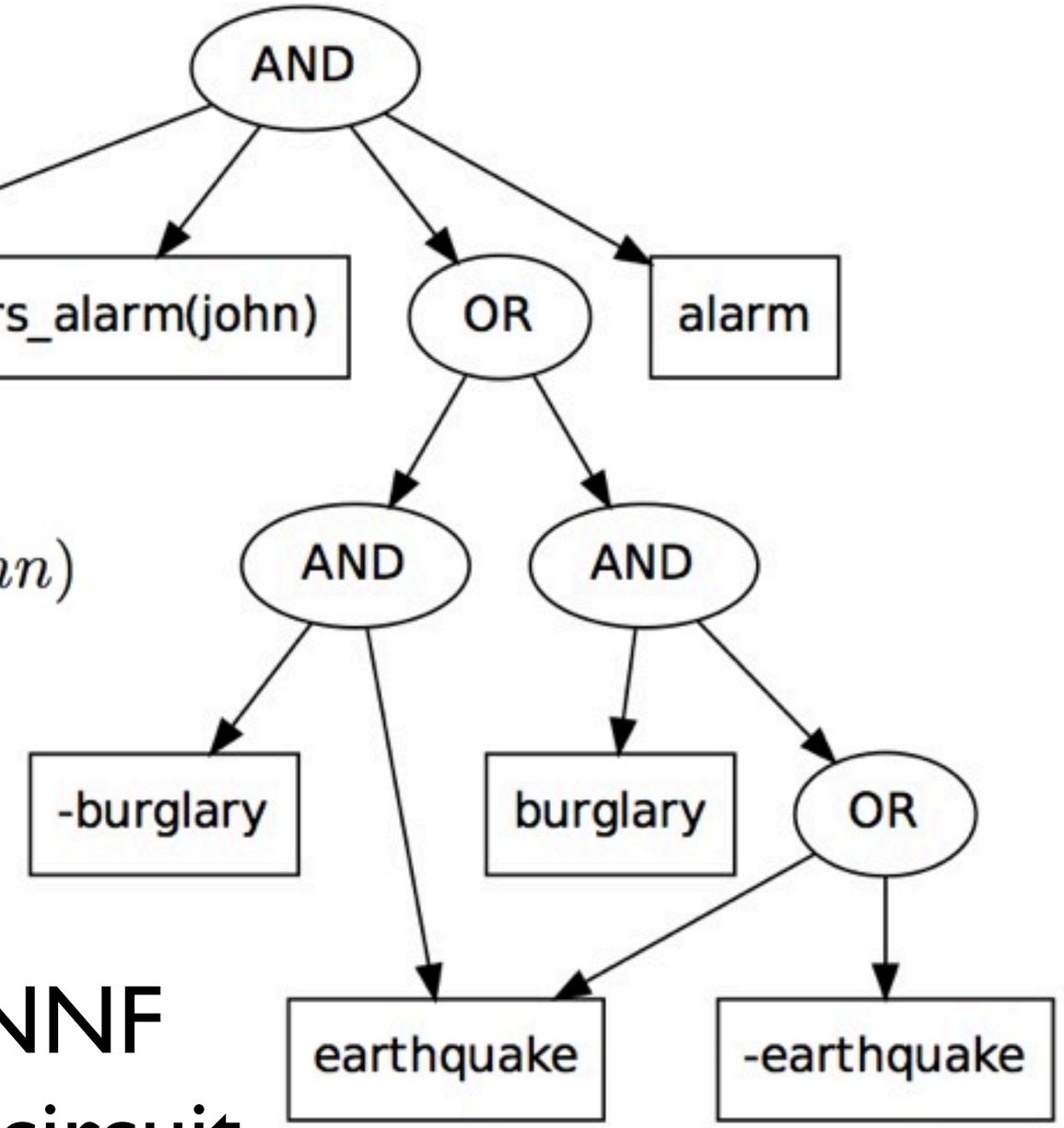
[Fierens et al, TPLP 14]

WMC using d-DNNFs

$$alarm \leftrightarrow burglary \vee earthquake$$

$$calls(john) \leftrightarrow alarm, hears\_alarm(john)$$

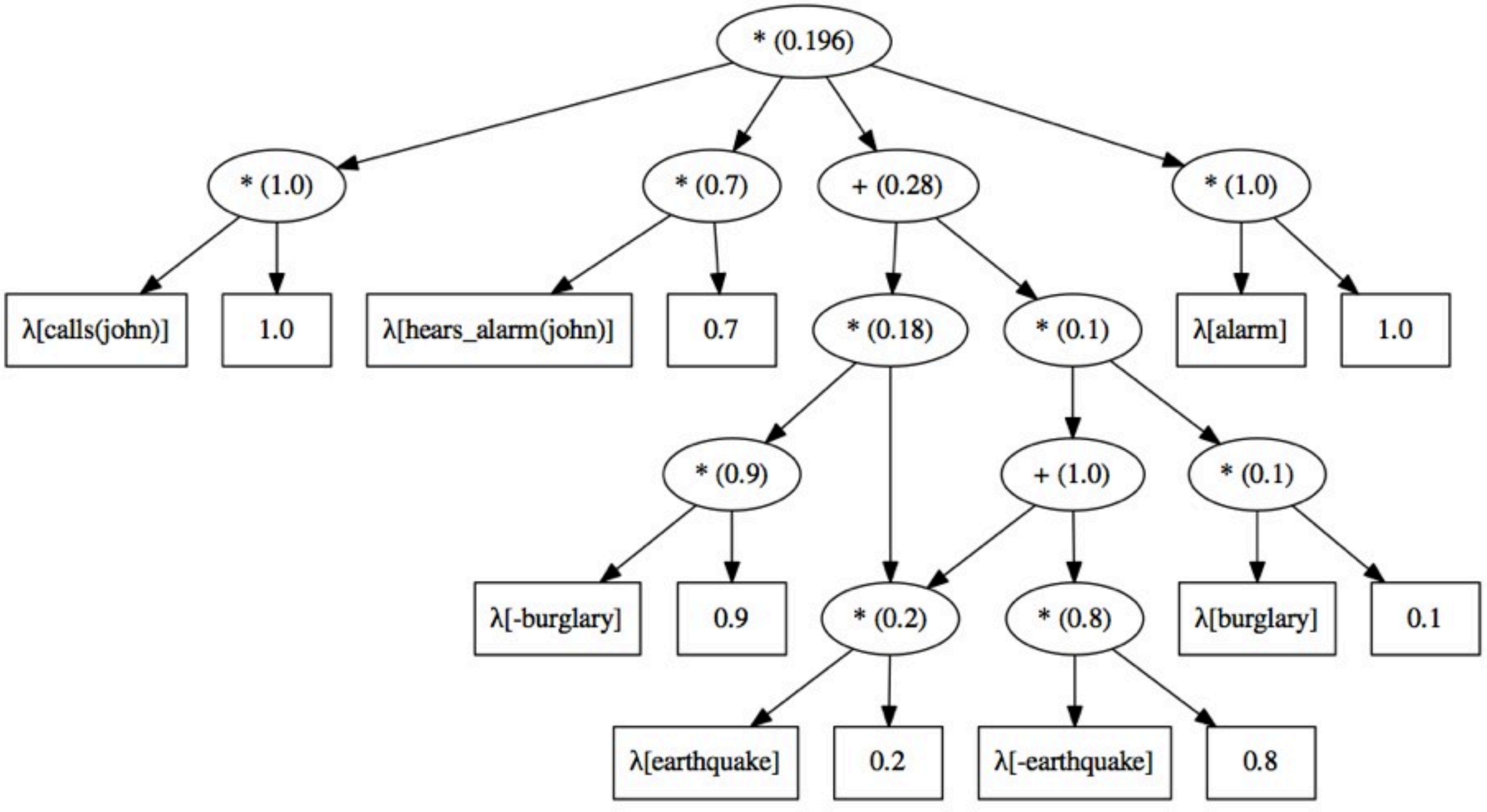
$$calls(john)$$



1. represent formula as d-DNNF
2. transform into arithmetic circuit
3. evaluate bottom-up

[Figure: Fierens et al, TPLP 14]

WMC using d-DNNFs



3. evaluate bottom-up

[Figure: Fierens et al, TPLP 14]

ProbLog Inference

- reduction to propositional formula
- addresses disjoint-sum-problem
- **but**: not all probabilistic logic programs face this problem! e.g., weather
- more generally: mutually exclusive proofs as assumed in PRISM

PRISM

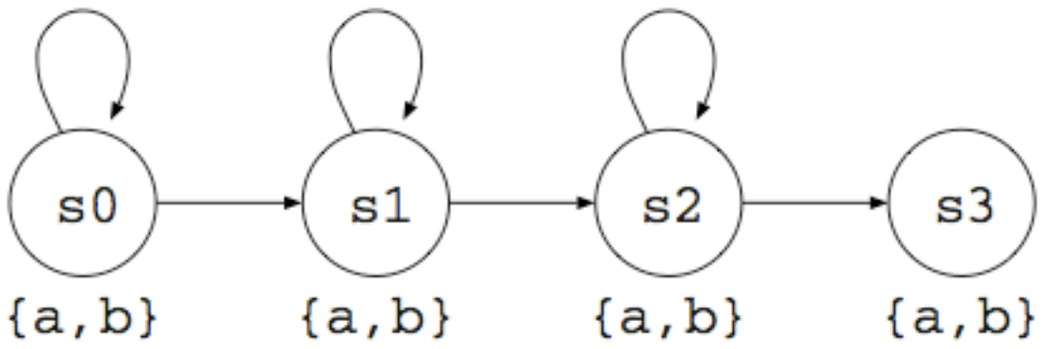


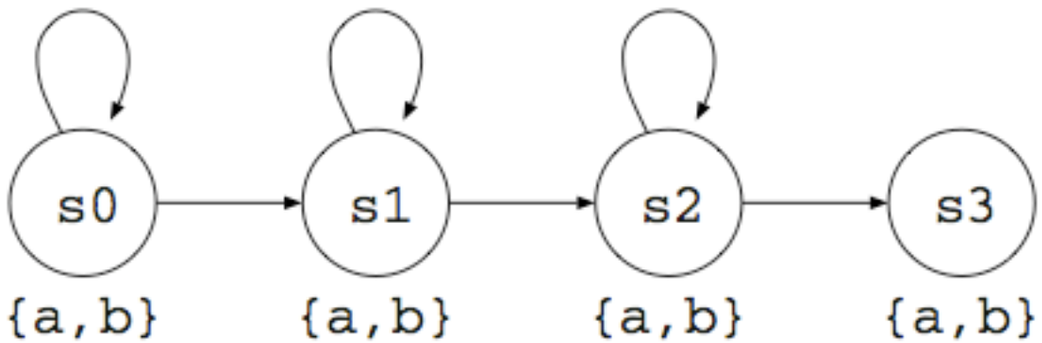
Fig. 1. Example of a left-to-right HMM with four states

```
target(hmm/1).
values(tr(s0),[s0,s1]).
values(tr(s1),[s1,s2]).
values(tr(s2),[s2,s3]).
values(out(_),[a,b]).

hmm(Cs):- hmm(0,s0,Cs).

hmm(T,s3,[C]):- msw(out(s3),  C). % If at the final state:
                                  %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,         % If not at the final state:
  msw(out(S),  C),                %   choose a symbol to be output,
  msw(tr(S),  Next),              %   choose the next state,
  T1 is T+1,                      %   Put the clock ahead,
  hmm(T1,Next,Cs).                %   and enter the next loop.
```

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM



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```

define switches

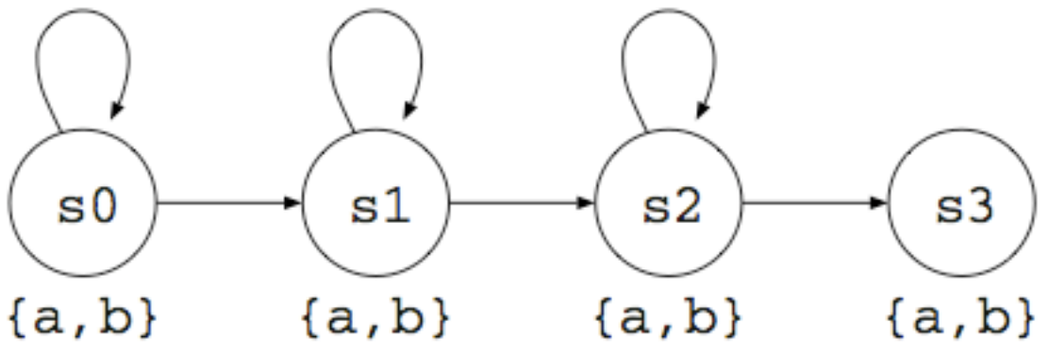
```
hmm(Cs):- hmm(0,s0,Cs).

hmm(T,s3,[C]):- msw(out(s3),  C). % If at the final state:
                                  %   output a symbol and then terminate.
hmm(T,S,[C|Cs]):- S\==s3,         % If not at the final state:
  msw(out(S),  C),                %   choose a symbol to be output,
  msw(tr(S),  Next),              %   choose the next state,
  T1 is T+1,                      %   Put the clock ahead,
  hmm(T1,Next,Cs).                %   and enter the next loop.
```

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM



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  msw(tr(S), Next),              %   choose the next state,
  T1 is T+1,                     %   Put the clock ahead,
  hmm(T1,Next,Cs).               %   and enter the next loop.
```
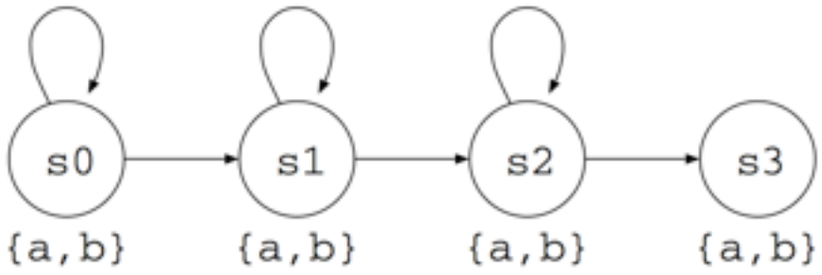

use switches

Fig. 2. PRISM program for the left-to-right HMM

[Figures: Sato and Kameya 08]

PRISM inference




```
hmm([a,b,b,b,b,a])
```

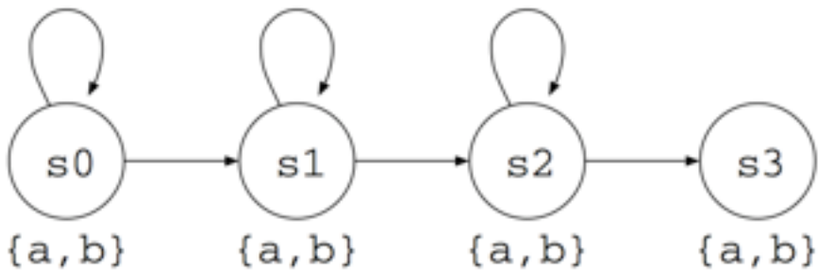
no memoization: two different RVS

$$
\begin{aligned}
E_1 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \\
& \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)} \\
E_2 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s0)} \wedge \mathtt{m(out(s0),b)} \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \\
& \wedge \mathtt{m(tr(s1),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)} \\
& \vdots \\
E_6 = {} & \mathtt{m(out(s0),a)} \wedge \mathtt{m(tr(s0),s1)} \wedge \mathtt{m(out(s1),b)} \wedge \mathtt{m(tr(s1),s2)} \wedge \mathtt{m(out(s2),b)} \\
& \wedge \mathtt{m(tr(s2),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s2)} \wedge \mathtt{m(out(s2),b)} \wedge \mathtt{m(tr(s2),s3)} \\
& \wedge \mathtt{m(out(s3),a)}
\end{aligned}
$$

Fig. 3. Six explanations for `hmm([a, b, b, b, b, a])`. Due to the space limit, the predicate name `msw` is abbreviated to `m`.

[Figures: Sato and Kameya 08]

PRISM inference



$$
\begin{aligned}
\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
\texttt{hmm}(0,\texttt{s0},[\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{a}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
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\texttt{hmm}(1,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s0}) \wedge \texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\\
&\quad \vee \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(2,\texttt{s1},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}])\ddagger\\
\texttt{hmm}(2,\texttt{s0},[\texttt{b},\texttt{b},\texttt{b},\texttt{a}]) &\Leftrightarrow \texttt{m}(\texttt{out}(\texttt{s0}),\texttt{b}) \wedge \texttt{m}(\texttt{tr}(\texttt{s0}),\texttt{s1}) \wedge \texttt{hmm}(3,\texttt{s1},[\texttt{b},\texttt{b},\texttt{a}])\\
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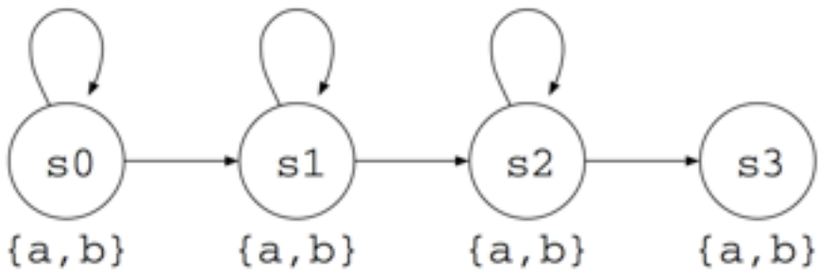
\end{aligned}
$$

Fig. 4. Factorized explanations for $\texttt{hmm}([\texttt{a},\texttt{b},\texttt{b},\texttt{b},\texttt{b},\texttt{a}])$

[Figures: Sato and Kameya 08]

PRISM inference



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[Figures: Sato and Kameya 08]

PRISM: compute probability by dynamic programming

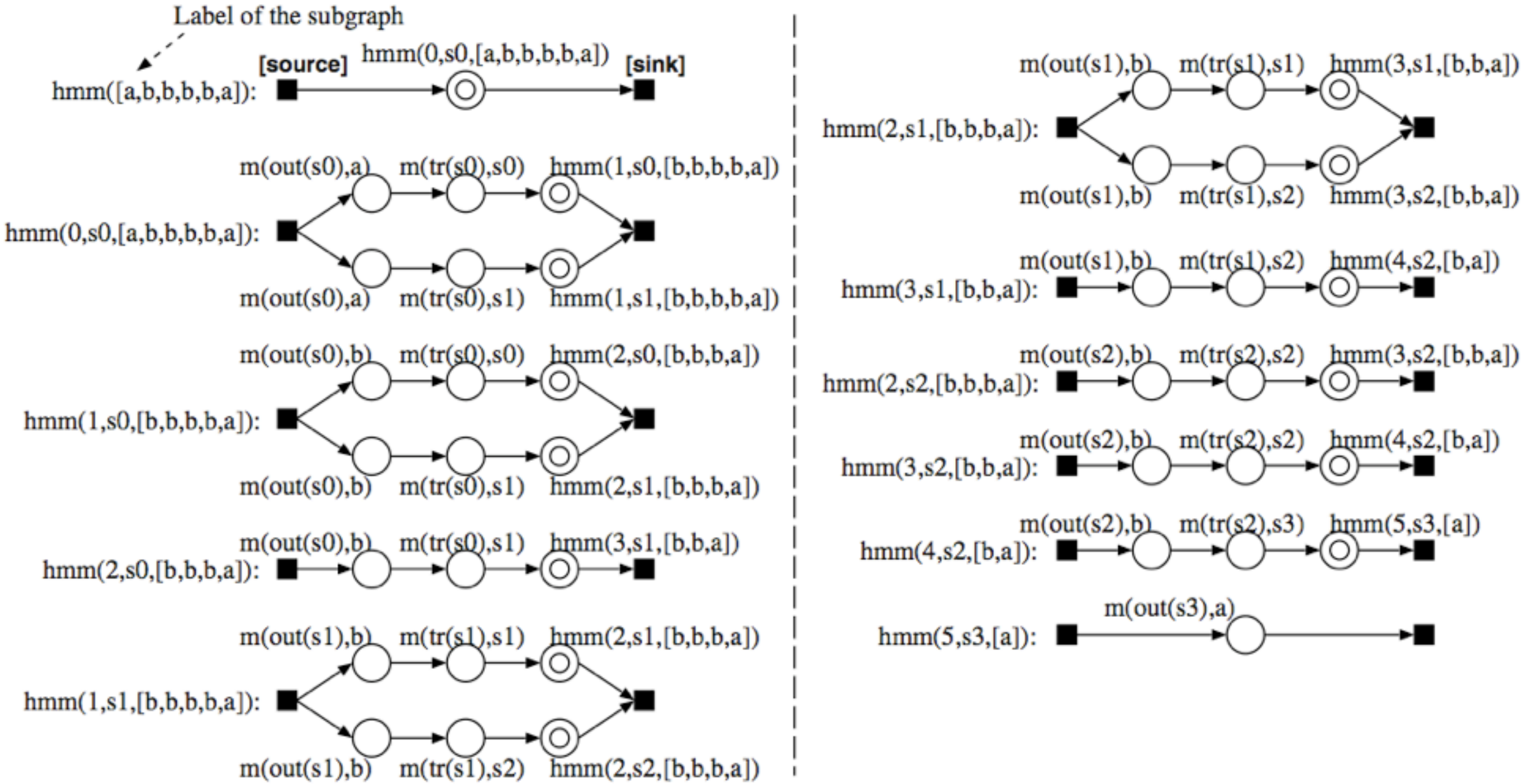


Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

PRISM: compute probability by dynamic programming

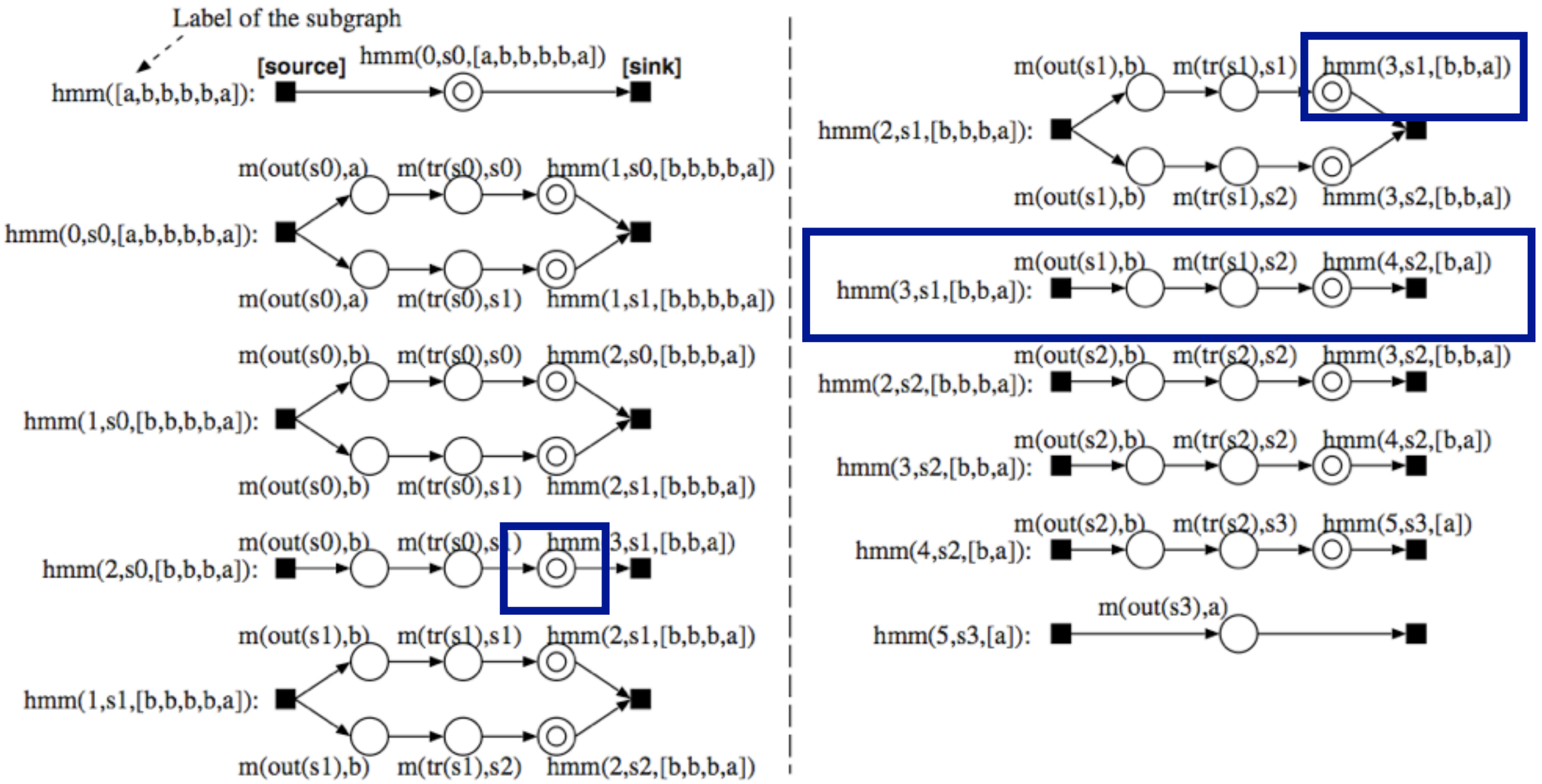


Fig. 5. Explanation graph

[Figures: Sato and Kameya 08]

Query Evaluation in PDB

- **Extensional** evaluation
  - guided by query expression only
  - exploit DB technology
  - for queries known to have polytime evaluation

Query Evaluation in PDB

- **Extensional** evaluation
  - guided by query expression only
  - exploit DB technology
  - for queries known to have polytime evaluation
- **Intensional** evaluation
  - construct **lineage** (= propositional formula)
  - compute probability of lineage
  - all queries

same idea as for ProbLog

Approximate Inference

- Lower and upper bounds

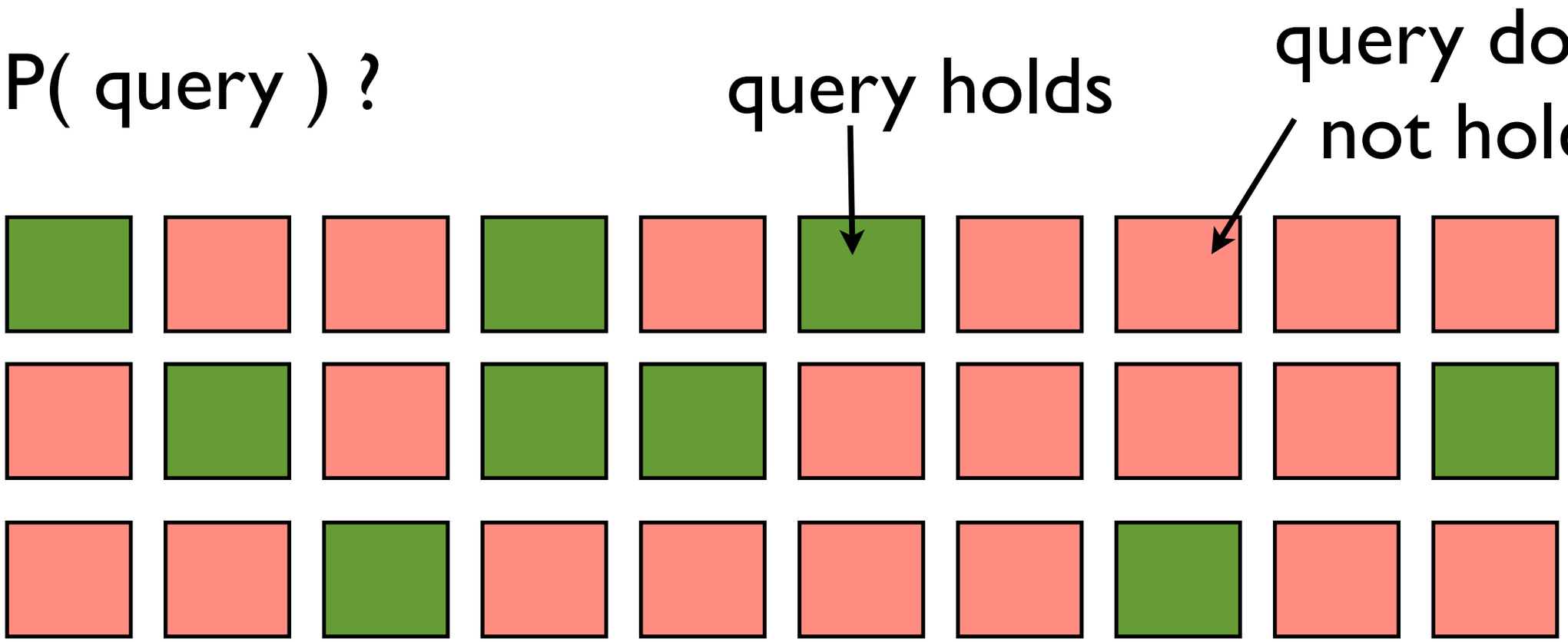
$$\phi_L \models \phi \models \phi_U$$

$$P(\phi_L) \leq P(\phi) \leq P(\phi_U)$$

- Sampling

Sampling

- P(query) ?



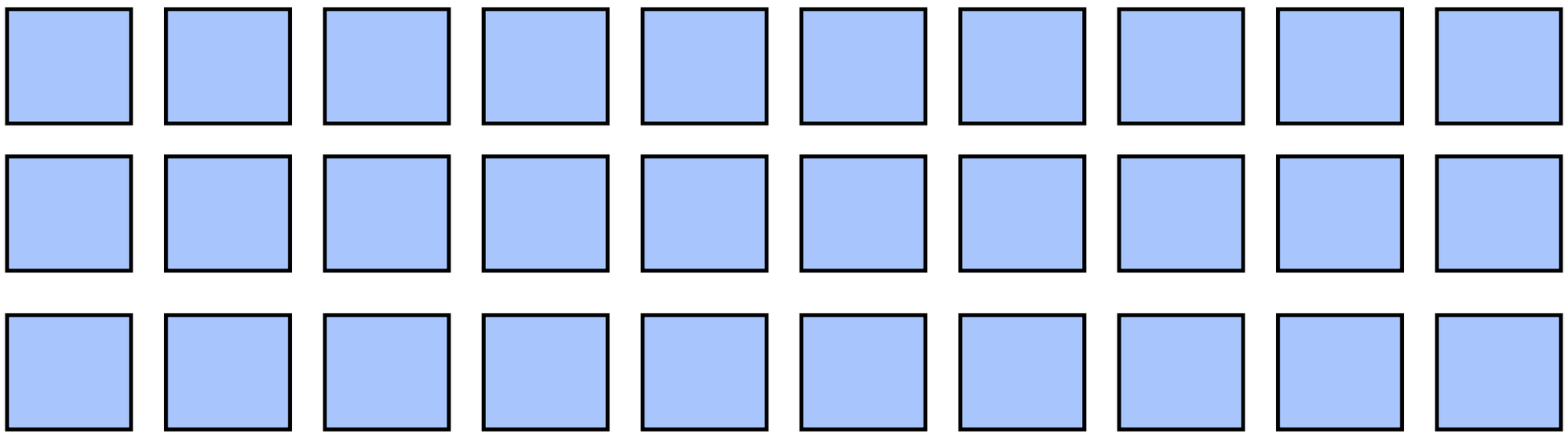
$$P(\text{query}) \approx \frac{\#\text{ query holds}}{\#\text{ worlds sampled}}$$

Rejection Sampling

- P(query | evidence) ?

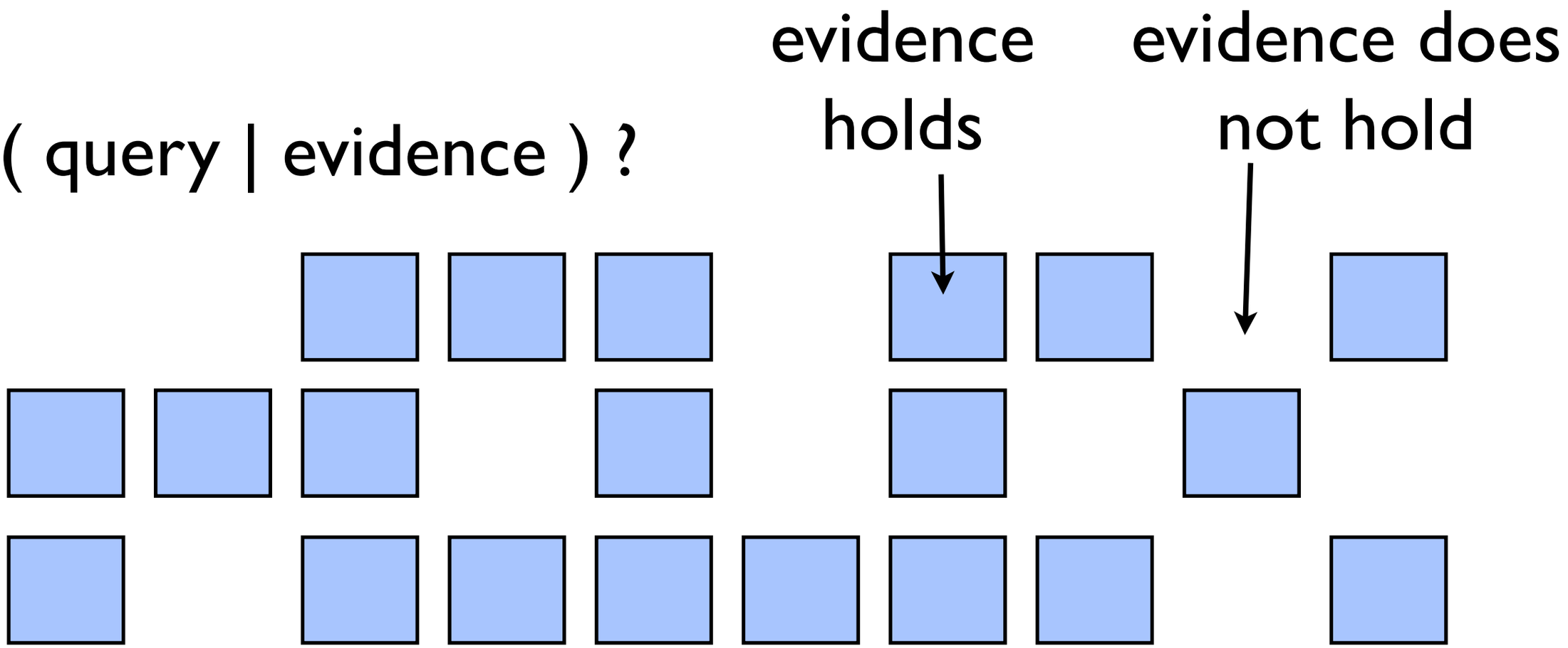
Rejection Sampling

- P(query | evidence) ?

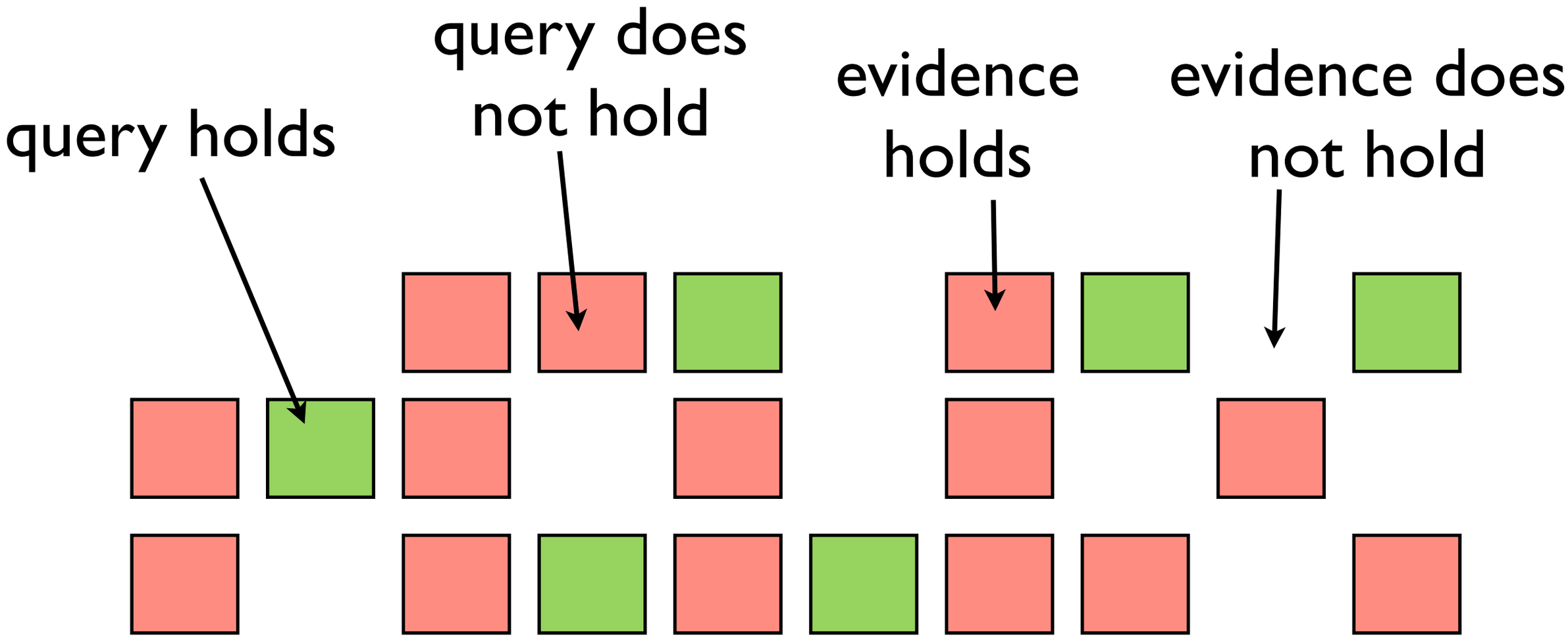


Rejection Sampling

- P(query | evidence) ?



Rejection Sampling



$$P(\text{query} \mid \text{evidence}) \approx \frac{\#\ \text{query \& evidence holds}}{\#\ \text{evidence holds}}$$

Markov Chain Monte Carlo (MCMC)

- Generate next sample by modifying current one
- Most common inference approach for PP languages such as Church, BLOG, ...
- Also considered for PRISM and ProbLog

Key challenges:
- how to propose next sample
- how to handle evidence

Parameter Learning

Parameter Learning

e.g., webpage classification model

for each ***CLASS1***, ***CLASS2*** and each ***WORD***

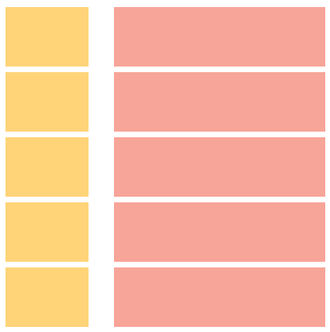
?? :: link_class(Source,Target,***CLASS1***,***CLASS2***).
?? :: word_class(***WORD***,***CLASS***).

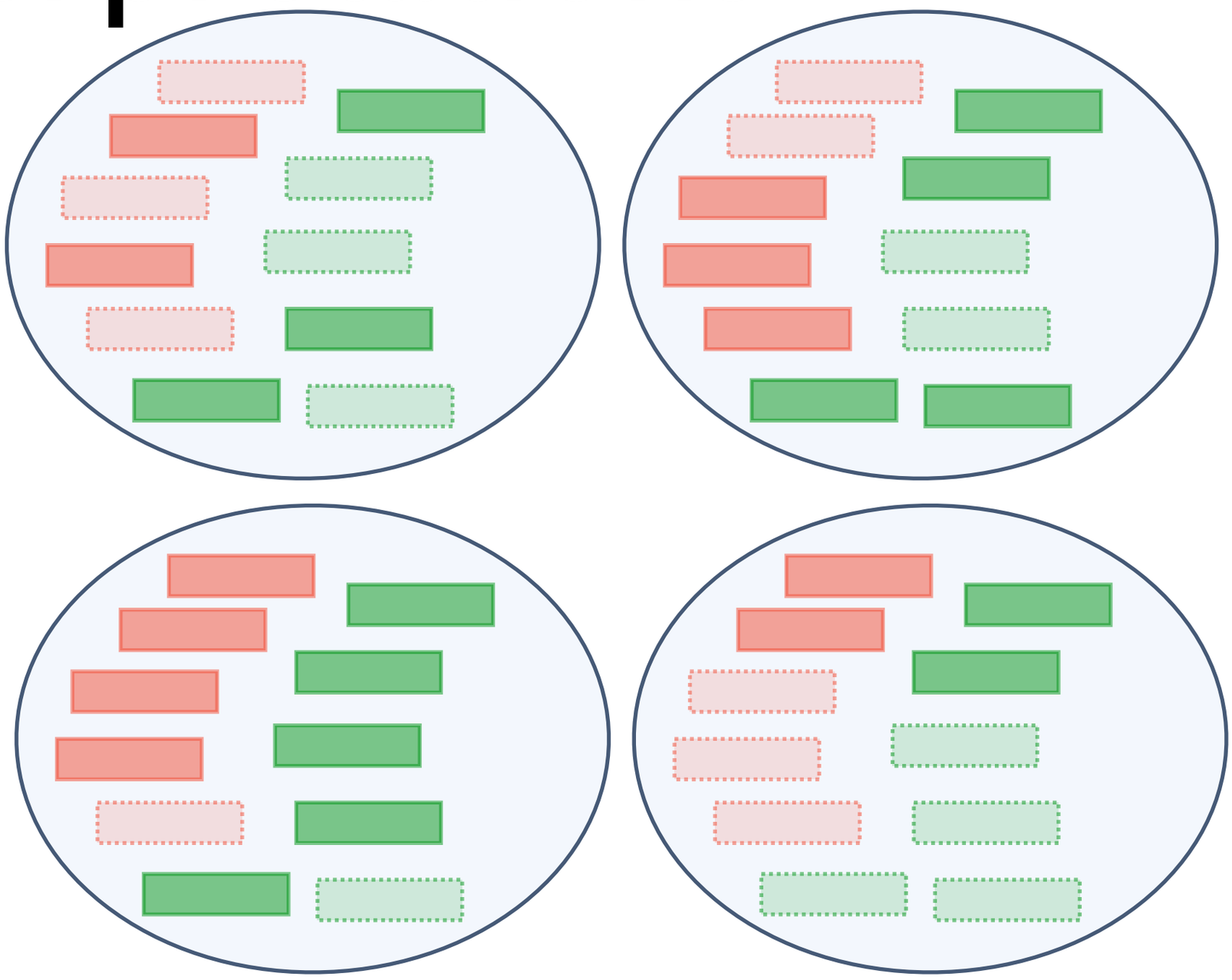
```
class(Page,C)    :-  has_word(Page,W), word_class(W,C).

class(Page,C)    :-  links_to(OtherPage,Page),
                     class(OtherPage,OtherClass),
                     link_class(OtherPage,Page,OtherClass,C).
```

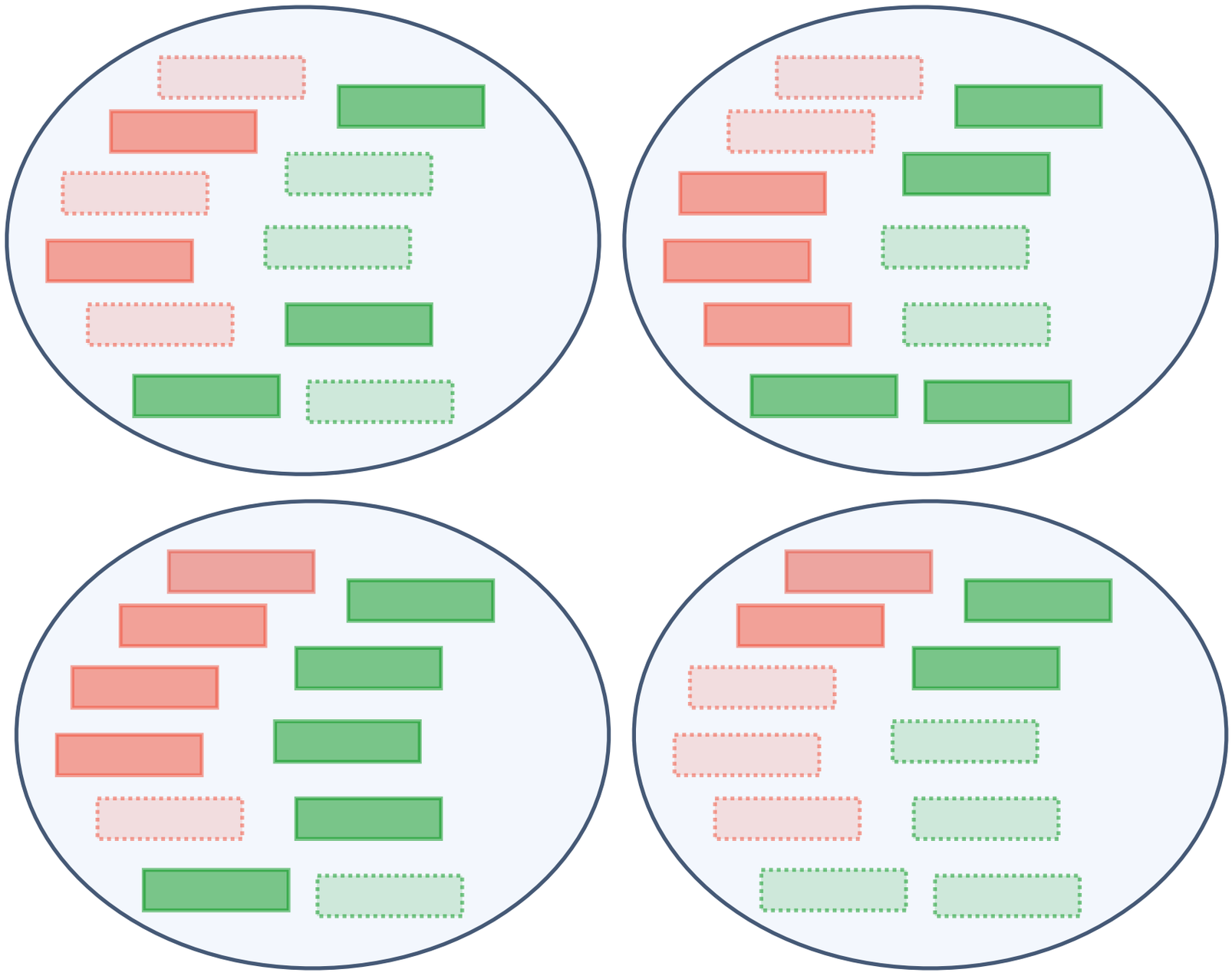
Sampling Interpretations

Sampling Interpretations

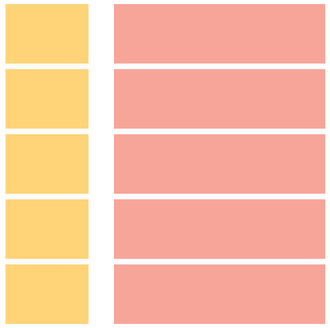


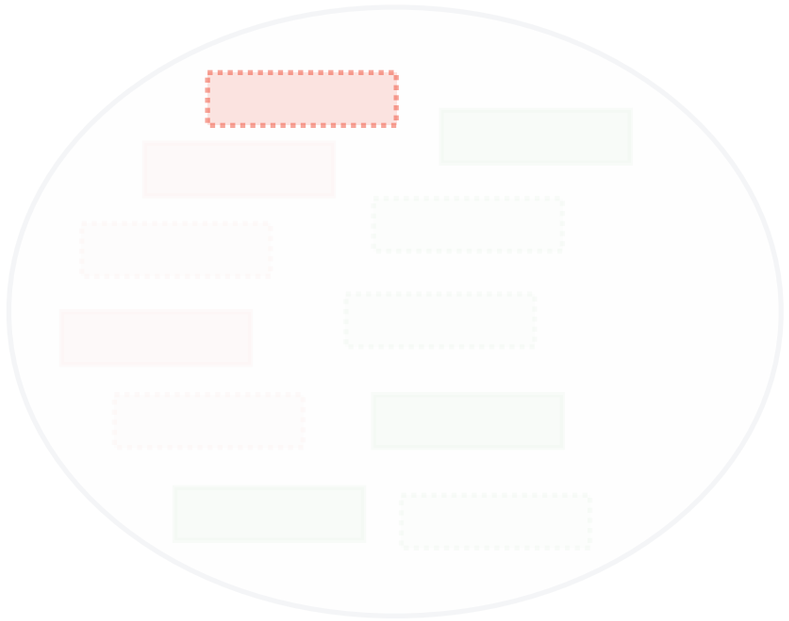


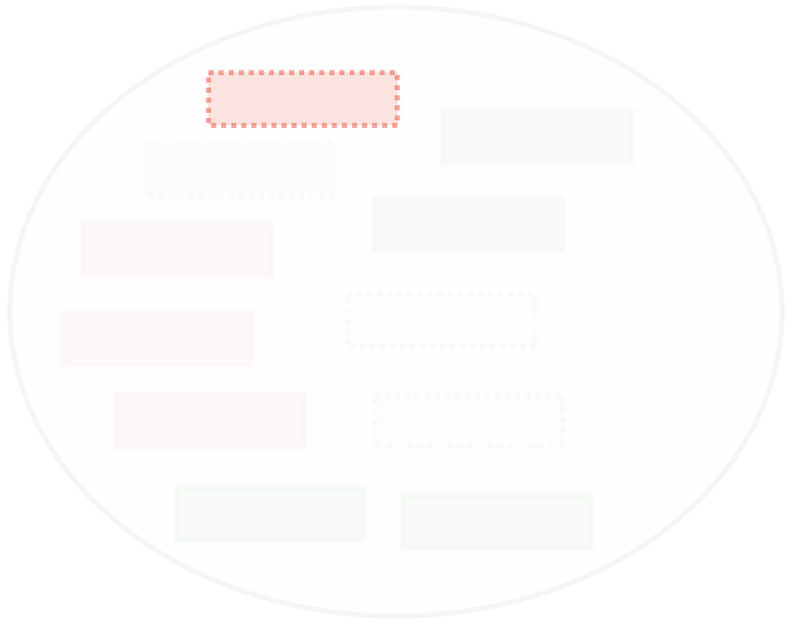
Parameter Estimation

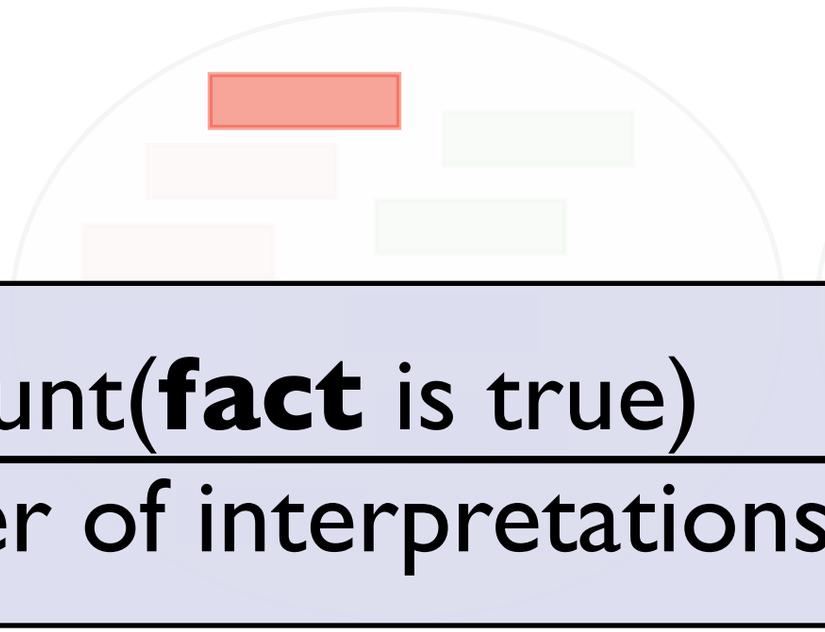


Parameter Estimation



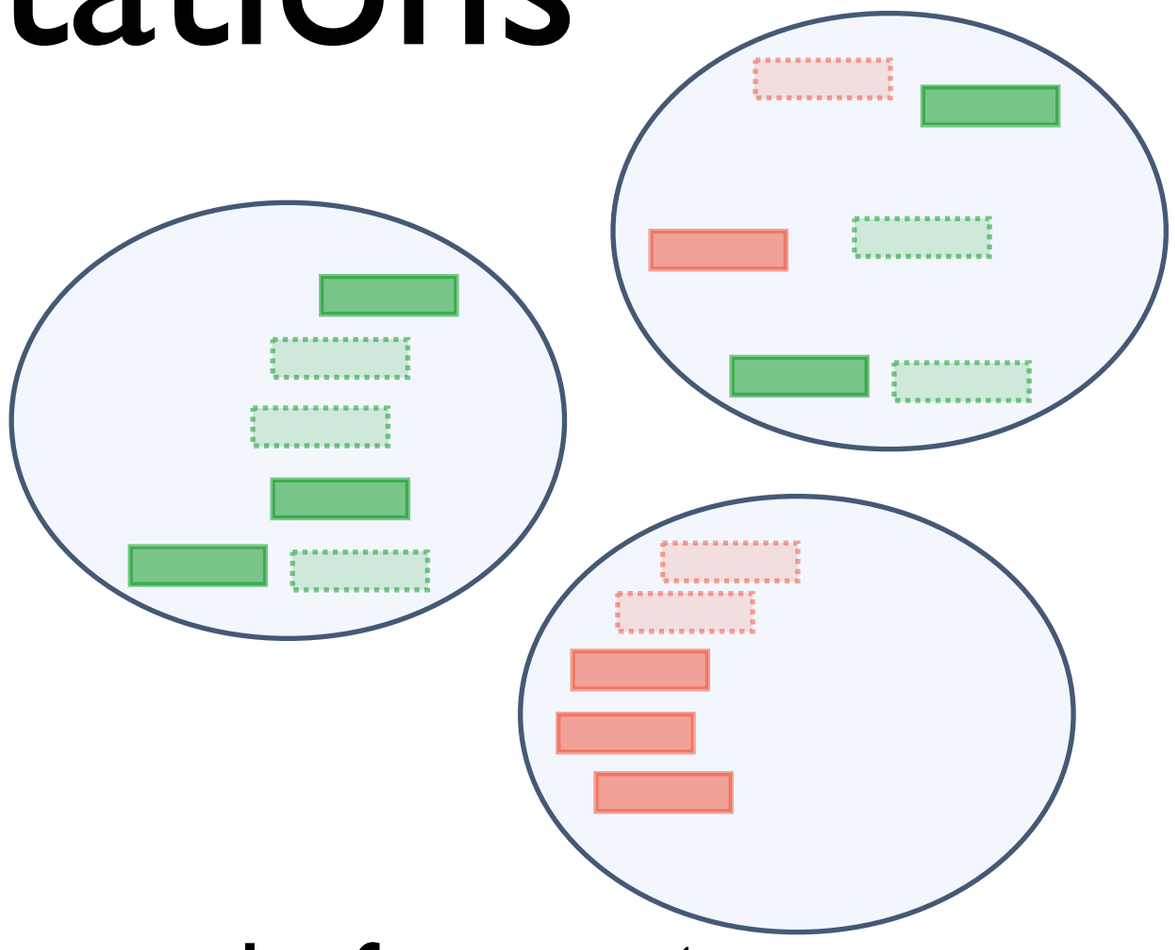






$$p(\textbf{fact}) = \frac{\text{count}(\textbf{fact} \text{ is true})}{\text{Number of interpretations}}$$

Learning from partial interpretations



- Not all facts observed
- Soft-EM
- use **expected count** instead of **count**
- **P(Q |E) -- conditional queries !**

[Gutmann et al, ECML 11; Fierens et al, TPLP 14]

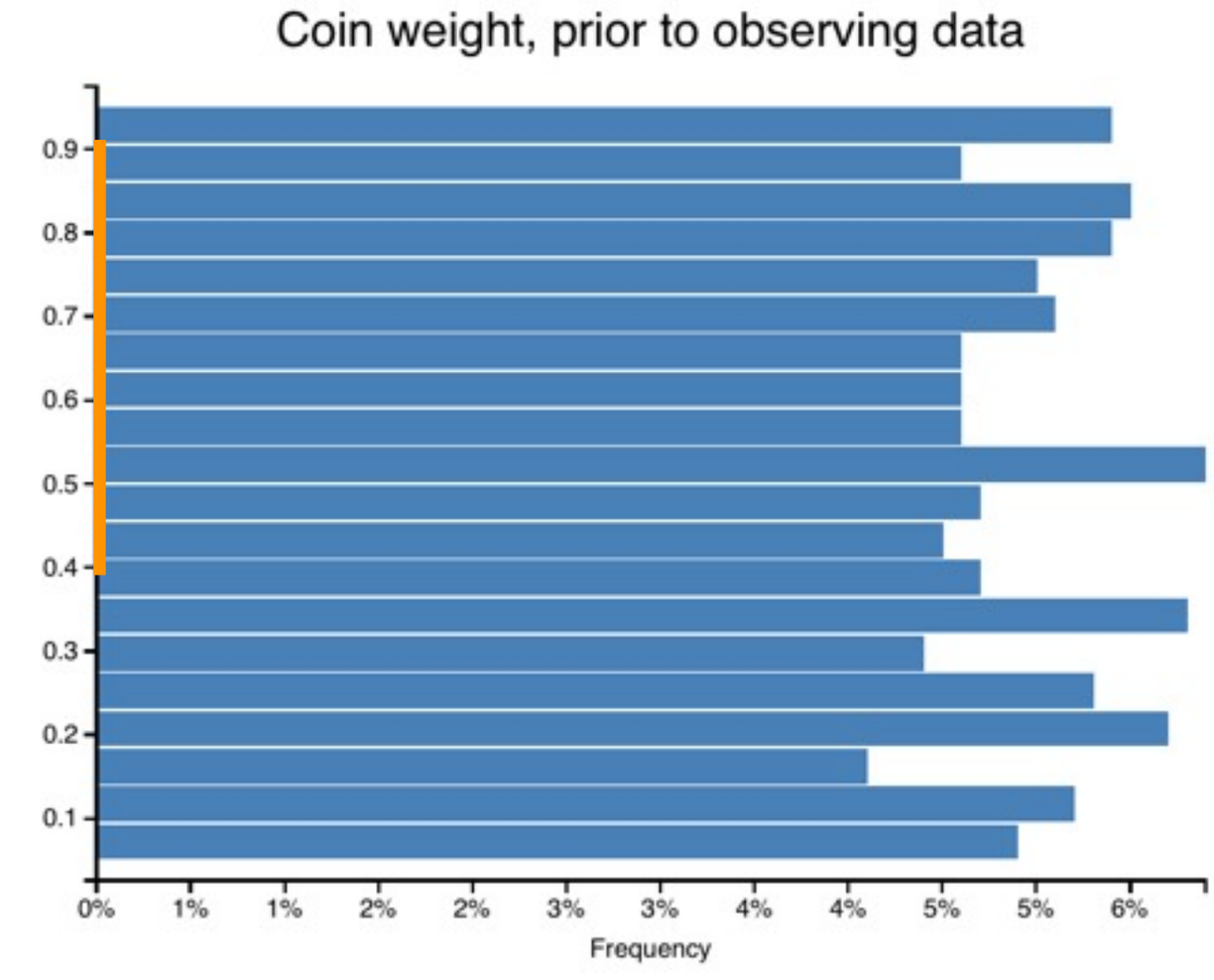
Bayesian Parameter Learning

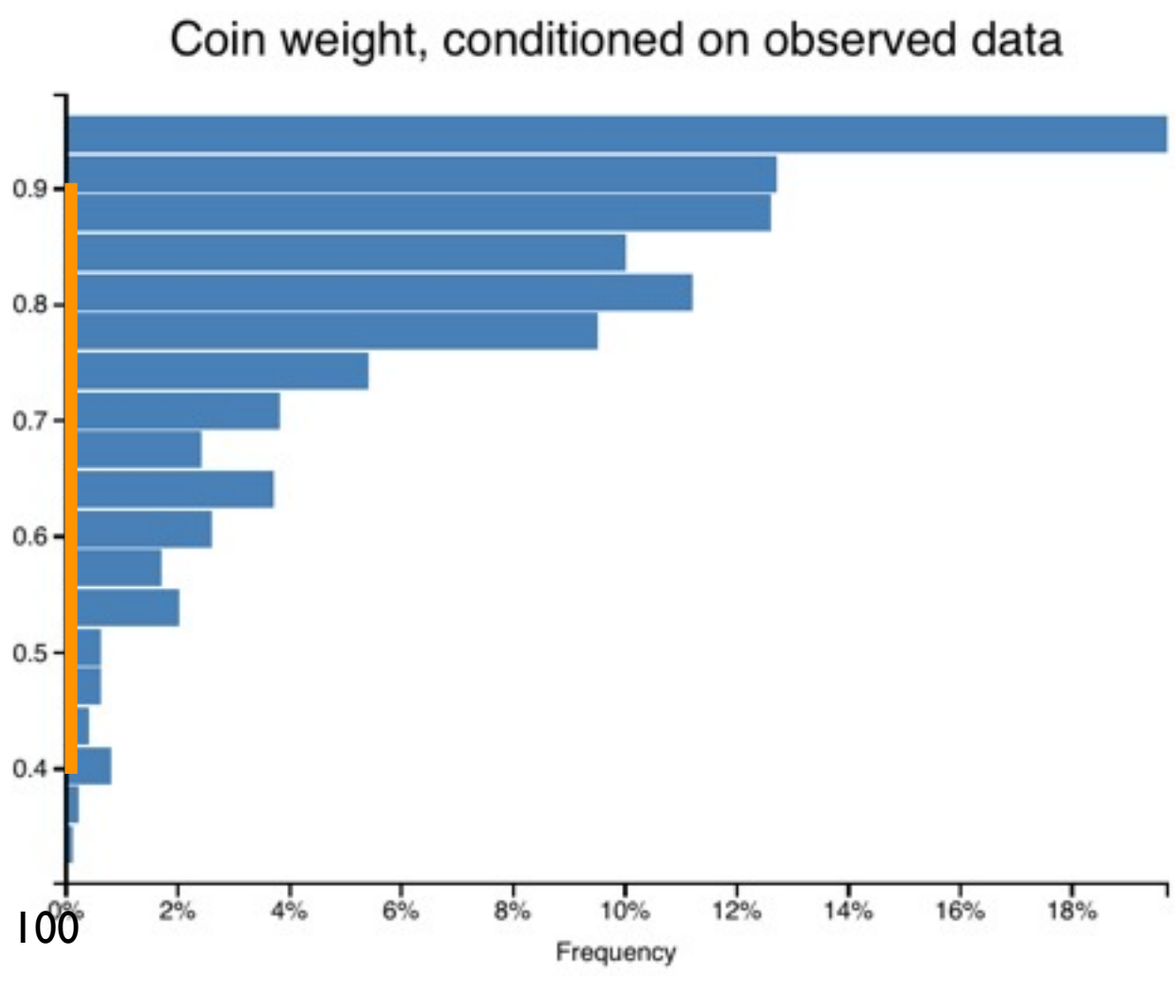
- Learning as inference (e.g., Church)
- Prior distributions for parameters
- Given data, find most likely parameter values

Example

[from probmods.org]

- Flipping a coin with unknown weight
- Prior: uniform distribution on [0,1]
- Observation: 5x heads in a row
- Sampling from Church model:





ProbLog Example

prior

```
0.05::weight(C,0.1); 0.2::weight(C,0.3); 0.5::weight(C,0.5);
                  0.2::weight(C,0.7); 0.05::weight(C,0.9) <- coin(C).
```

```
Param::toss(_,Param,_).
heads(C,R) :- weight(C,Param),toss(C,Param,R).
tails(C,R) :- weight(C,Param),\+toss(C,Param,R).

data(C,[]).
data(C,[h|R]) :- heads(C,R), data(C,R).
data(C,[t|R]) :- tails(C,R), data(C,R).
```

```
coin(c1).
coin(c2).
param(0.1).
param(0.3).
param(0.5).
param(0.7).
param(0.9).
```

```
query(weight(C,X)) :- coin(C),param(X).
```

ask for posterior

```
evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
evidence(data(c2,[h,t,h,h,h,h,h,t,t,h,t,t,h]),true).
```

data

ProbLog Example

```
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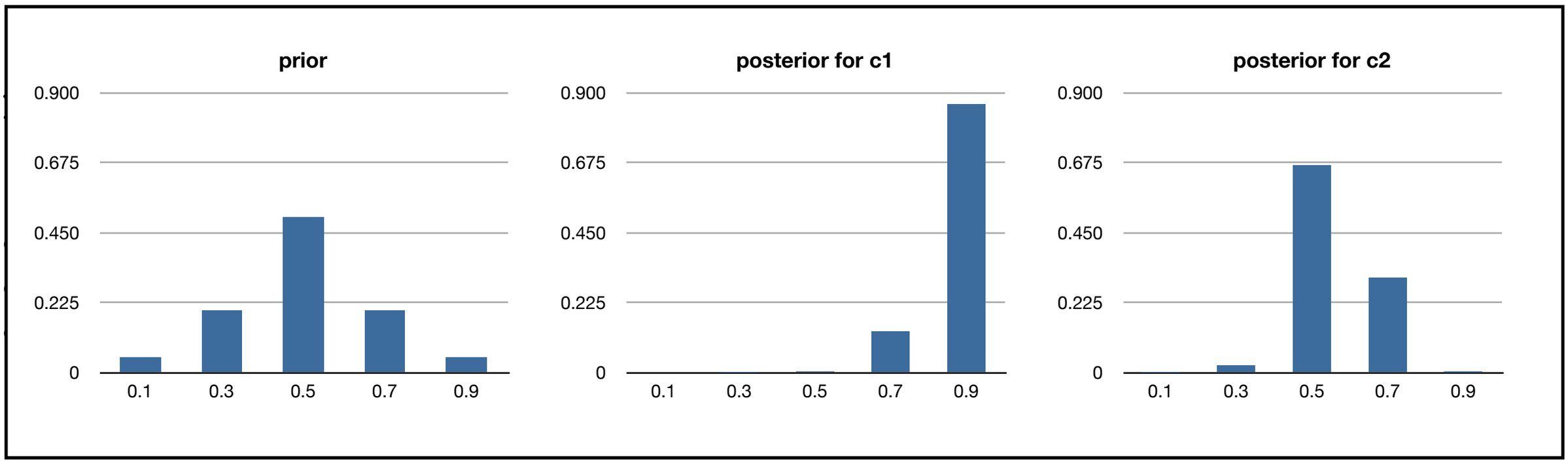


```
query(weight(C,X)) :- coin(C),param(X).

evidence(data(c1,[h,h,h,h,h,h,h,h,h,h,h,h,h,h]),true).
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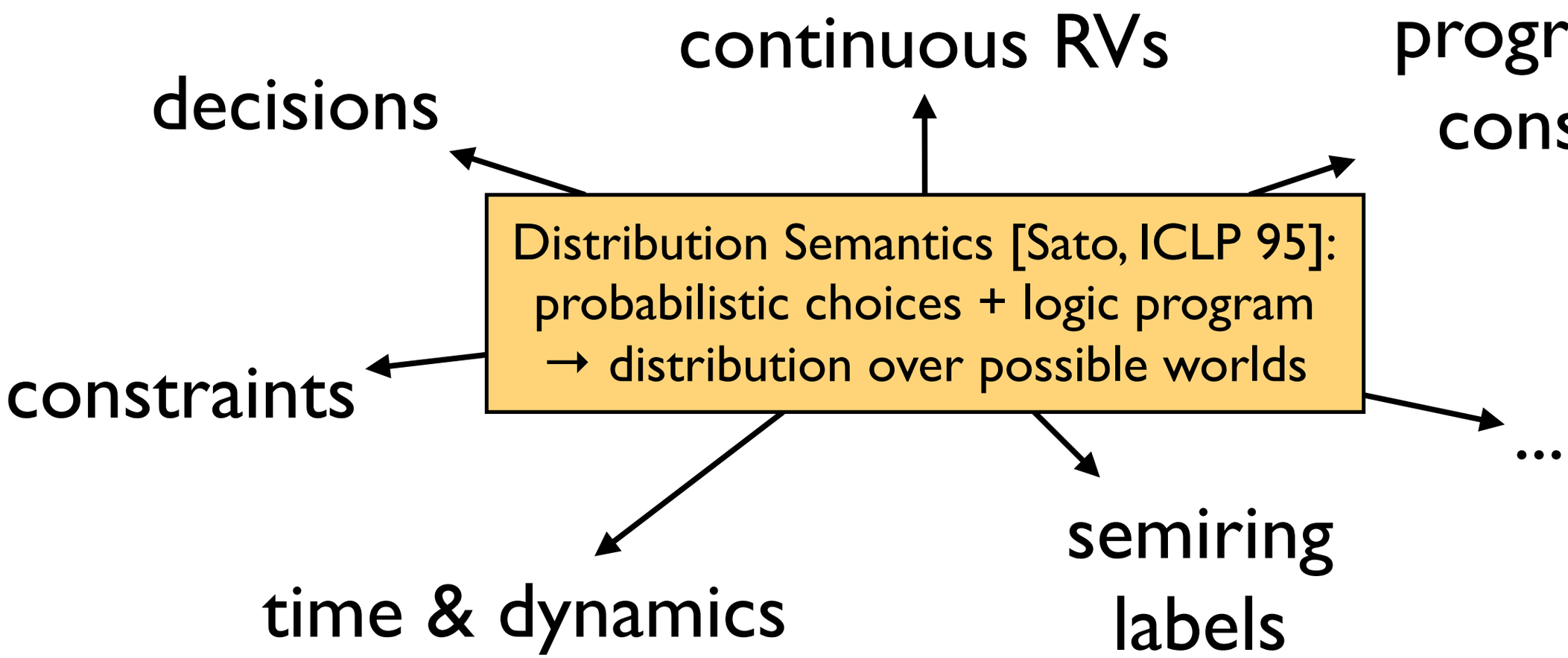
```

Roadmap

- Modeling (with detours to related work)
- Reasoning (and a bit of learning)
- Language extensions

Extensions of basic PLP



Extensions and Variants of ProbLog

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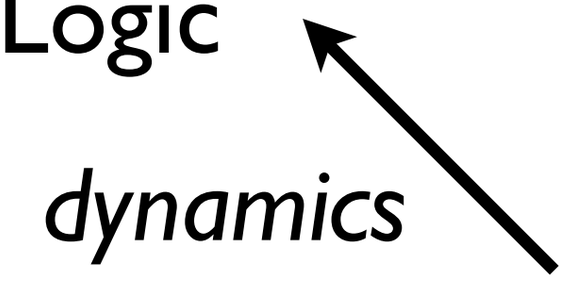
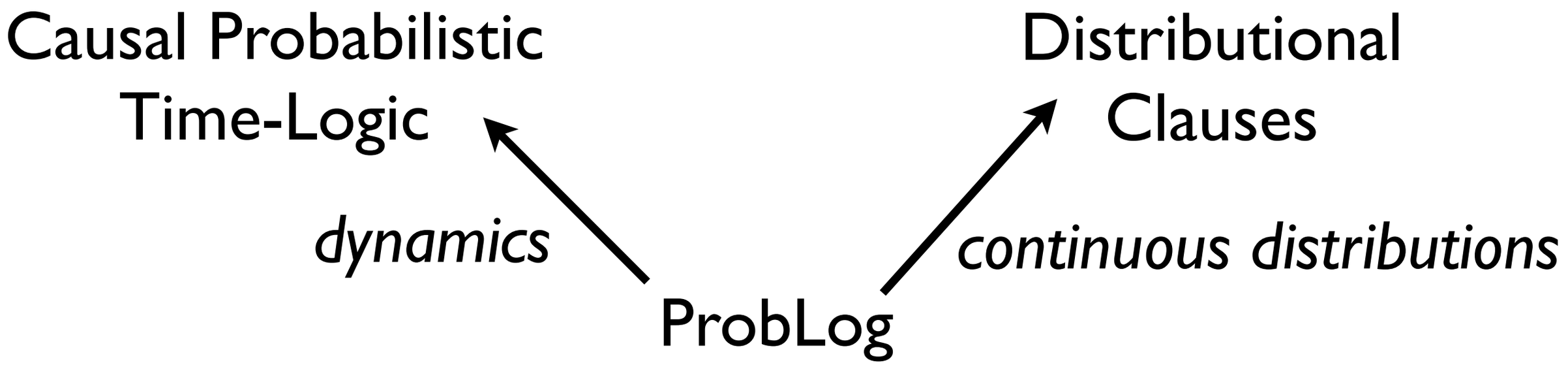
Causal Probabilistic Time-Logic *dynamics*



ProbLog

Extensions and Variants of ProbLog



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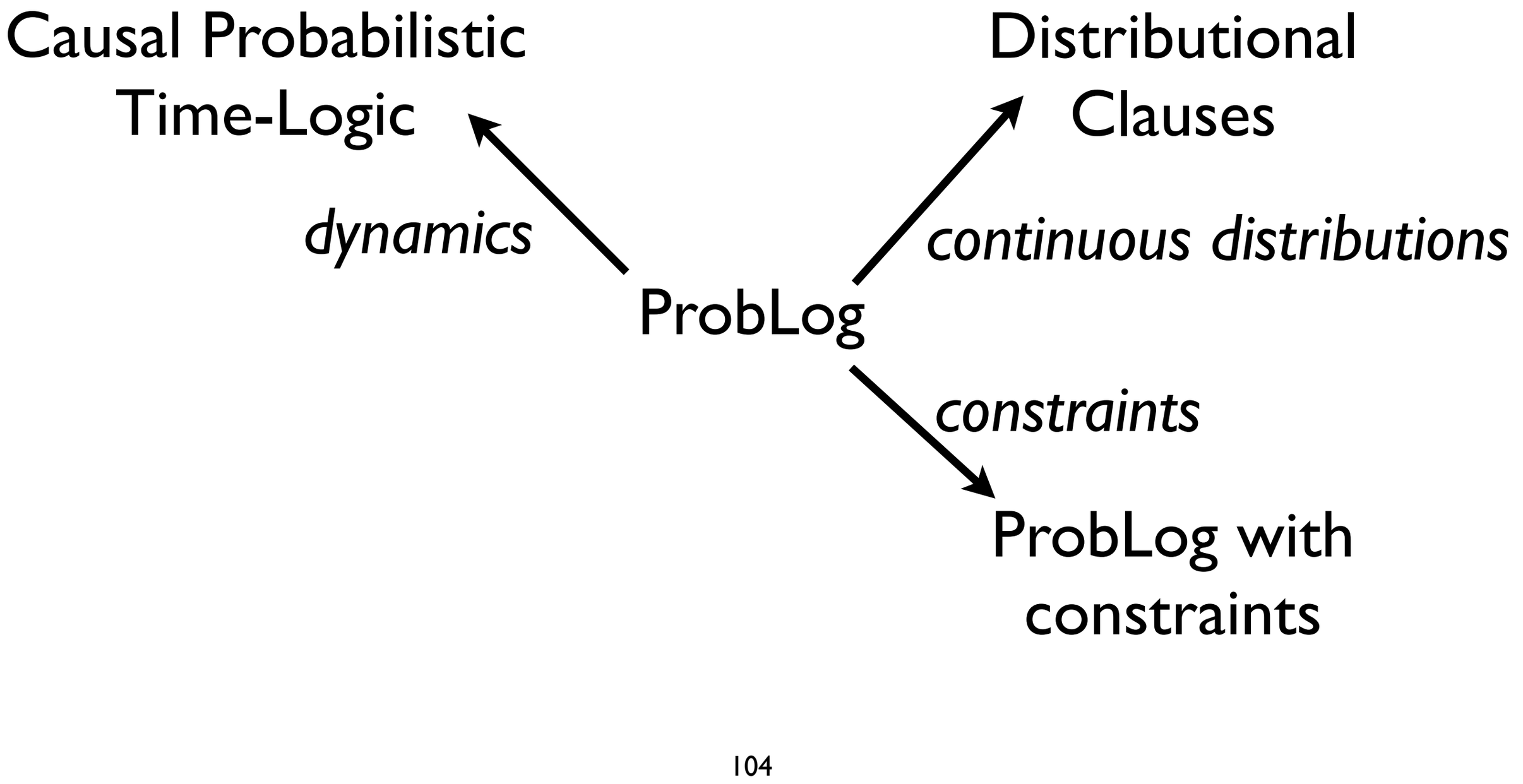

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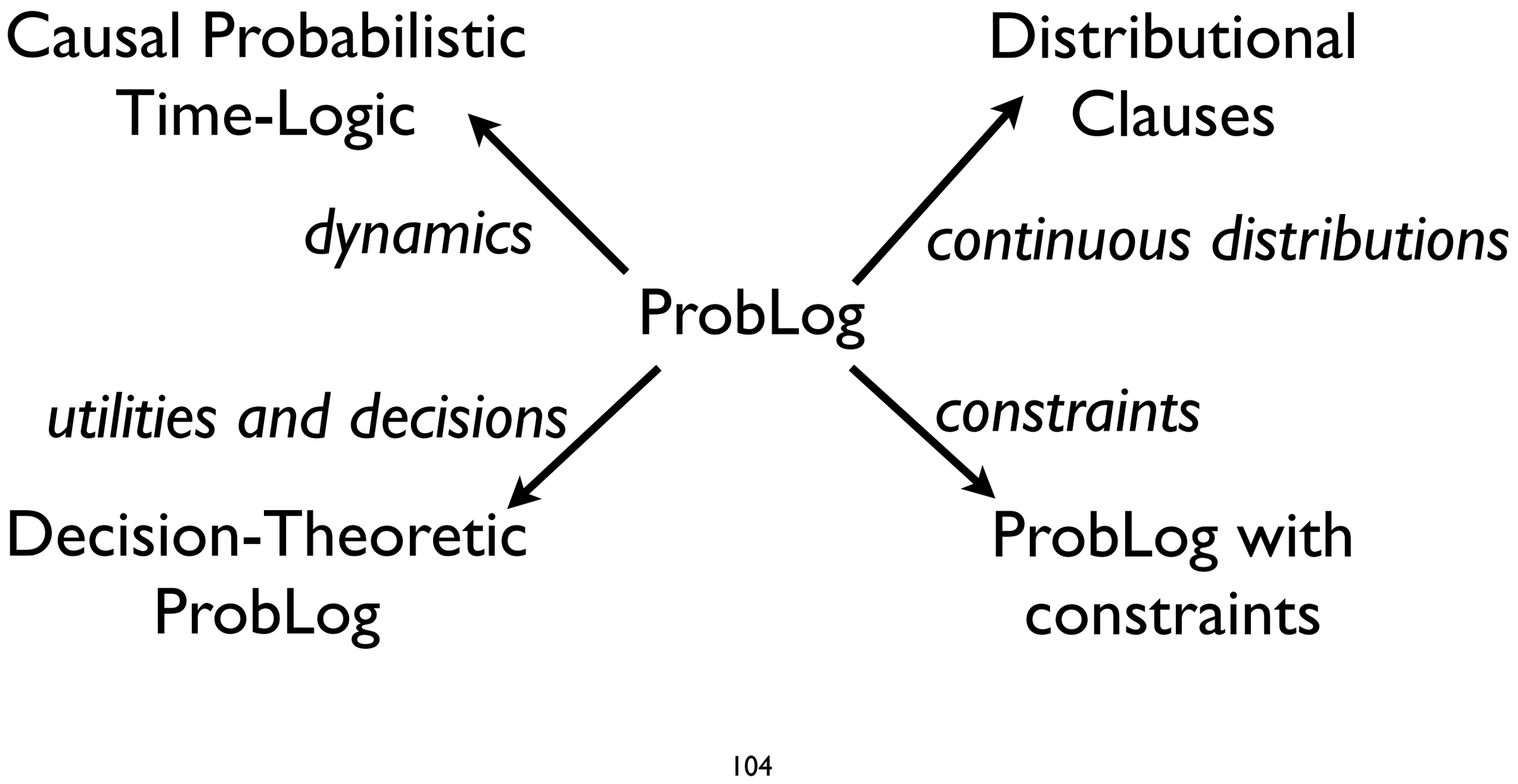

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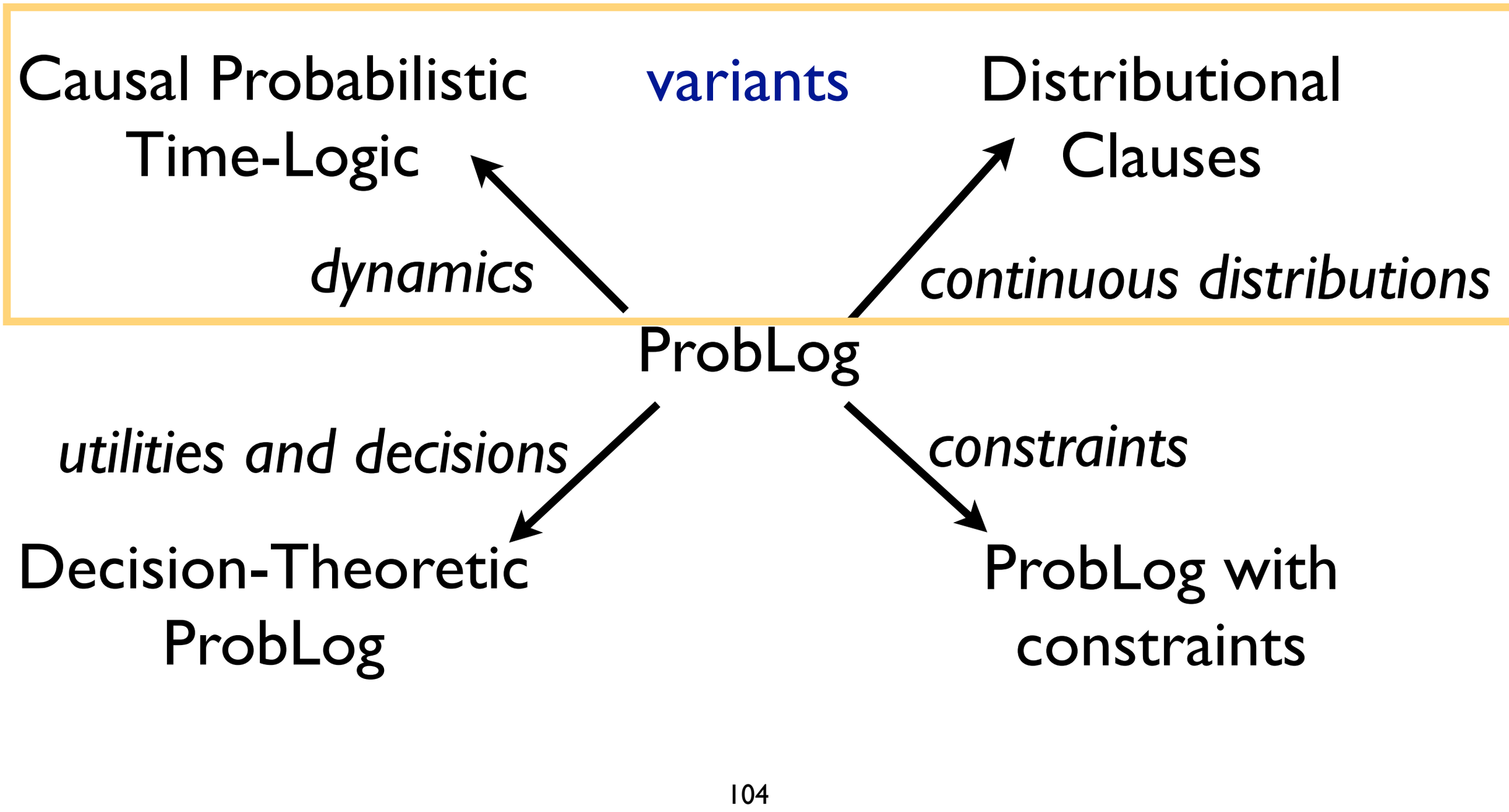

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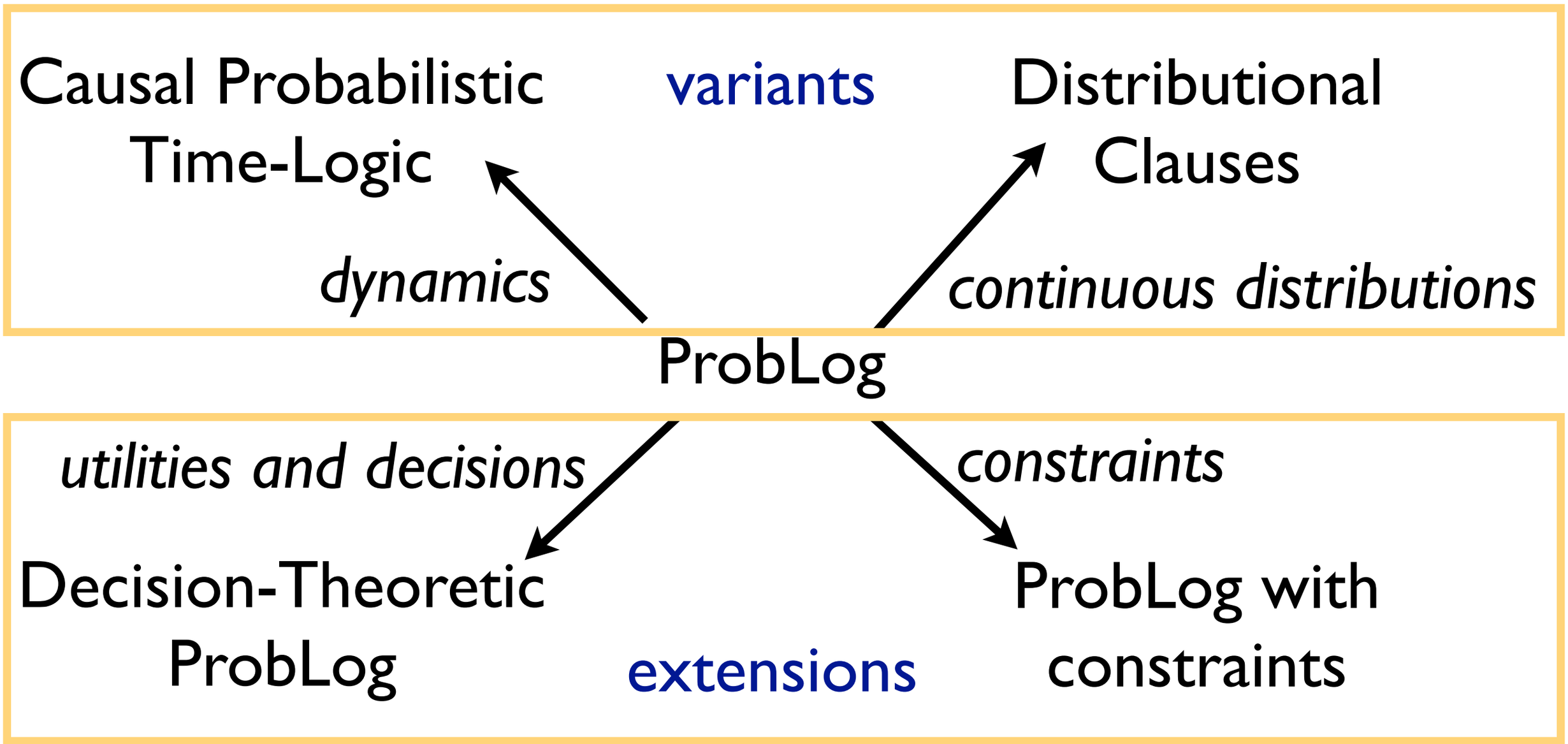
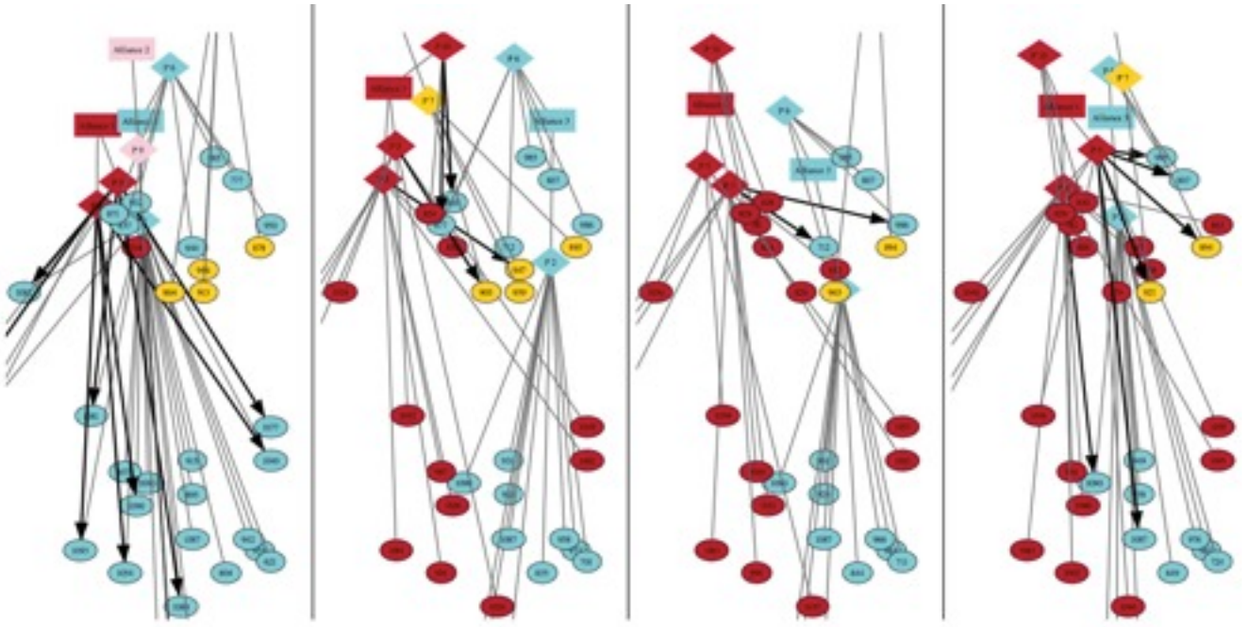


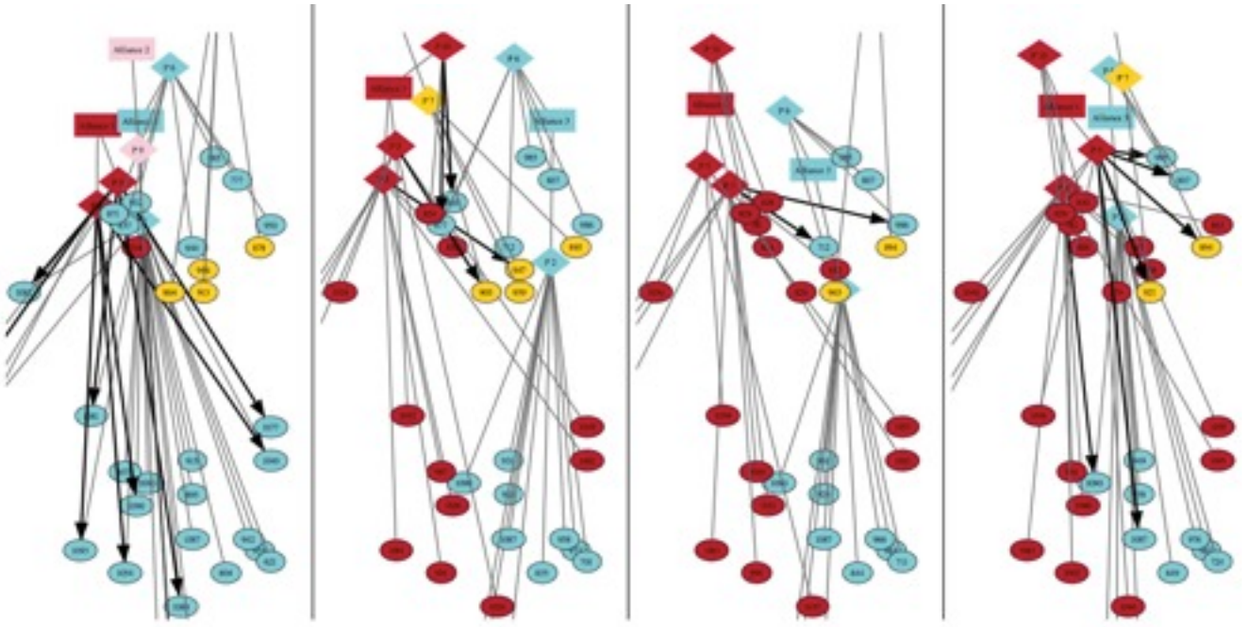
Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

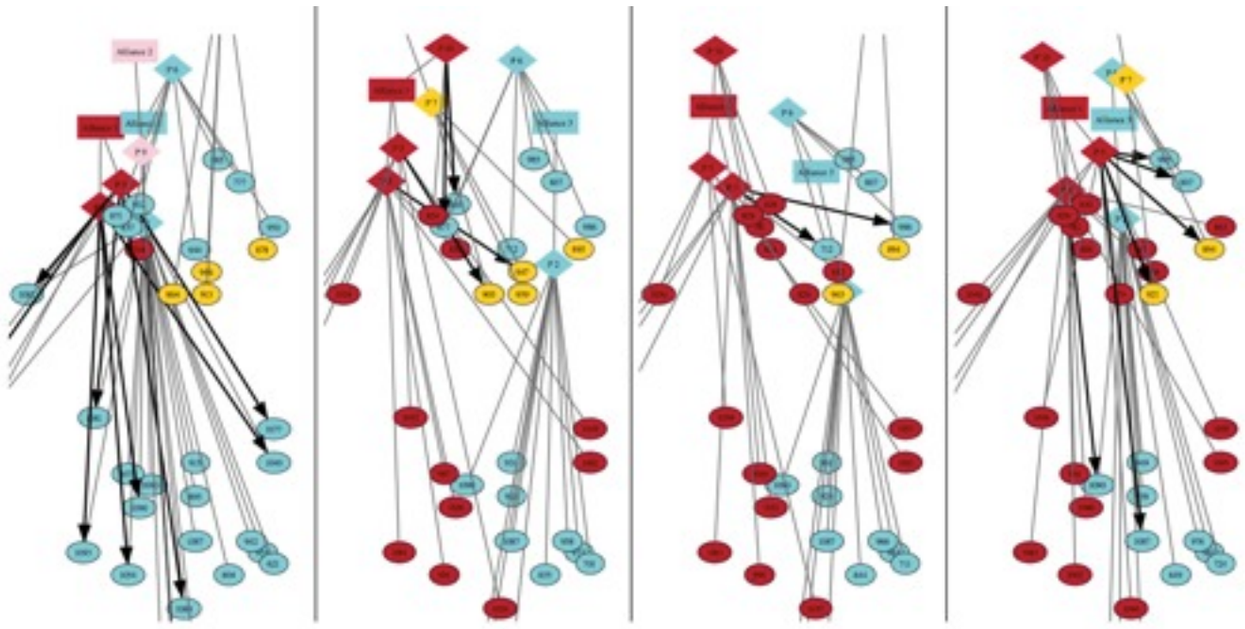
```
0.4::conquest(Attacker,C); 0.6::nil <-
```

```
city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

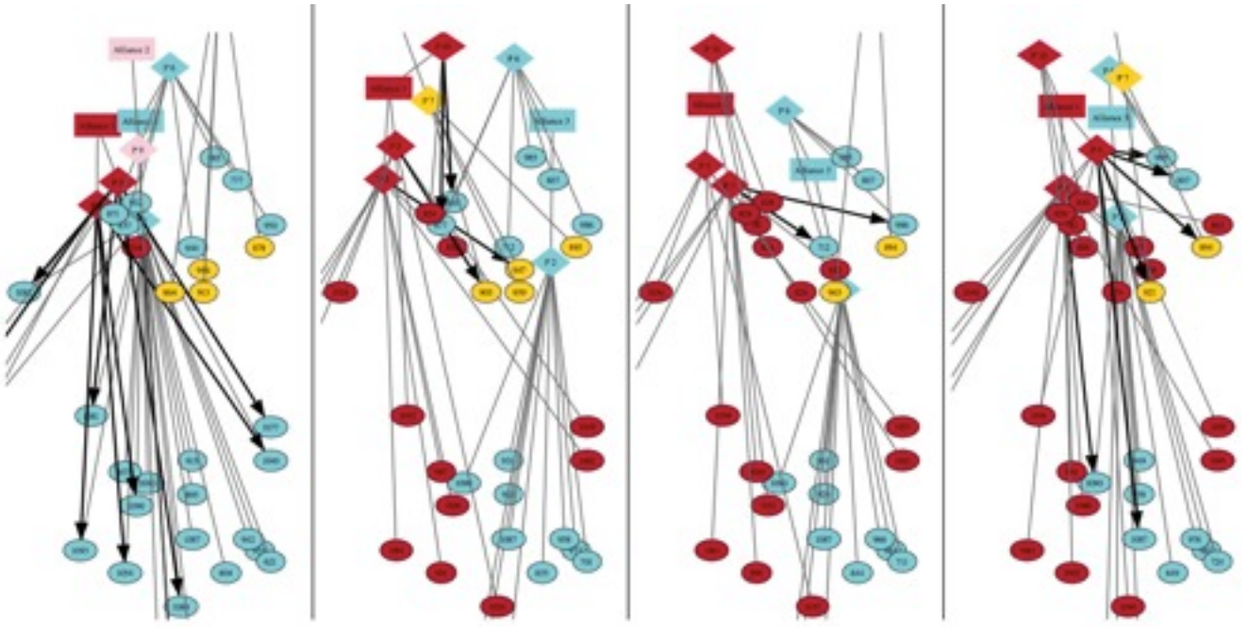
one of the **effects** holds at time T+1

```
0.4::conquest(Attacker,C); 0.6::nil <-
        city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Causal Probabilistic Time-Logic (CPT-L)



how does the world change over time?

one of the **effects** holds at time T+1

```
0.4::conquest(Attacker,C); 0.6::nil <-

                city(C,Owner),city(C2,Attacker),close(C,C2).
```

if **cause** holds at time T

[Thon et al, MLJ 11]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

random variable with Gaussian distribution

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
```

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
    ≃length(OBot) ≥ ≃length(OTop),
    ≃width(OBot) ≥ ≃width(OTop).
```

comparing values of random variables

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
      ≃length(OBot) ≥ ≃length(OTop),
      ≃width(OBot) ≥ ≃width(OTop).
ontype(Obj,plate) ~ finite([0 : glass, 0.0024 : cup,
                            0 : pitcher, 0.8676 : plate,
                            0.0284 : bowl, 0 : serving,
                            0.1016 : none])
                          :- obj(Obj), on(Obj,O2), type(O2,plate).
```

random variable with discrete distribution

[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

Distributional Clauses (DC)

- Discrete- and continuous-valued random variables
- Inference: particle filter

```
length(Obj) ~ gaussian(6.0,0.45) :- type(Obj,glass).
stackable(OBot,OTop) :-
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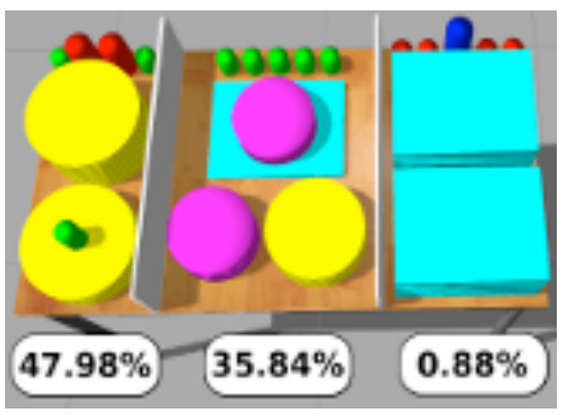
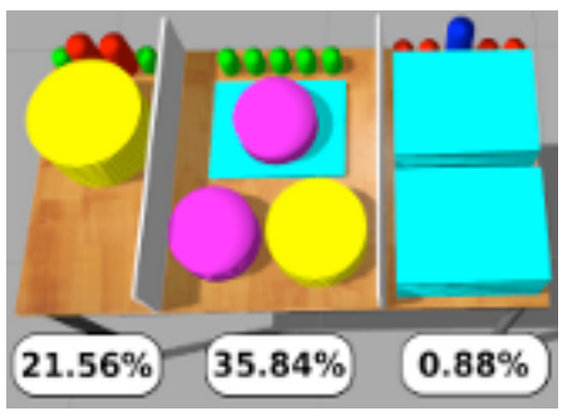
                            0.1016 : none])
                    :- obj(Obj), on(Obj,O2), type(O2,plate).
```

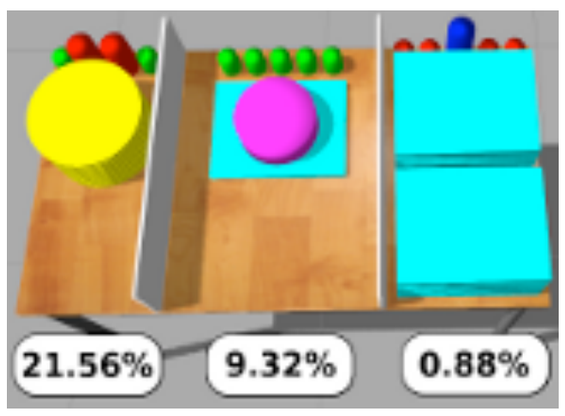
[Gutmann et al, TPLP 11; Nitti et al, IROS 13]

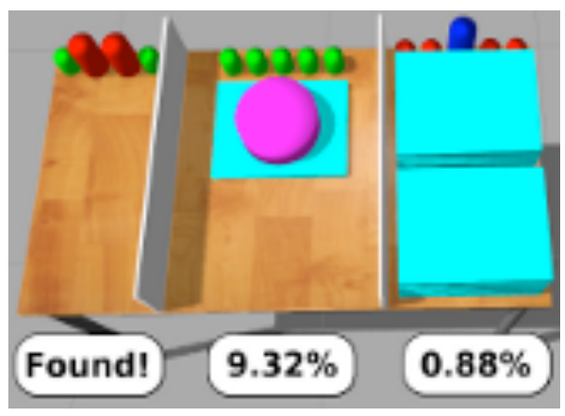
Occluded Object Search

- DC model of objects and their spatial arrangement
- different types of objects suitable for different tasks
- shelves with objects of different shape and size
- given a task, find an object to perform that task







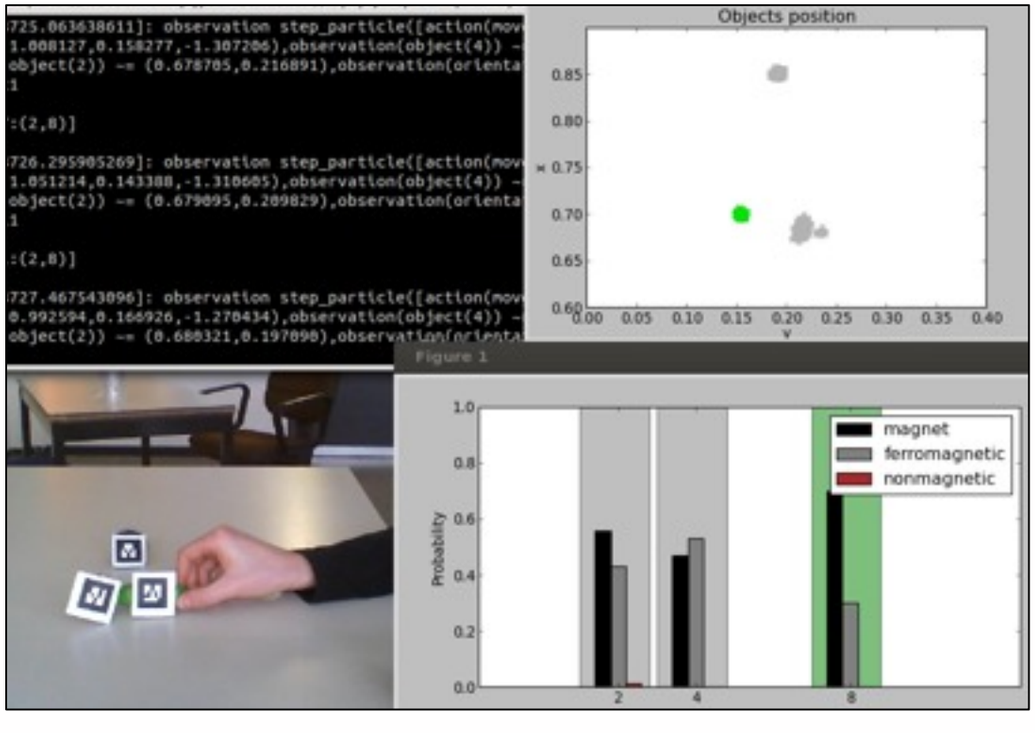


[Moldovan et al, ICRA 12, 14]

Relational State Estimation over Time

Magnetism scenario

- object tracking
- category estimation from interactions

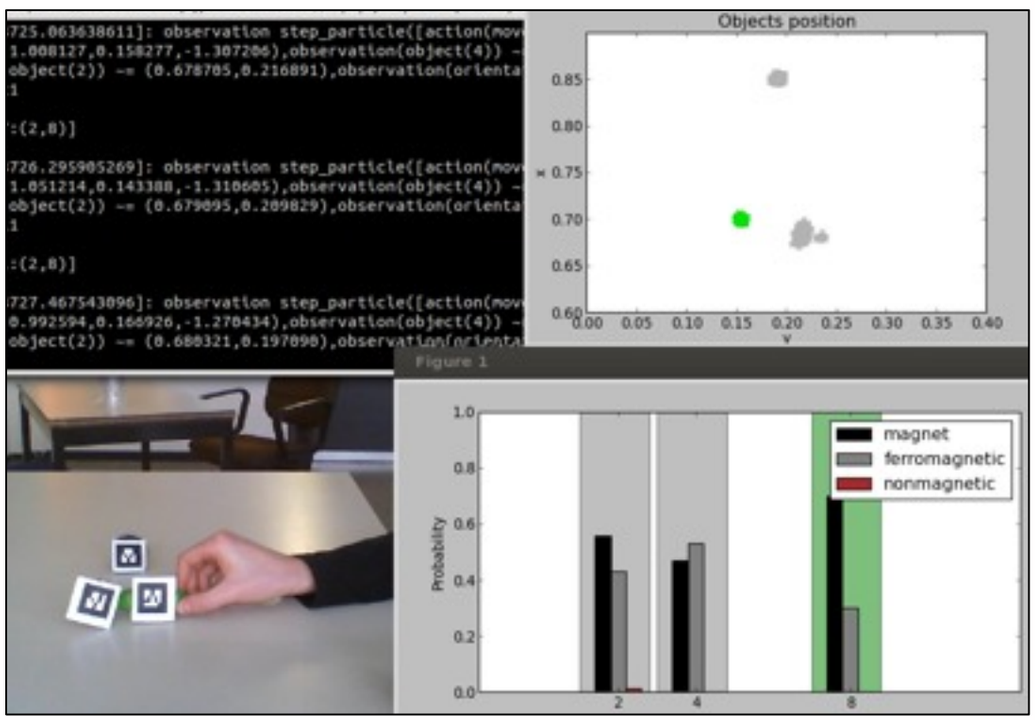


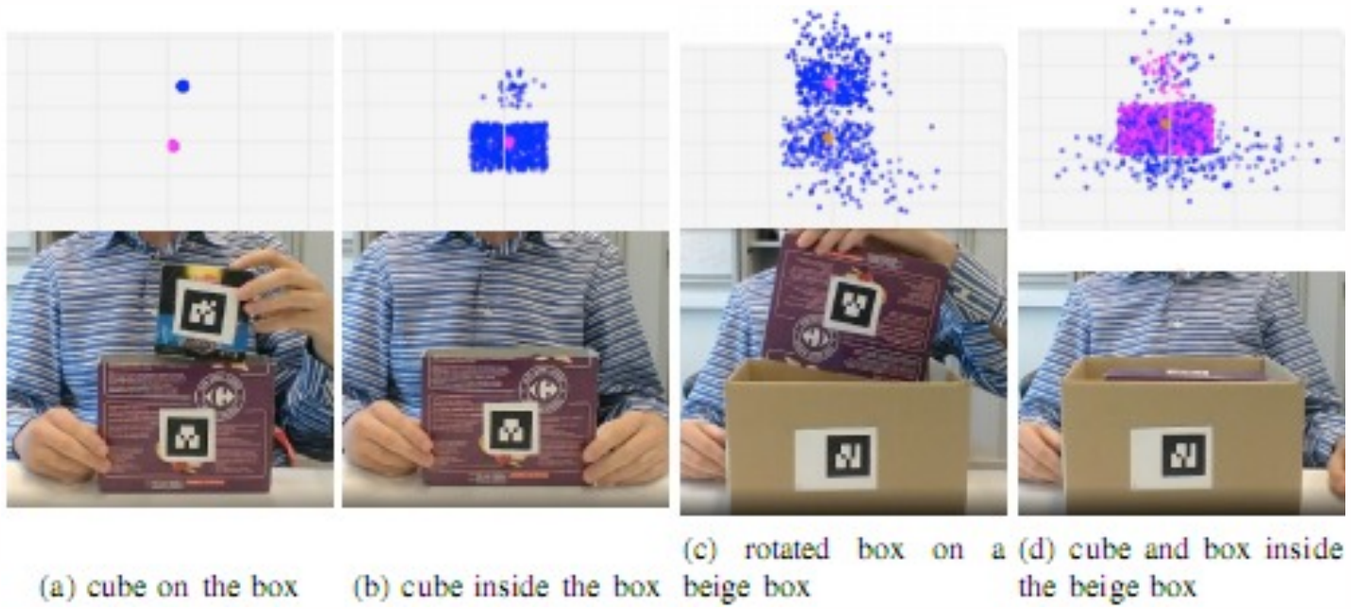
[Nitti et al, IROS 13]

Relational State Estimation over Time

Magnetism scenario

- object tracking
- category estimation from interactions



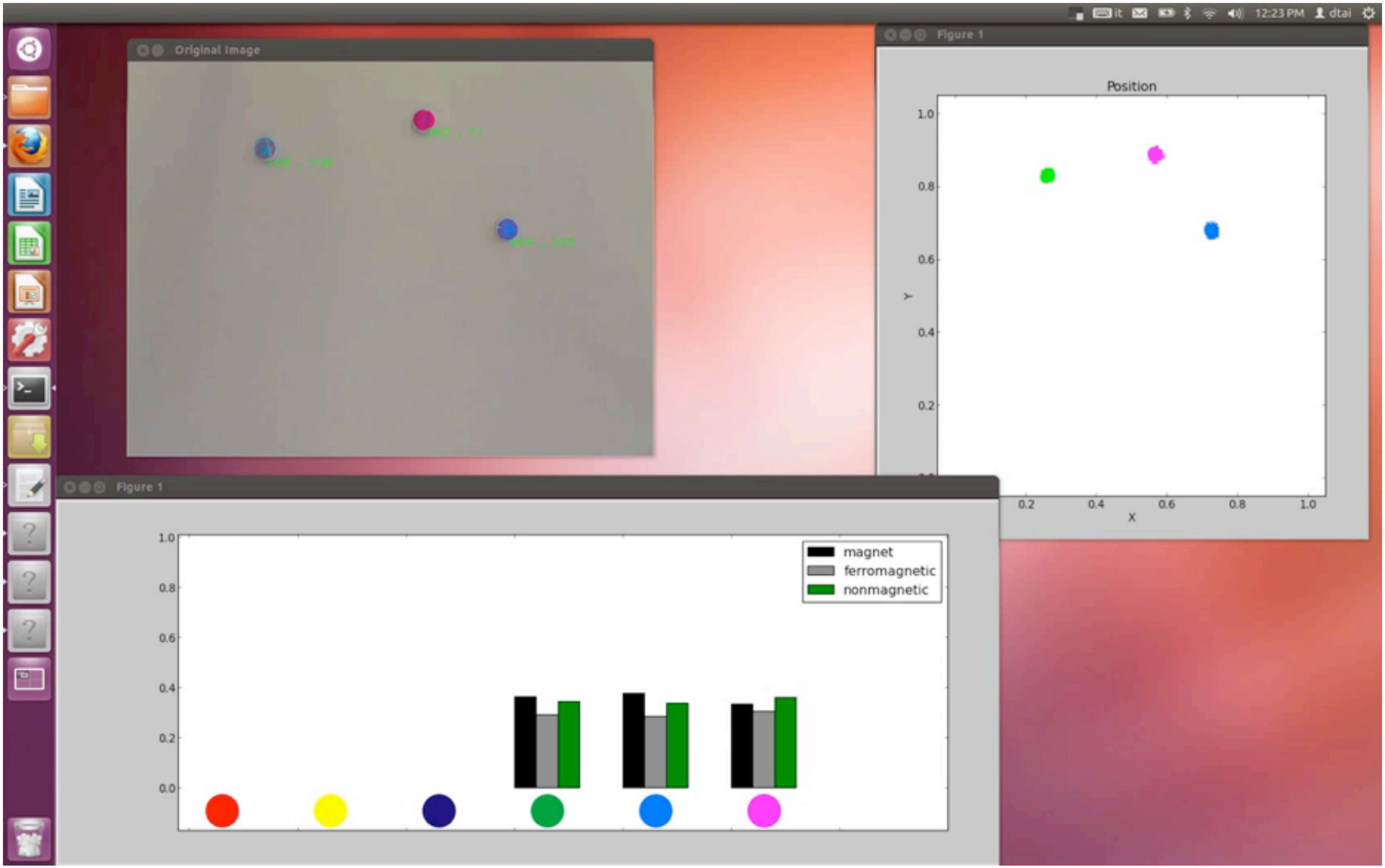


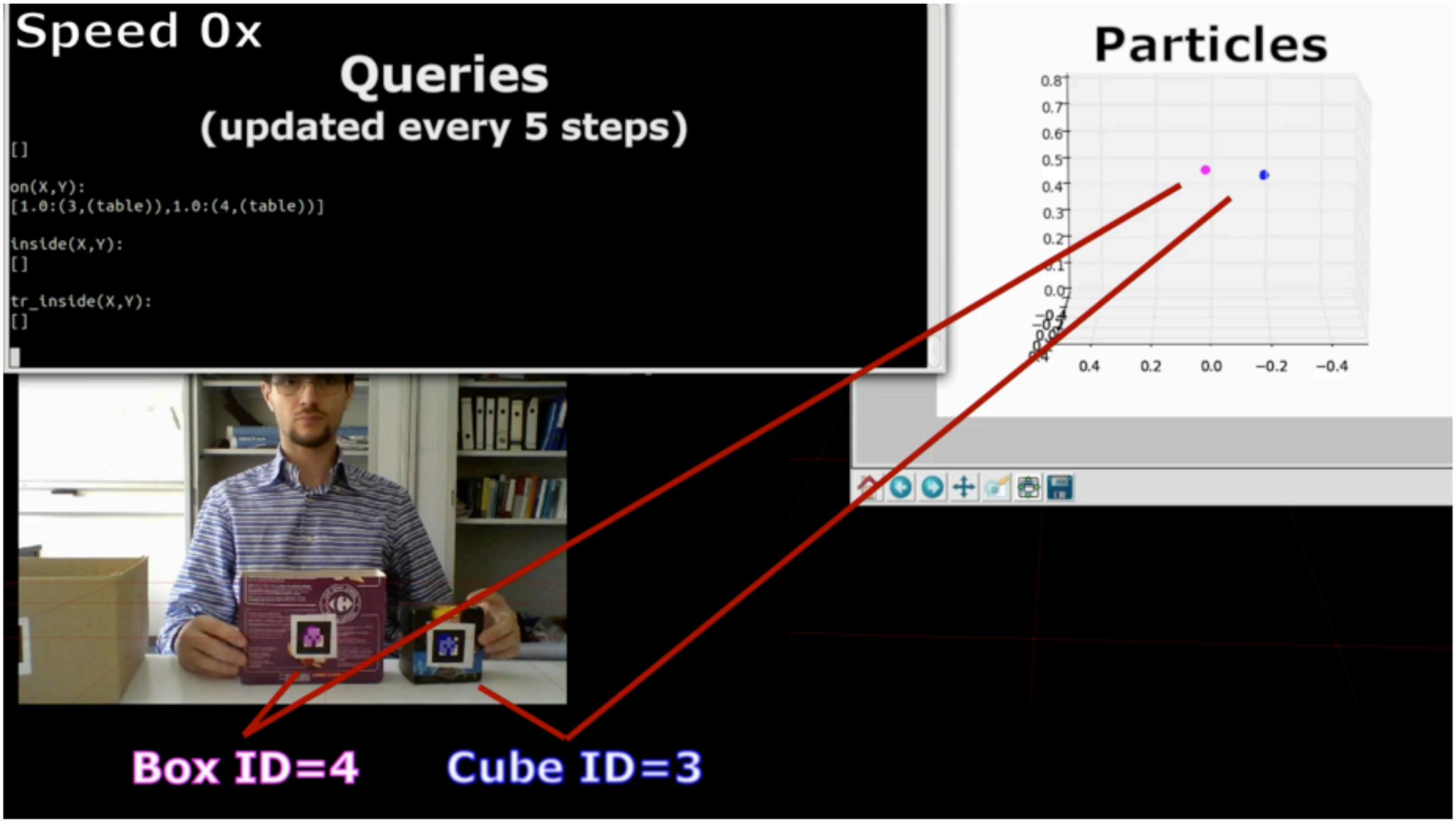
(a) cube on the box (b) cube inside the box (c) rotated box on a beige box (d) cube and box inside the beige box

Box scenario

- object tracking even when invisible
- estimate spatial relations

[Nitti et al, IROS 13]


Original Image
Figure 1
Position
1.0
0.8
0.6
0.4
0.2
Y
0.2
0.4
0.6
0.8
1.0
X
Figure 1
1.0
0.8
0.6
0.4
0.2
0.0
magnet
ferromagnetic
nonmagnetic


Speed 0x
Queries
(updated every 5 steps)
[]
on(X,Y):
[1.0:(3,(table)),1.0:(4,(table))]
inside(X,Y):
[]
tr_inside(X,Y):
[]
Particles
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.4
0.2
0.0
−0.2
−0.4
Box ID=4
Cube ID=3

IROS 13

cProbLog: constraints on possible worlds

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).

P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```


[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

```
weight(skis,6).
weight(boots,4).
weight(helmet,3).
weight(gloves,2).
```

distribution over **all** possible worlds

```
P::pack(Item) :-
  weight(Item,Weight),
  P is 1.0/Weight.

excess(Limit) :- ...

not excess(10).
pack(helmet) v pack(boots).
```

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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constraints as first-order logic formulas

sbhg e(10)	sb g e(10)	sbh e(10)	sb
s hg e(10)	s g	s h	s
bhg	b g	bh	b
hg	g	h	

[Fierens et al, PP 12; Shterionov et al]

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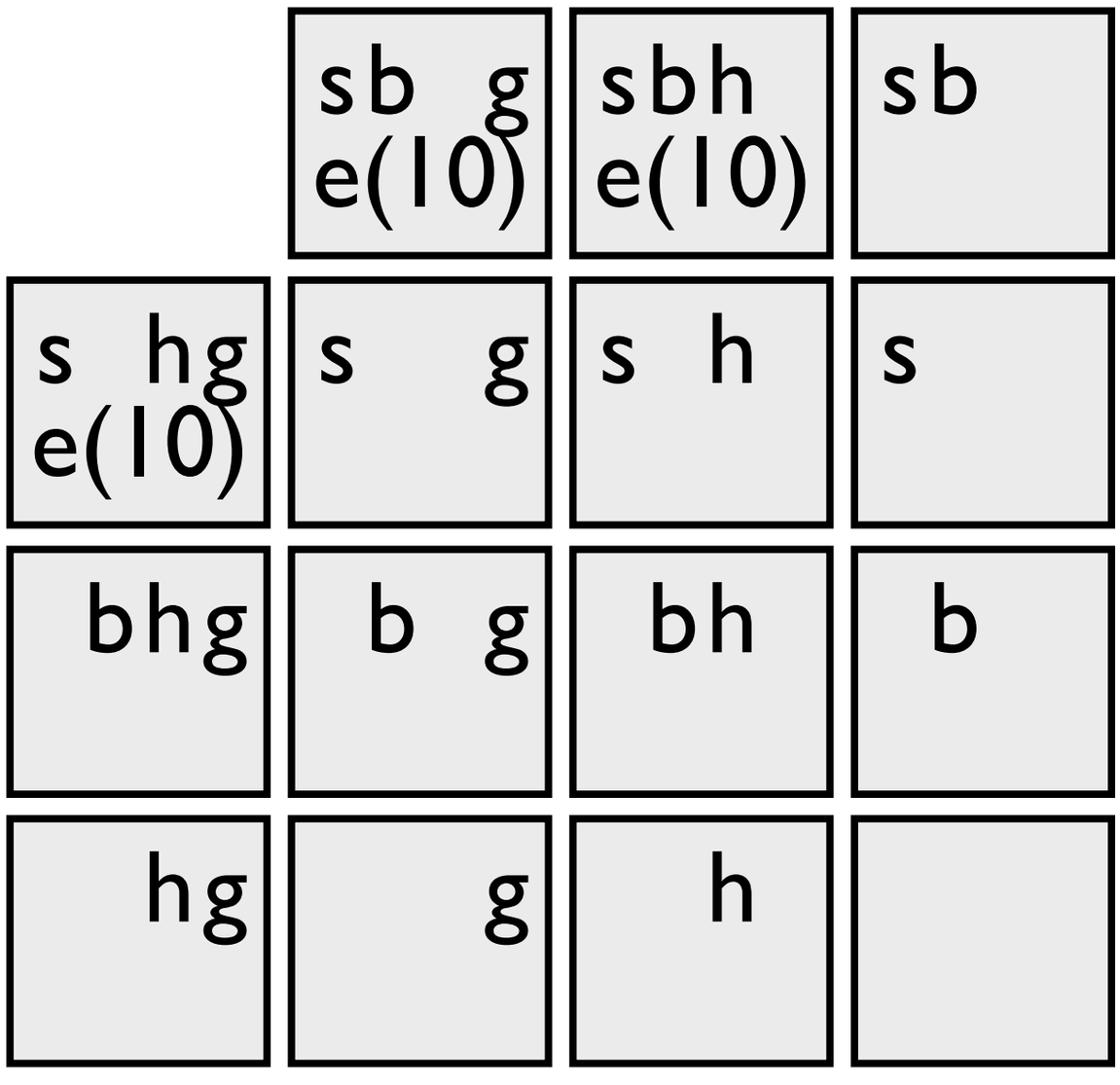
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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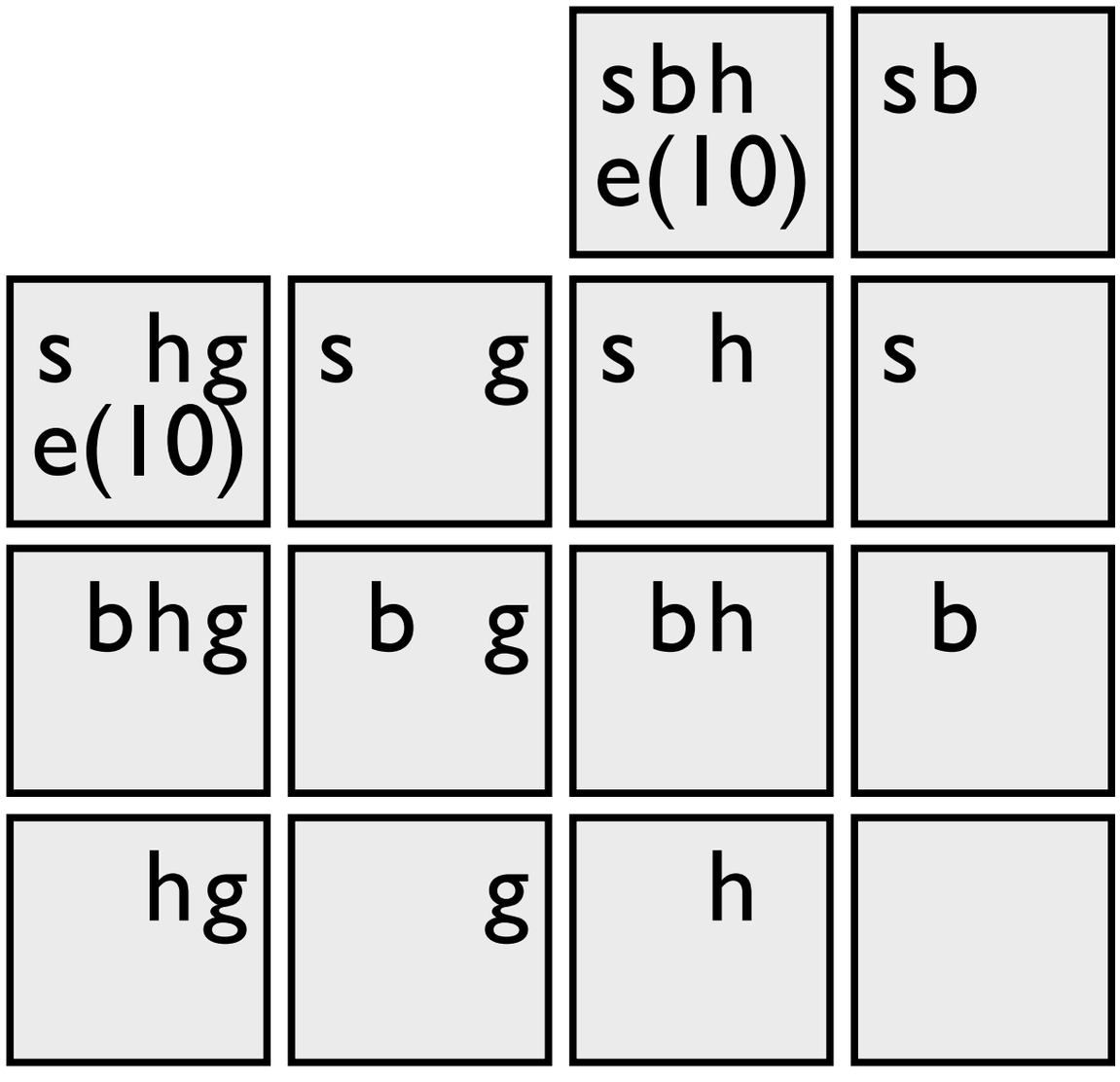
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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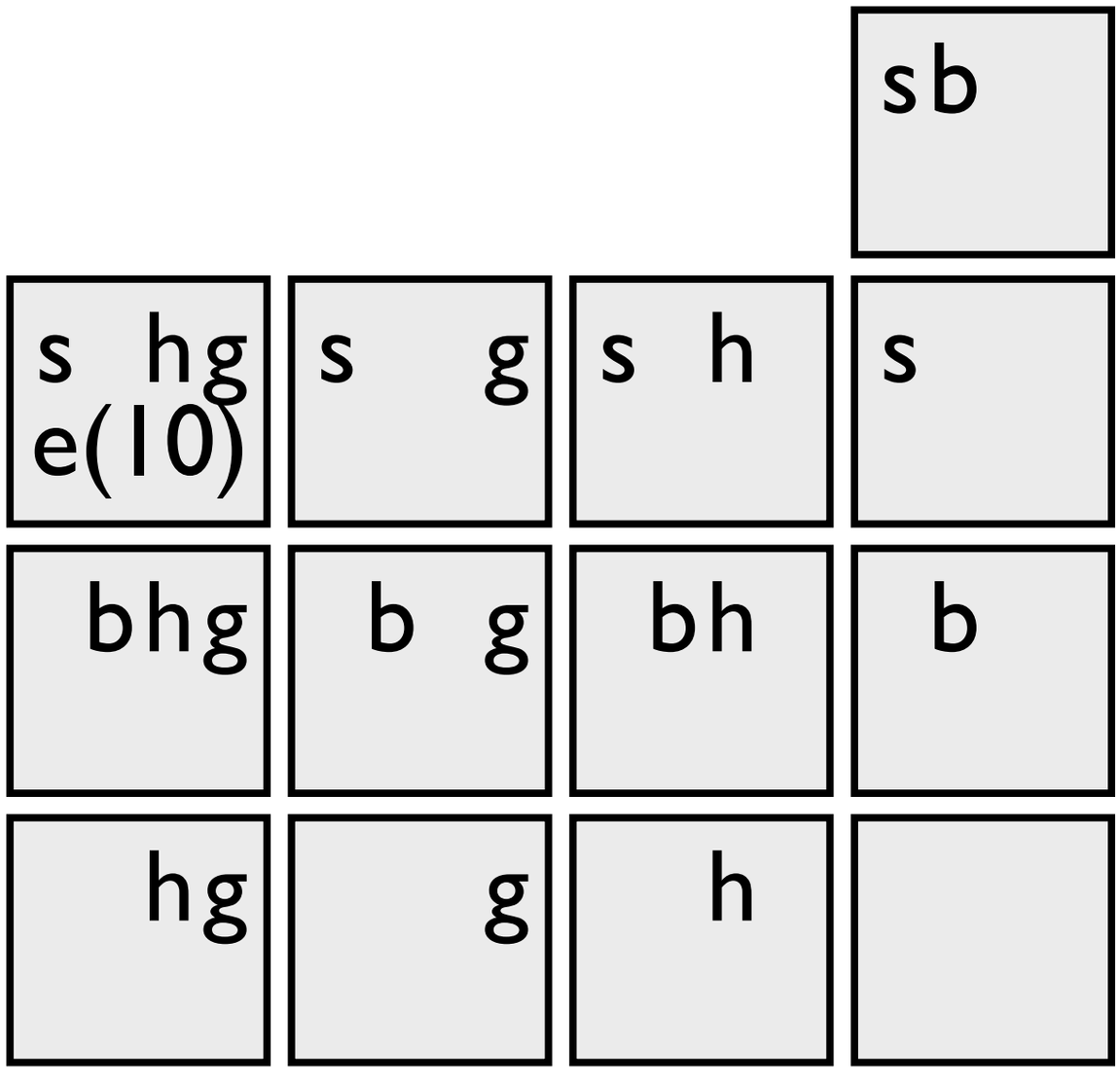
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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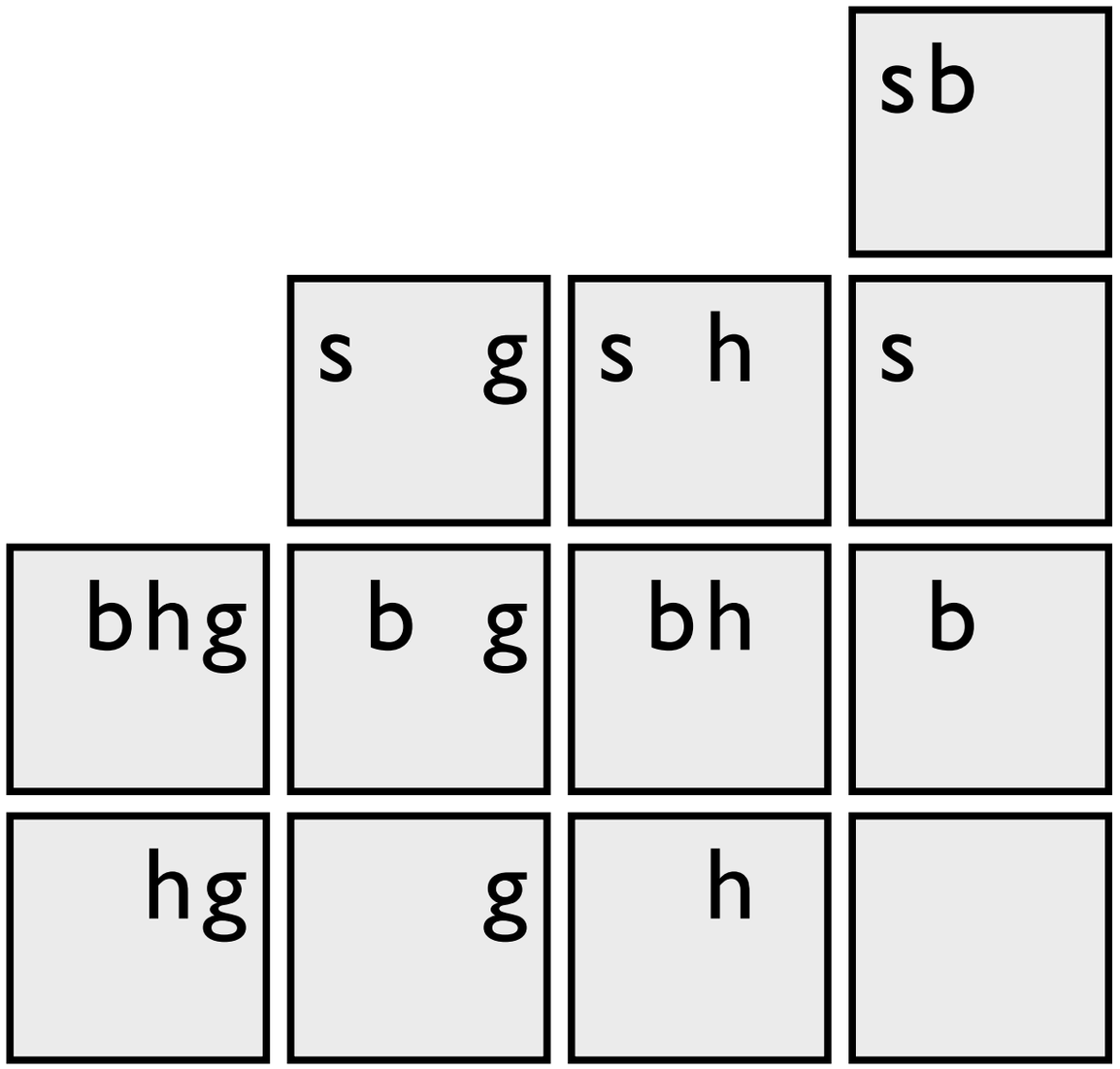
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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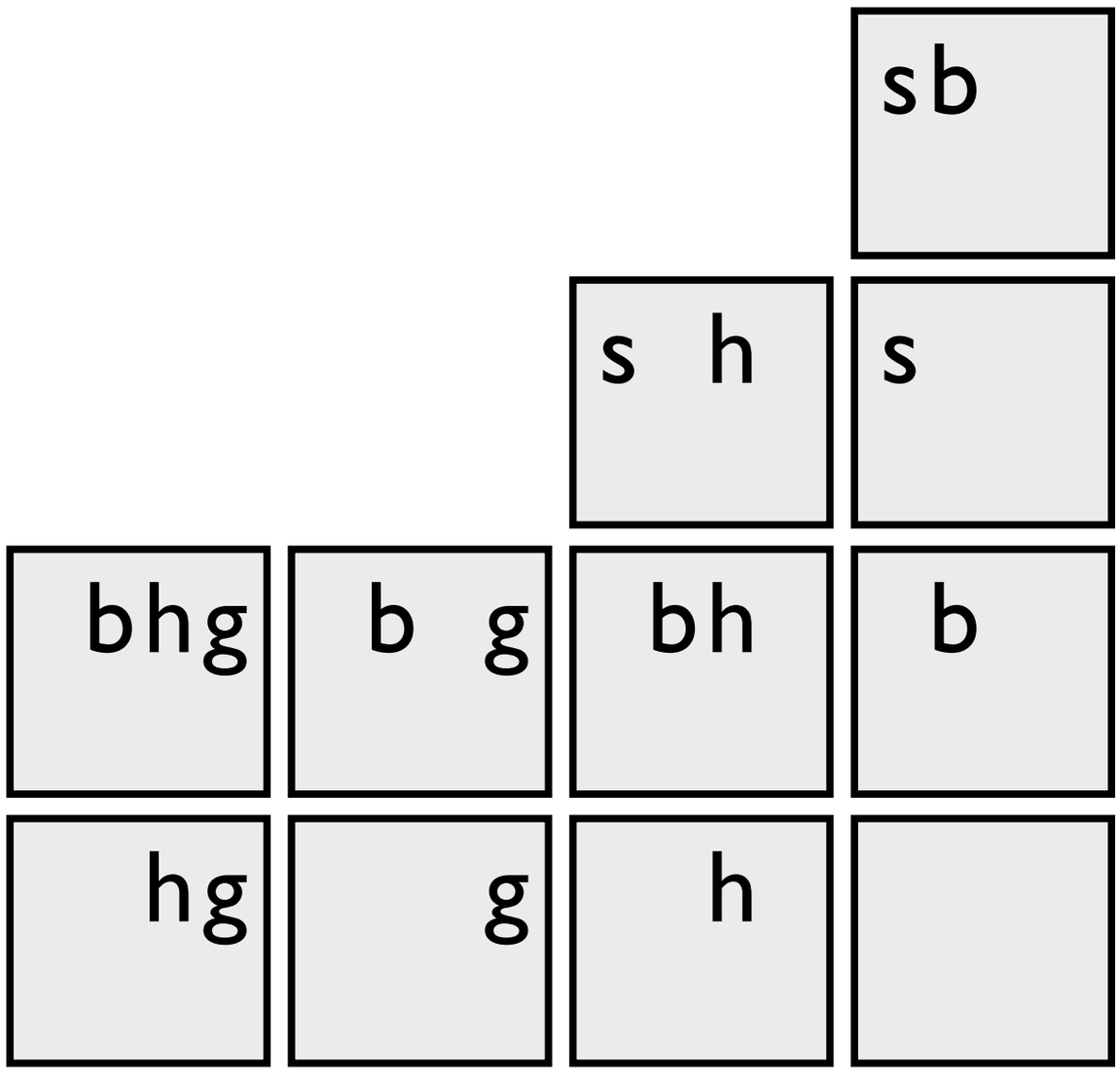
constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

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```
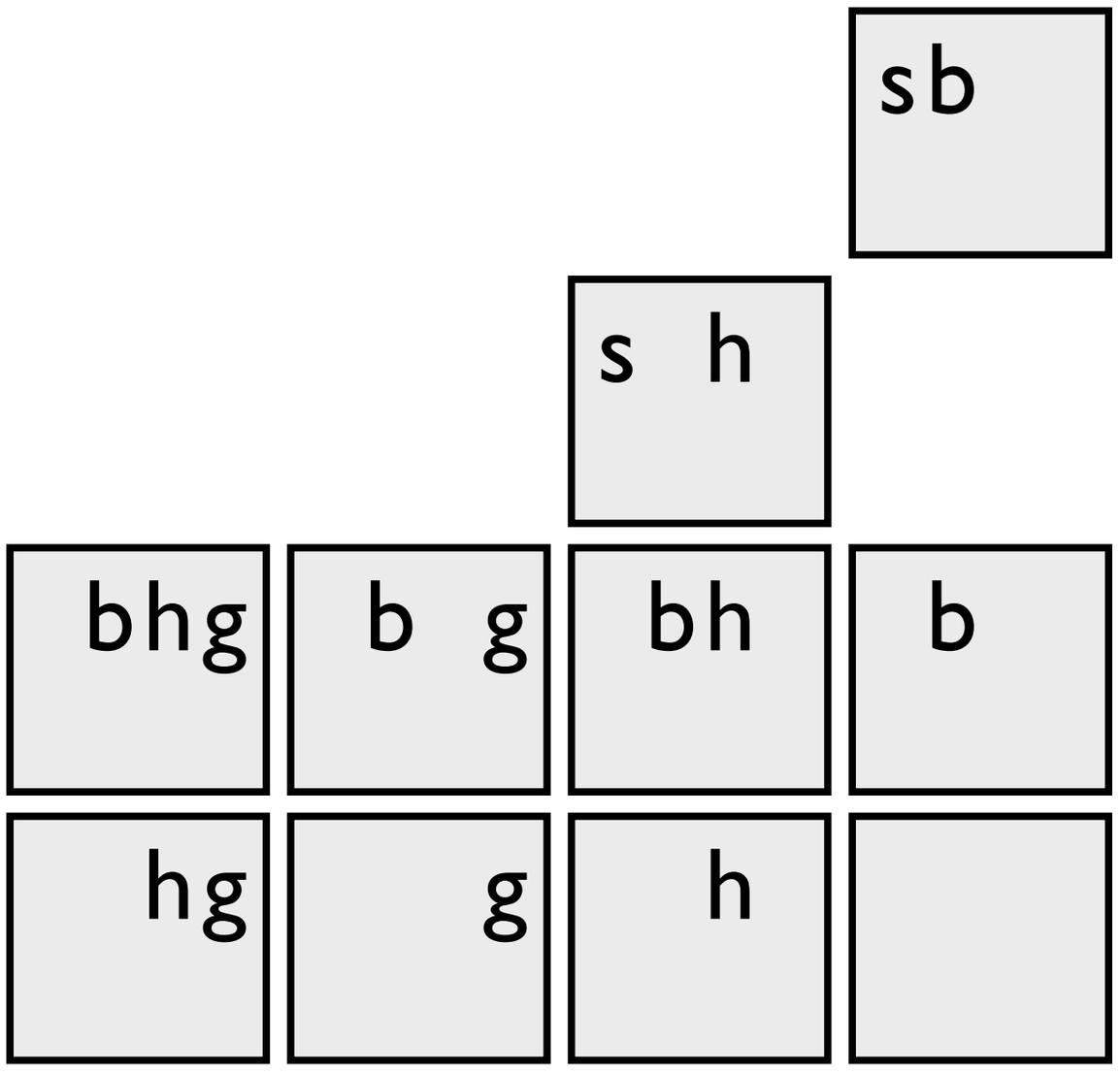

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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```

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```
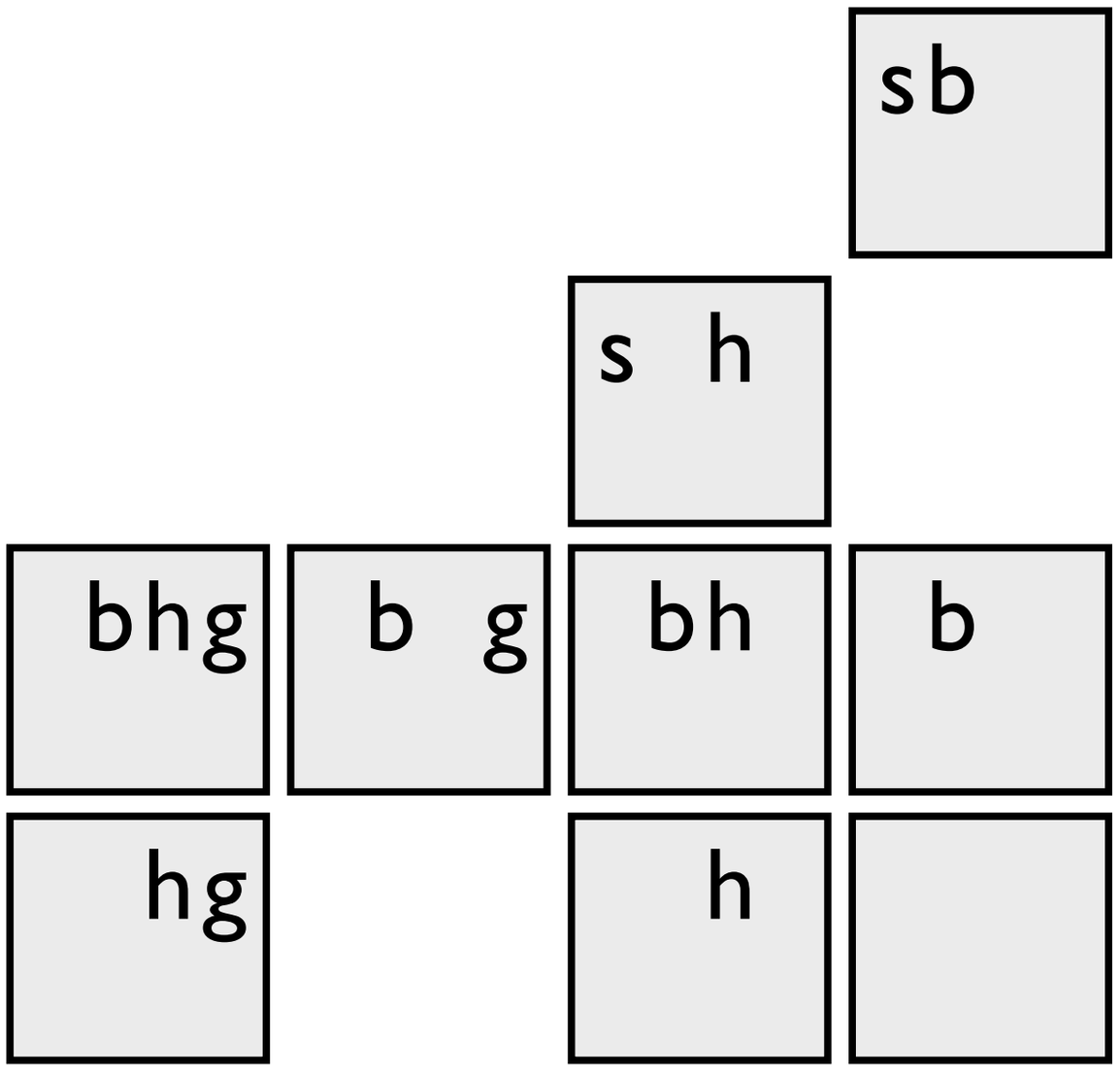

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

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excess(Limit) :- ...
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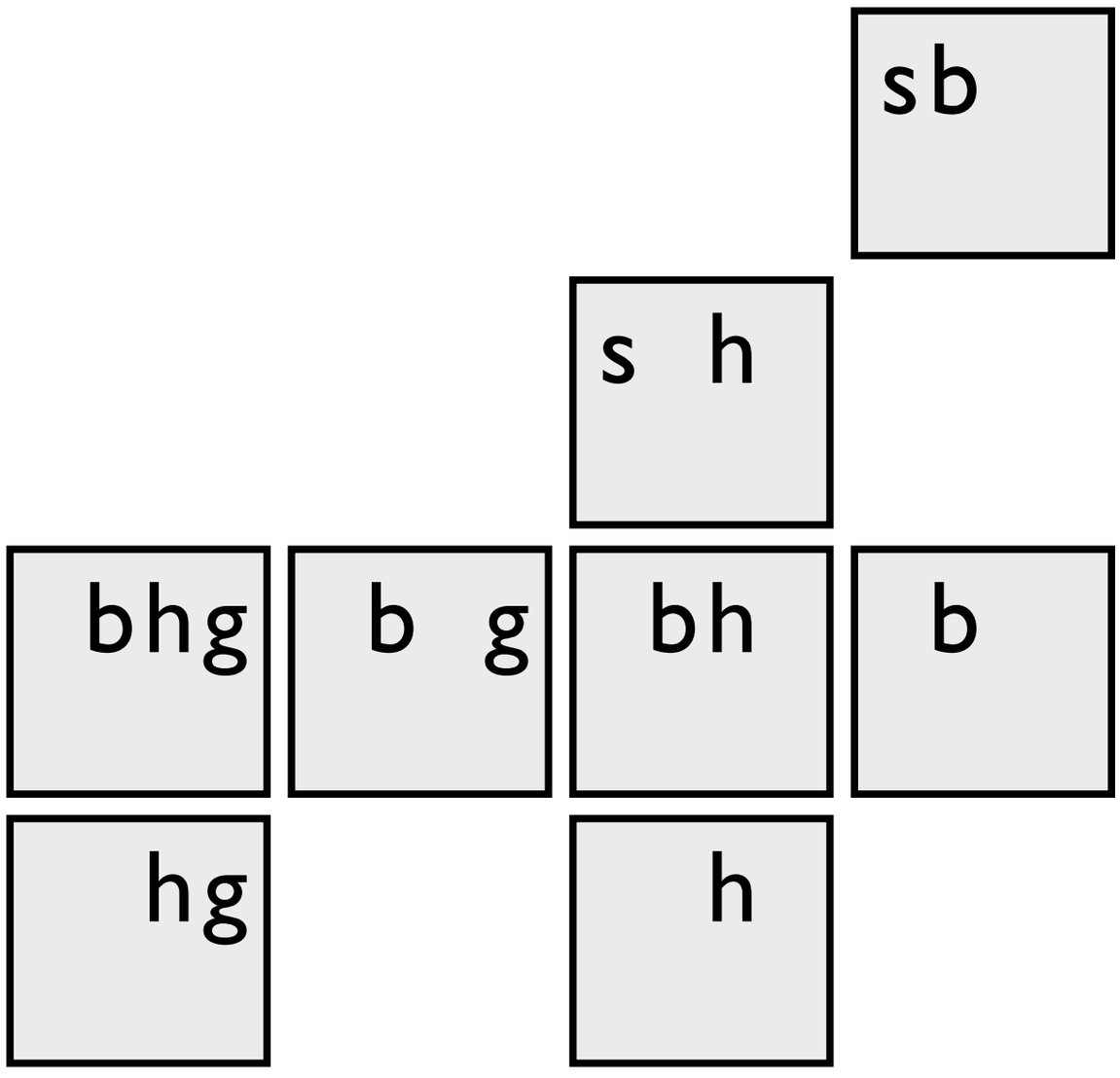
pack(helmet) v pack(boots).
```

constraints as first-order logic formulas



[Fierens et al, PP 12; Shterionov et al]

cProbLog: constraints on possible worlds

```
weight(skis,6).
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  weight(Item,Weight),
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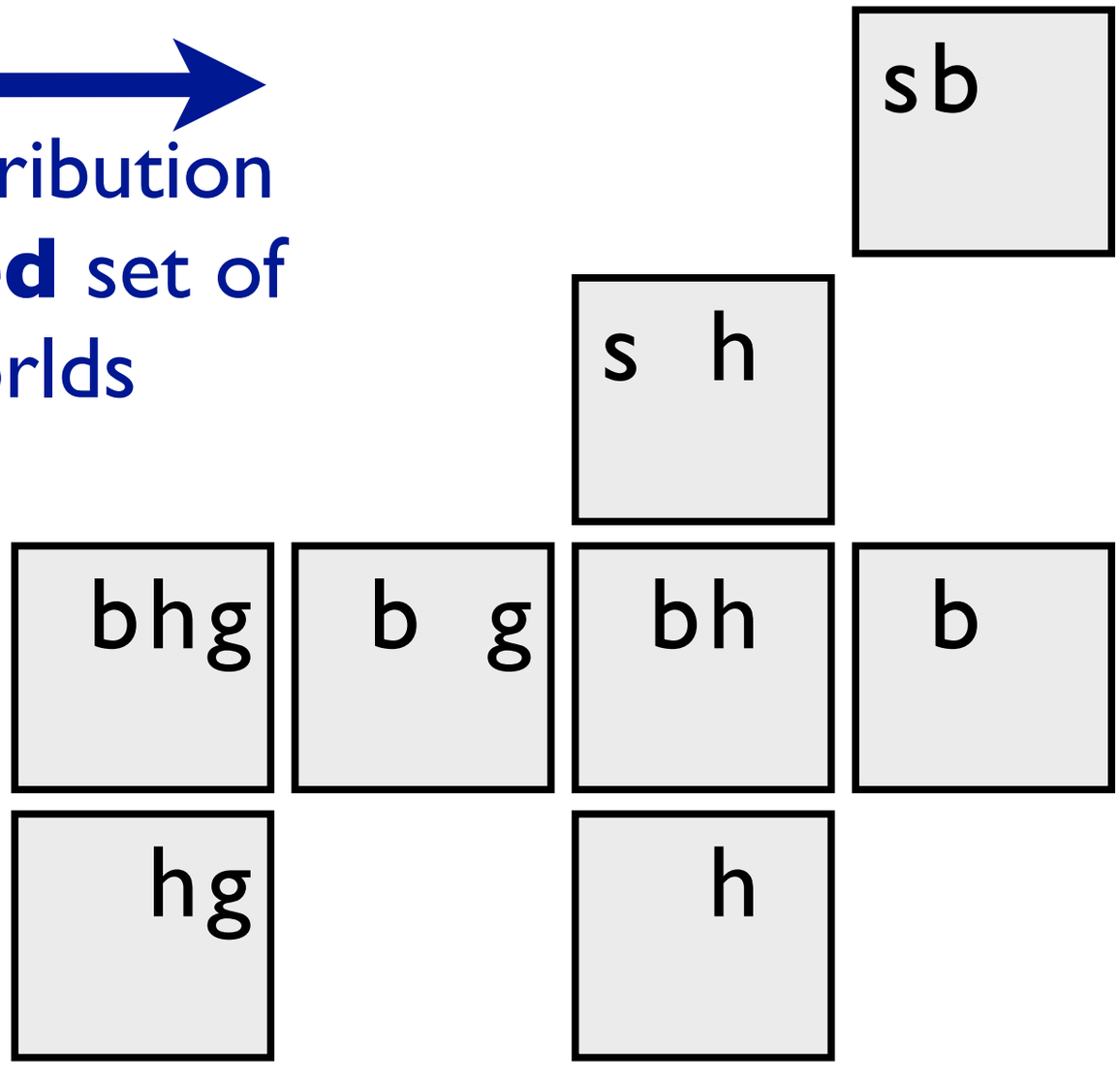
excess(Limit) :- ...
```

```
not excess(10).
pack(helmet) v pack(boots).
```

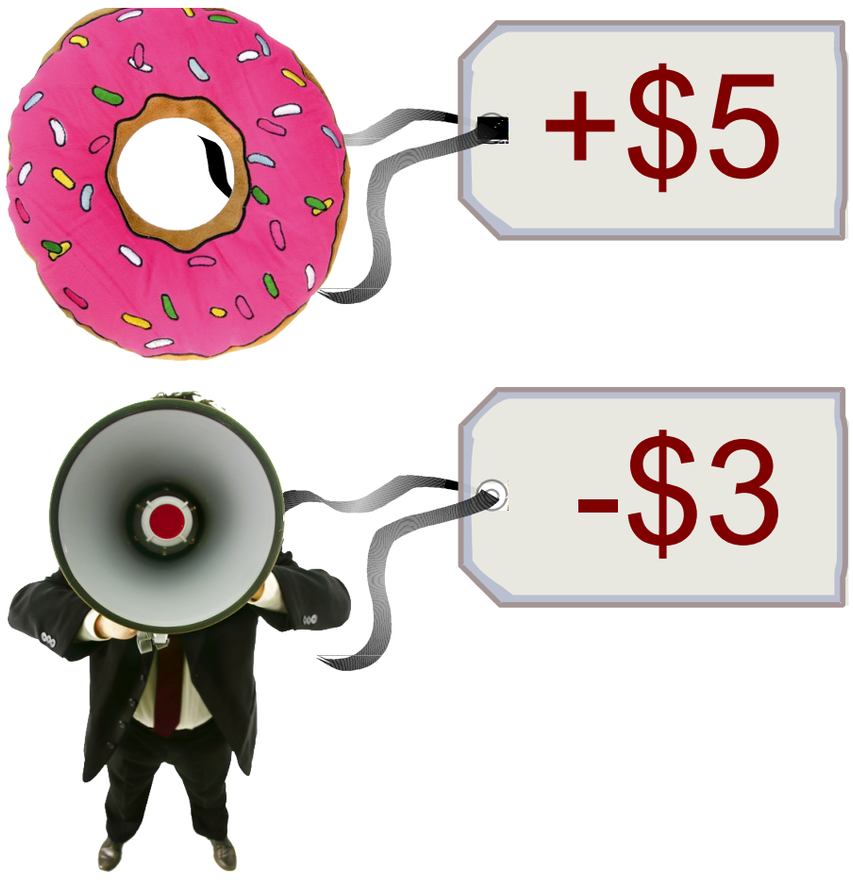
constraints as first-order logic formulas

normalized distribution over **restricted** set of possible worlds

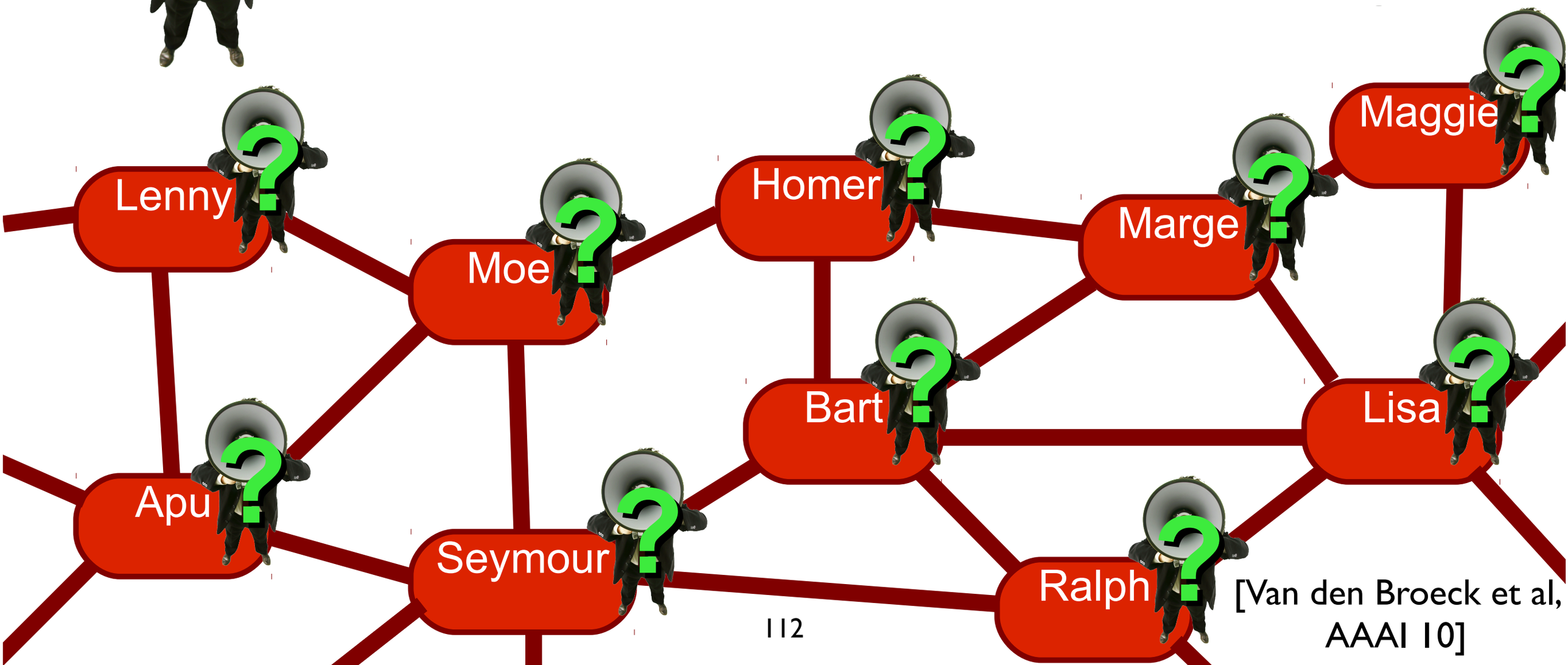


[Fierens et al, PP 12; Shterionov et al]

Viral Marketing

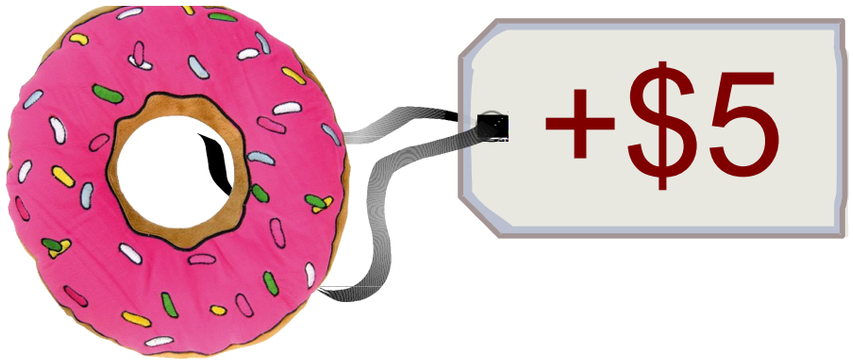


Which advertising strategy maximizes expected profit?

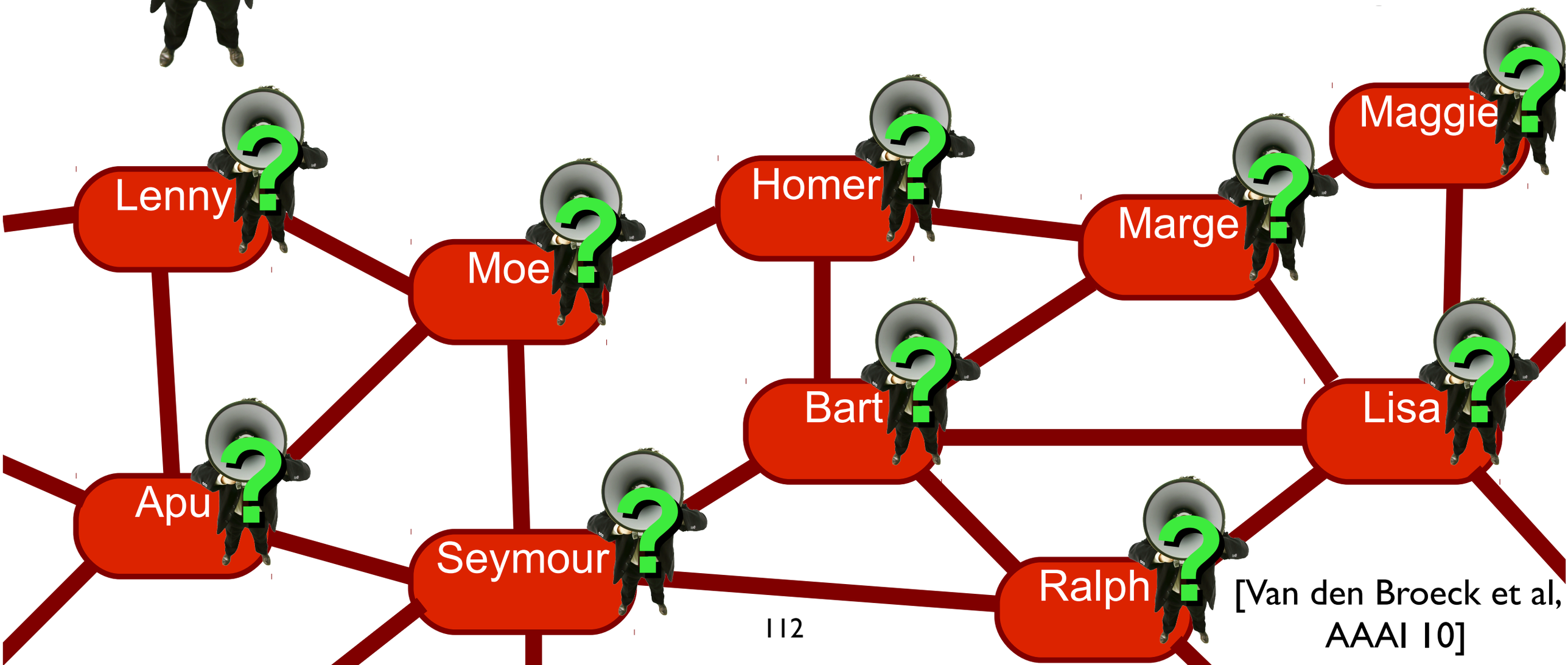


[Van den Broeck et al, AAAI 10]

Viral Marketing

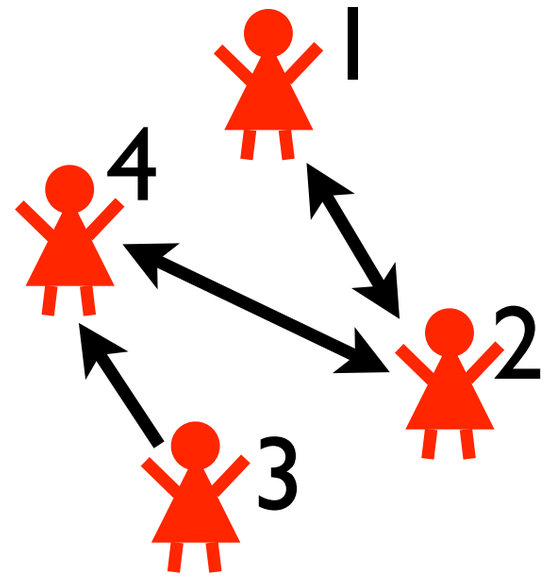


decide truth values of some atoms



[Van den Broeck et al, AAAI 10]

DTProbLog



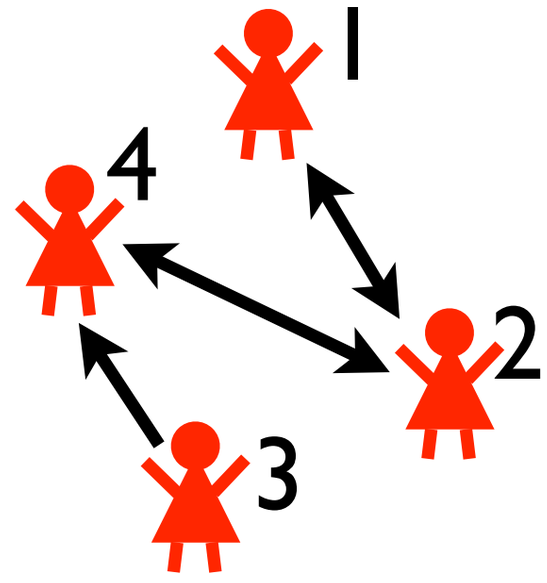

```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

DTProbLog

```
? :: marketed(P) :- person(P).
```

decision fact: true or false?




```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
friend(2,4).
friend(3,4).
friend(4,2).
```

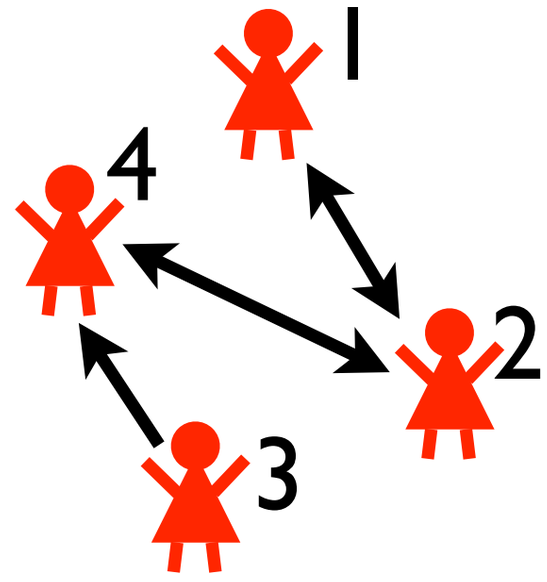
DTProbLog

```
? :: marketed(P) :- person(P).
```

```
0.3 :: buy_trust(X,Y) :- friend(X,Y).
0.2 :: buy_marketing(P) :- person(P).

buys(X) :- friend(X,Y), buys(Y), buy_trust(X,Y).
buys(X) :- marketed(X), buy_marketing(X).
```

probabilistic facts
+ logical rules




```
person(1).
person(2).
person(3).
person(4).
```

```
friend(1,2).
friend(2,1).
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friend(3,4).
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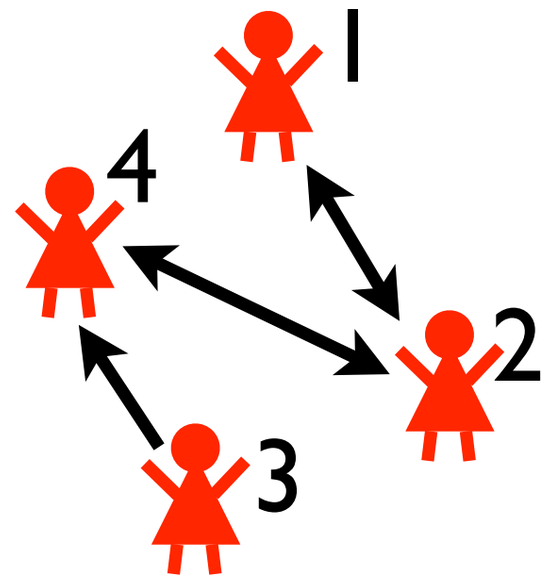
```

DTProbLog

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person(2).
person(3).
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friend(1,2).
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```
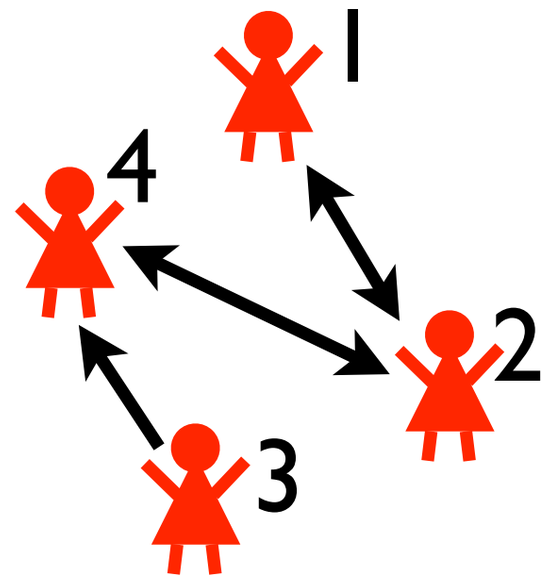

utility facts: cost/reward if true

DTProbLog

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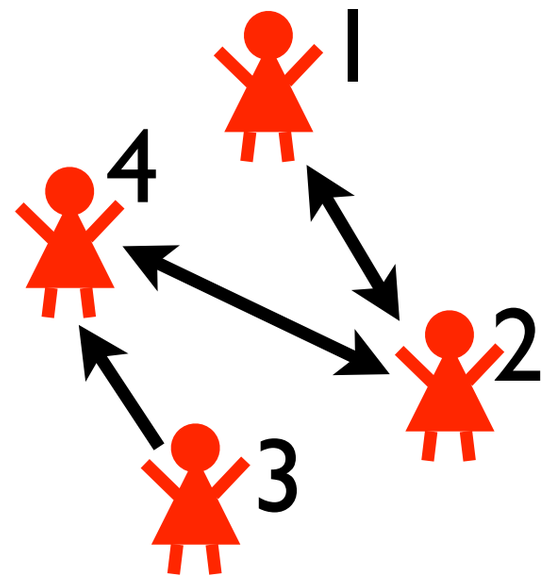


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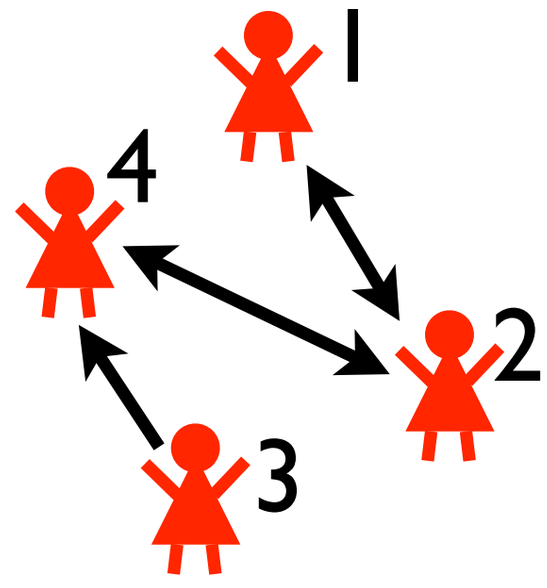


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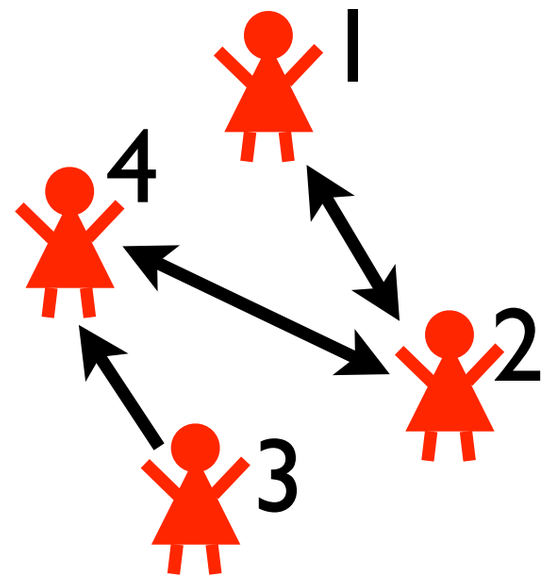
```
marketed(1)        marketed(3)
```

DTProbLog

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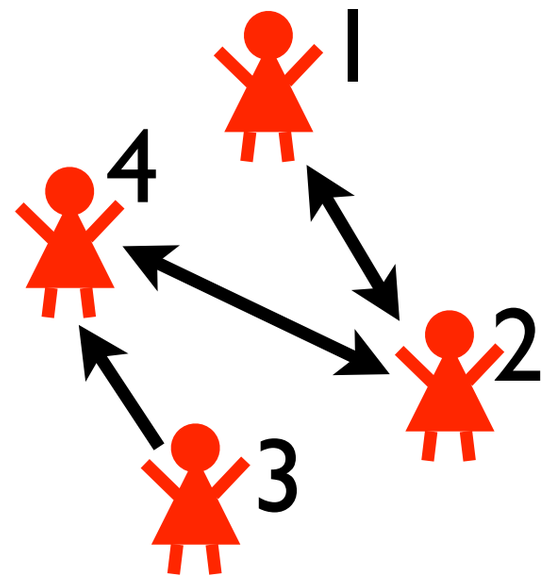
```
marketed(1)        marketed(3)

    bt(2,1)   bt(2,4)          bm(1)
```

DTProbLog

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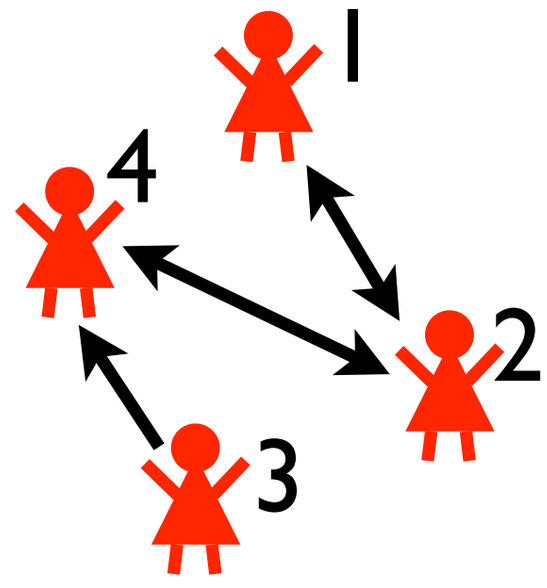
DTProbLog

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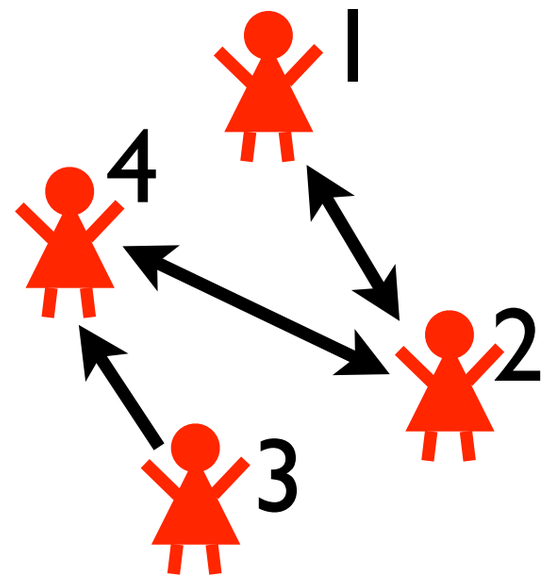
utility = −3 + −3 + 5 + 5 = 4

probability = 0.0032

DTProbLog

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probability = 0.0032

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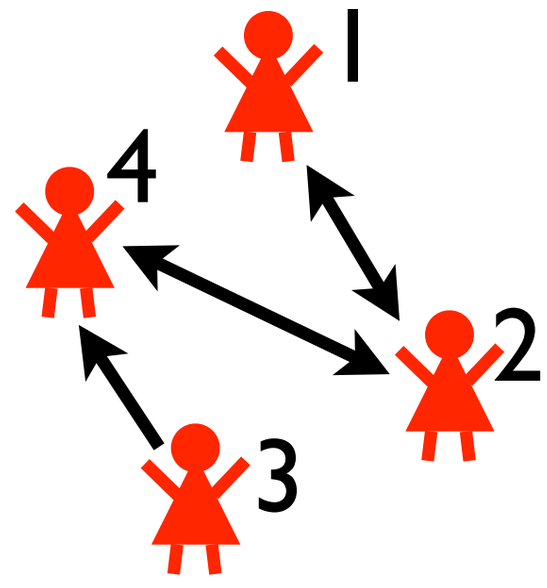
marketed(1)        marketed(3)

   bt(2,1)   bt(2,4)          bm(1)

buys(1)    buys(2)
```

world contributes 0.0032×4 to expected utility of strategy

DTProbLog

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```
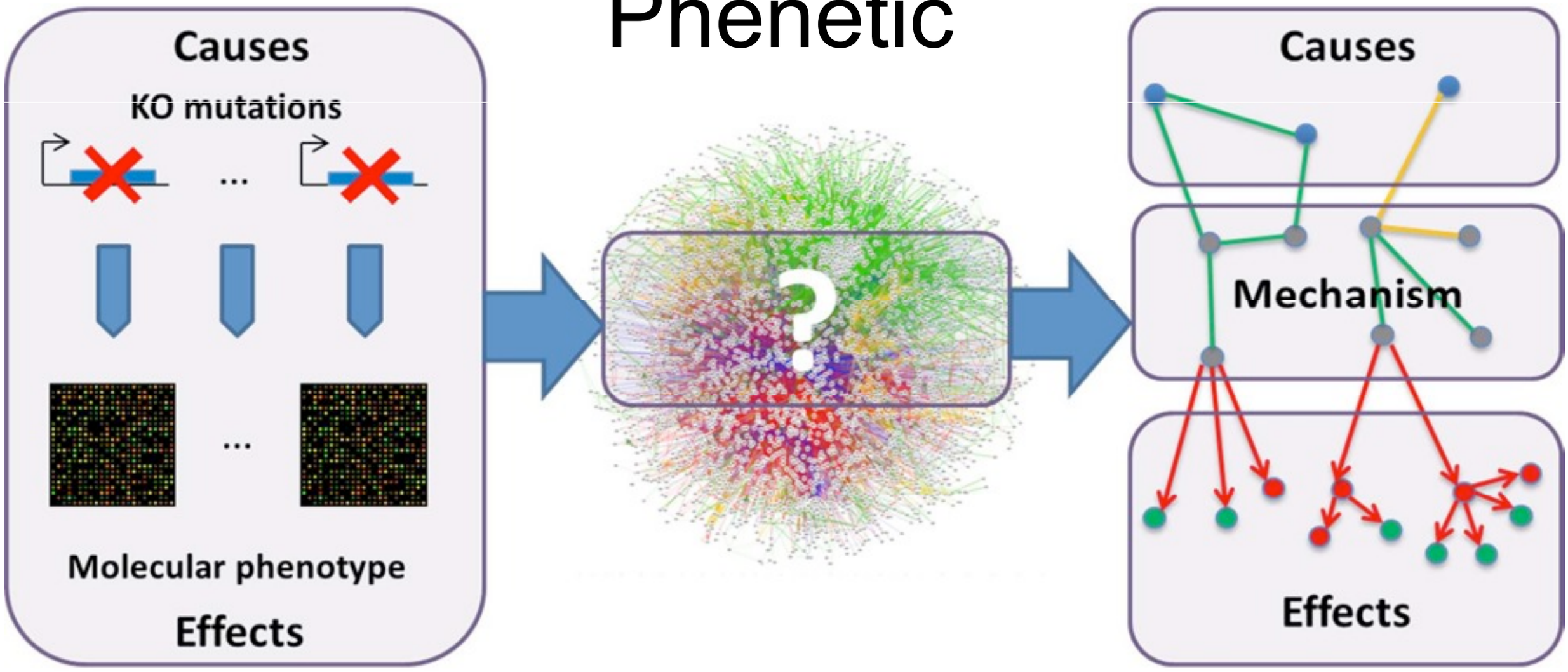

task: find strategy that maximizes expected utility
solution: using ProbLog technology

Phenetic



- Causes: Mutations
  - All related to similar phenotype
- Effects: Differentially expressed genes
- 27 000 cause effect pairs

- Interaction network:
  - 3063 nodes
    - Genes
    - Proteins
  - 16794 edges
    - Molecular interactions
    - Uncertain

- Goal: connect causes to effects through common subnetwork
  - = Find mechanism
- Techniques:
  - DTProbLog
  - Approximate inference

[De Maeyer et al., Molecular Biosystems 13]

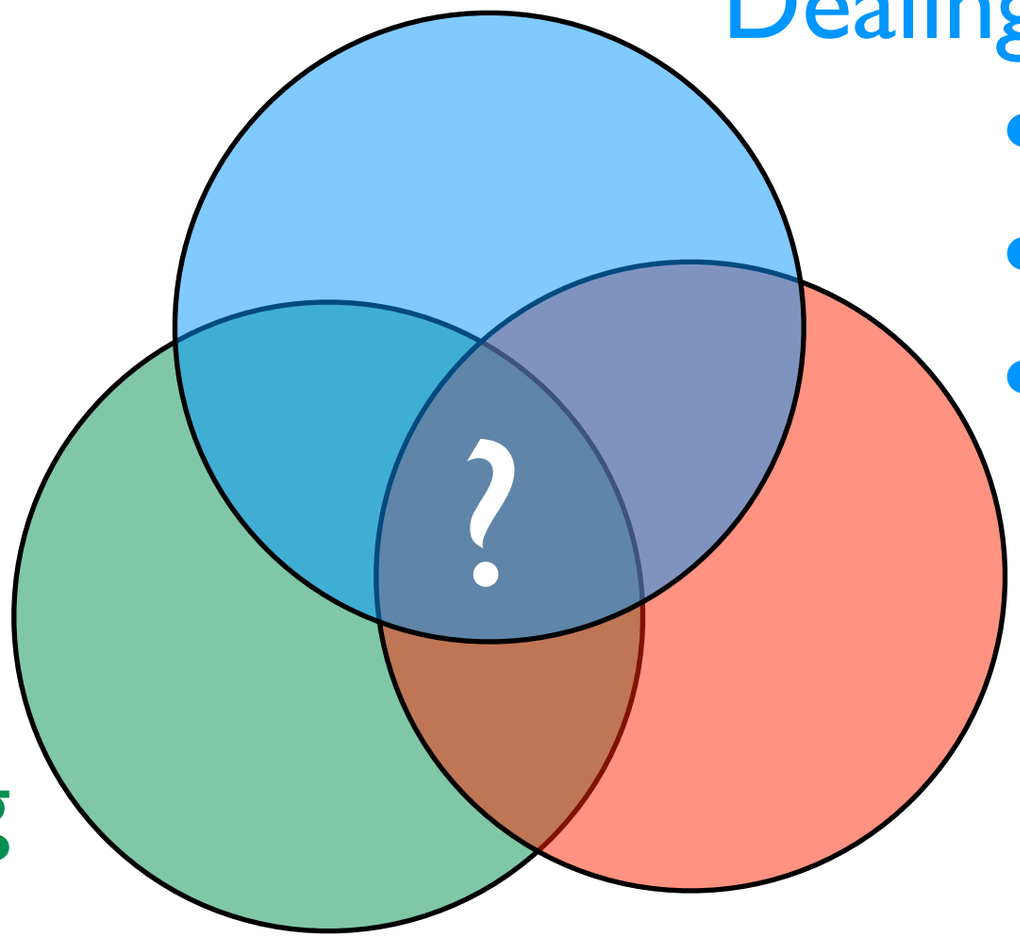
Roadmap

- Modeling (with detours to related work)
- Reasoning (and a bit of learning)
- Language extensions

A key question in AI:

Reasoning with relational data

- logic
- databases
- programming
- ...



Dealing with uncertainty

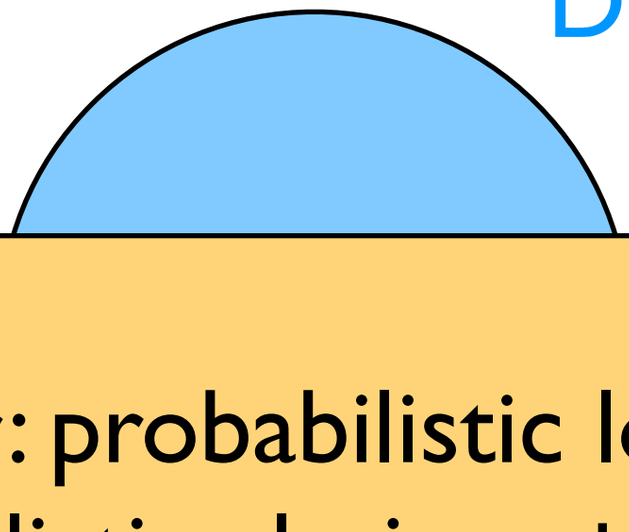
- probability theory
- graphical models
- ...

Learning

- parameters
- structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

A key question in AI:



Dealing with uncertainty
- probability theory

Reas
relat
- log
- da
- pr
- ...

dels

- Our answer: probabilistic logic programming
  = probabilistic choices + logic program
- Many languages, systems, applications, ...
- ... and much more to do!

structure

Statistical relational learning, probabilistic logic learning, probabilistic programming, ...

PLP Systems

- **PRISM** http://sato-www.cs.titech.ac.jp/prism/
- **ProbLog2** http://dtai.cs.kuleuven.be/problog/
- Yap Prolog http://www.dcc.fc.up.pt/~vsc/Yap/ includes
  - **ProbLog1**
  - **cplint** https://sites.google.com/a/unife.it/ml/cplint
  - **CLP(BN)**
  - **LP2**
- **PITA** in XSB Prolog http://xsb.sourceforge.net/
- **AILog2** http://artint.info/code/ailog/ailog2.html
- **SLPs** http://stoics.org.uk/~nicos/sware/pepl
- **contdist** http://www.cs.sunysb.edu/~cram/contdist/
- **DC** https://code.google.com/p/distributional-clauses
- **WFOMC** http://dtai.cs.kuleuven.be/ml/systems/wfomc

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